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Development of the super high angular resolution principle for X-ray imaging *

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Abstract Development of the Super High Angular Resolution Principle (SHARP) for coded-mask X-ray imaging is presented. We prove that SHARP can be considered as a generalized coded mask imaging method with a coding pattern comprised of diffraction-interference fringes in the mask pattern. The angular resolution of SHARP can be improved by detecting the fringes more precisely than the mask's element size, i.e. by using a detector with a pixel size smaller than the mask's element size. The proposed mission SHARP-X for solar X-ray observations is also briefly discussed.

Key words: instrumentation: high angular resolution — techniques: image processing — telescopes

1 INTRODUCTION

The Super High Angular Resolution Principle (SHARP) is proposed for high angular resolution X-ray imaging in astrophysics and solar physics. In our previous work (Zhang & Zhang 2009), the diffraction and interference effects have been studied for SHARP with a long distance between the mask and detector (the so-called baseline), in the range of tens of meters. The feasibility of coded mask imaging beyond the diffraction limit of a single pinhole has been demonstrated by simulations. The previously achieved angular resolution is 0.36 arcsec at 1.24 keV ($\lambda = 1$ nm), which is better than the 0.5 arcsec resolution of the *Chandra* X-ray Observatory (Weisskopf et al. 2000). A potential mission called SHARP-X based on SHARP has been proposed to observe solar activities in the 1–100 keV energy band with sub-arcsec angular resolution. The development of SHARP-X has been supported by the Chinese Academy of Sciences through an R&D technology demonstration project. The main scientific objectives of SHARP-X are understanding detailed aspects of coronal mass ejections (CMEs) and solar flares, such as fine structures and evolutions of the solar flares, nonlinear solar flare dynamics, solar particle acceleration mechanisms, etc. SHARP-X is foreseen to make significant progress on studies of solar high-energy explosive events and space weather forecast models. The basic mission concept of SHARP-X will employ a cyclic configuration (Zand 1996) with a mask element size of $a_{\rm m} = 50\,\mu{\rm m}$, a mask-detector distance of $D = 50\,{\rm m}$ and a detector

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pixel size of $a_{\rm m} = 50\,\mu{\rm m}$ (the so-called C50-1 configuration). The mast technology will be applied to make such a long baseline fesible. In this work, we prove that SHARP can be considered as a generalized coded mask imaging method with a coding pattern comprised of diffraction-interference fringes of the mask pattern. A potential way to further improve the angular resolution of SHARP is also discussed.

2 PROOF OF SHARP

The coded mask imaging method was reviewed by Zand (1996). In the limit of geometrical optics, i.e., where the diffraction effect is not considered, the detection of the X-ray flux can be described with Equation (1) (Fenimore et al. 1978),

$$P = O * M + N, \tag{1}$$

where O is the X-ray flux spatial distribution, M is the encoding pattern of the mask, N is noise, and $*$ means cross correlation. Prince et al. (1988) and Skinner (2004) pointed out that the pinhole diffraction effect could not be ignored when a coded mask telescope approached sub-arcsec angular resolution. The Fresnel-Huygens principle is a good approximation to describe the X-ray diffraction-interference pattern on the detector plane (Fig. 1),

$$E_R(x,y) = C\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E_i(x_0,y_0)A(x_0,y_0)\exp[(i\boldsymbol{k}\cdot\boldsymbol{r})/r]dx_0dy_0, \tag{2}$$

where A is the mask pattern, $\boldsymbol{r}$ is the vector from (x_0, y_0) on the mask to (x, y) on the detector, $\boldsymbol{k}$ is the wave vector, $C = -i/\lambda$ is a constant, E_i describes the incident flux, and $r = \sqrt{(x-x_0)^2 + (y-y_0)^2 + D^2}$ is the length of the vector $\boldsymbol{r}$. In case the mask size is far less than the baseline, r can be approximated as

$$r = D + \frac{x^2 + x_0^2 + y^2 + y_0^2 - 2xx_0 - 2yy_0}{2D}\,.$$

Then

$$E_R(x,y) = C_2\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E_iM{\rm e}^{-2i\pi(f_xx_0+f_yy_0)}dx_0dy_0, \tag{3}$$

where

$$\begin{aligned} C_2 &= \frac{-i}{\lambda D}{\rm e}^{ikD}{\rm e}^{\frac{ik}{2D}(x^2+y^2)}, \\ f_x &= x/\lambda D, \\ f_y &= y/\lambda D, \end{aligned}$$

and

$$M = A(x_0,y_0){\rm e}^{\frac{ik}{2D}(x_0^2+y_0^2)}.$$

For a point source $E{\rm p}_i$, E_i is expressed as a plane wave,

$$E_i = E{\rm p}_i = A{\rm p}_i{\rm e}^{\frac{2\pi i}{\lambda}[\tan(\theta_i)\cos(\varphi_i)x_0+\tan(\theta_i)\sin(\varphi_i)y_0]}, \tag{4}$$

where $A{\rm p}_i$ is the amplitude of the plane wave and (θ_i,φ_i) are the azimuth angles of the incident X-ray. Assuming

$$f_{xi} = \tan(\theta_i)\cos(\varphi_i)/\lambda$$

and

$$f_{yi} = \tan(\theta_i)\sin(\varphi_i)/\lambda,$$

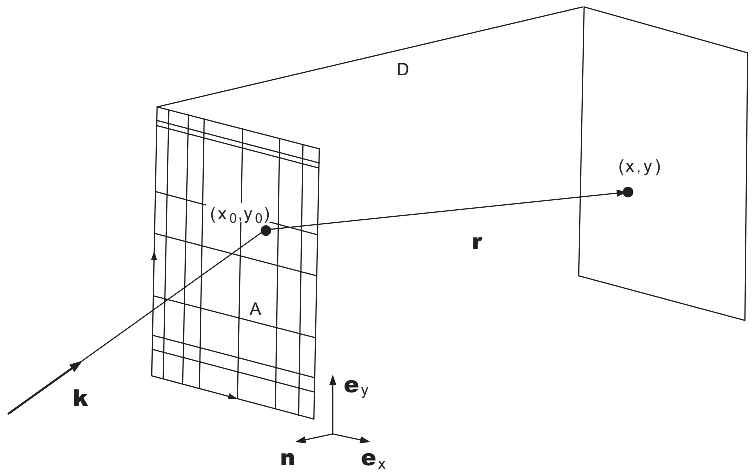

Fig. 1 Monochromatic X-ray with wave vector $\boldsymbol{k}$ illuminating a mask in the x_0–y_0 plane produces a diffraction pattern in the x–y plane (Lindsey 1978).

then the point source $E\mathrm{p}_i$ can be represented as

$$E\mathrm{p}_i = \mathrm{Ap}_i \mathrm{e}^{2\pi i(f_{xi}x_0 + f_{yi}y_0)}.$$

Substituting $E\mathrm{p}_i$ into Equation (3) gives

$$E_R(f_x, f_y) = C_2 A\mathrm{p}_i \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} M\mathrm{e}^{-2i\pi[(f_x - f_{xi})x_0 + (f_y - f_{yi})y_0]} dx_0 dy_0. \qquad (5)$$

Let

$$E_{R0} = C_2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} M\mathrm{e}^{2i\pi(f_x x_0 + f_y y_0)} dx_0 dy_0$$

and

$$G_R = E_{R0}E_{R0}^*,$$

then the detected pattern on the detector can be expressed as

$$P_R = \mathrm{Ap}_i^2 * G_R(f_{xi} - f_x, f_{yi} - f_y)$$

for a point source located at (f_{xi}, f_{yi}). The sky's flux distribution can be expressed as

$$O(f_{xi}, f_{yi}) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} O(-f_x, -f_y)\delta(f_x + f_{xi}, f_y + f_{yi}) df_x df_y,$$

so the pattern on the detector including noise is

$$P = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} O(-f_x, -f_y) G_R(-f_x - f_{xi}, -f_y - f_{yi}) df_x df_y + N = O * G_R + N. \qquad (6)$$

Therefore SHARP can be considered as a generalized coded mask imaging method with a coding pattern G_R, which is comprised of the diffraction-interference fringes of the mask pattern A. In principle, a reconstruction method applied to the traditional coded mask imaging can be applied for SHARP, such as the Diffraction-Interference Cross-Correlation (DICC) method proposed by Zhang & Zhang (2009).

3 IMPROVEMENT OF ANGULAR RESOLUTION OF SHARP

Zhang & Zhang (2009) showed that the angular resolution of SHARP with DICC is almost the same as that in the limit of geometrical optics. However, it is possible to benefit from the diffraction-interference information of the mask to improve the imaging quality. The diffraction-interference fringes can be finer than the pinhole size due to interference between photons through different pinholes. By detecting the fringes with a detector of better spatial resolution than the mask element size, we obtain an angular resolution better than that in the limit of geometrical optics. A SHARP-X like system with a pinhole size of $a_{\rm m} = 50\,\mu{\rm m}$, baseline of $D = 50\,{\rm m}$ and detector pixel size of $a_d = a_{\rm m}/4 = 12.5\,\mu{\rm m}$ (the so-called C50-4 configuration) is simulated. The reconstructed image of two monochromatic point sources at 1.24 keV with an angular separation of $\Delta_i = 0.21\,{\rm arcsec}$ (with about 20 000 photons collected) is shown in Figure 2(a), which is reconstructed with a cross-correlation method in the limit of geometrical optics. Figure 2(b) shows the reconstructed image of the same sources as in Figure 2(a) by the DICC method with a significant diffraction-interference effect. A slight improvement from DICC to cross-correlation in the limit of geometrical optics is observed. The angular resolutions at 1.24 keV and in the limit of geometrical optics are calculated to be 0.25 and 0.27 arcsec, respectively, which is improved from that of 0.36 and 0.32 arcsec with the C50-1 configuration by Zhang & Zhang (2009). The improvement of angular resolution from the C50-1 configuration to the C50-4 configuration in the limit of geometrical optics is due to the improvement of the detector's spatial resolution, and the further improvement from Figure 2(a) to Figure 2(b) is due to the interference effect.

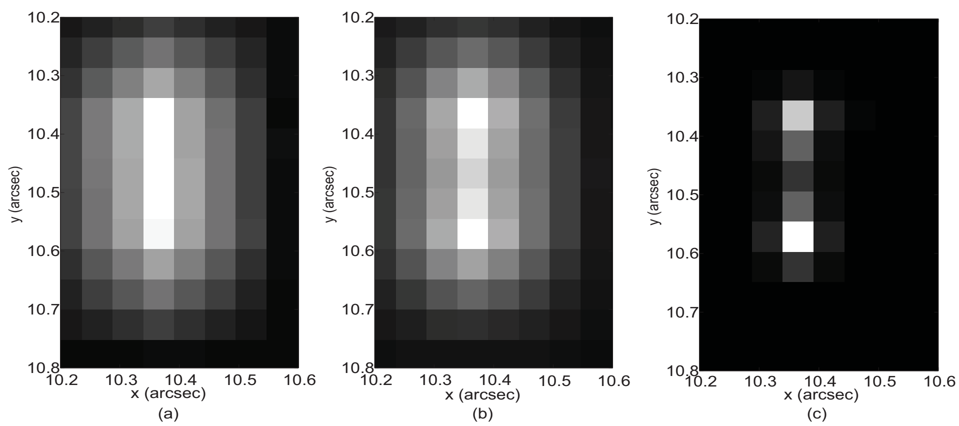

Fig. 2 Reconstructed images of two monochromatic point sources at 1.24 keV with an angular separation of about 0.21 arcsec by (a) cross-correlation in the limit of geometrical optics, (b) DICC, and (c) LR with C50-4 configuration, i.e., the detector pixel size of $a_{\rm d} = a_{\rm m}/4 = 12.5\,\mu{\rm m}$ and the baseline of 50 m. A slight improvement in angular resolution from (a) to (b) and a significant improvement from (b) to (c) are observed.

By applying a detector with a smaller pixel size than the mask element, we significantly decrease the baseline of the SHARP-X mission to achieve approximately the same angular resolution as that of the previous configuration C50-1. A new configuration is proposed with a baseline of 30 m and the mask element size of 50 $\mu{\rm m}$. The detector's energy resolution limits the detector's spatial resolution, which is the system's main constraint (Zhang & Zhang 2009). For a silicon detector, the idealized energy resolution (FWHM) at 1.24 keV, i.e., silicon Fano noise, is about 53 eV (Lechner et al. 1996), which limits the detector's pixel size to about $a_{\rm d} = a_{\rm m}/2 = 25\ \mu{\rm m}$ (the so-called C30-2 configuration). The calculated angular resolution of 0.48 arcsec both at 1.24 keV and in the limit of geometrical optics can be achieved with cross-correlation. Although the angular resolution degrades from 0.36 arcsec at 1 keV and 0.32 arcsec in the limit of geometrical optics to 0.48 arcsec, the difficulty of engineering to fabricate and operate the mast decreases significantly.

4 DISCUSSION

A further improvement of the angular resolution can be achieved with some nonlinear reconstruction methods beyond the limits of the imaging system, which are the so-called super-resolution methods. These methods can partially compensate for the high frequency information loss of the coding process, based on analytical continuation theory. The Lucy-Richardson method (LR method) described by Richardson (1972), which is in the image domain based on Bayes' Theorem, is commonly applied as a super-resolution method. The reconstructed image by using the LR method with a C50-4 configuration of the same sources as in Figure 2(b) is shown in Figure 2(c). A significant improvement in angular resolution is observed. In the C30-2 configuration with a baseline of 30 m, the angular resolution can be further improved from 0.48 arcsec (refer to Sec. 3) to 0.34 arcsec by the LR method, which is the same as that in the C50-1 configuration with a baseline of 50 m. Although the LR method shows a great improvement in angular resolution, the LR algorithm has some serious shortcomings. The known problems with the LR method are noise amplification, a spatially variable convergence rate and ringing. Some developed methods based on the LR algorithm, such as the Wiener-Lucy algorithm (Tsumuraya 1996) and the Maximum Likelihood Frequency Domain Correction Super-resolution Algorithm (Zhen et al. 2009), have been proposed to overcome the disadvantages of the LR algorithm.

An experiment in the optical band with a sodium lamp at 589 nm to demonstrate SHARP has been set up in Tsinghua Univ., which is called SHARP-O. The basic parameters of SHARP-O are the mask pinhole size of $169\ \mu{\rm m}$, the mask-detector distance of 1 m and the detector pixel size of 9 μm. SHARP-O is equivalent to SHARP-X with the C50-1 configuration at 1.24 keV. The first reconstructed image of two point sources with an angular separation of 26 arcsec is shown in Figure 3. The two sources can be identified and the angular resolution is 26 arcsec with the Wiener-Lucy re-

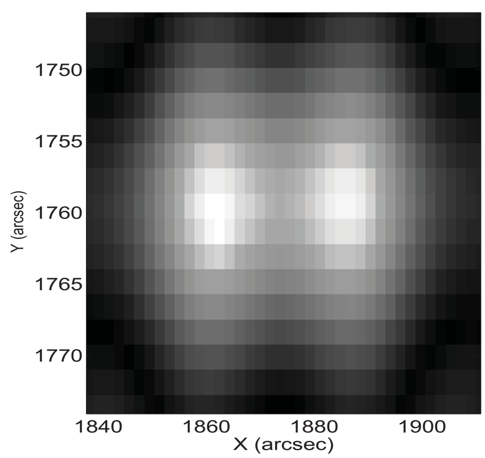

Fig. 3 Reconstructed images of two point sources with an angular separation of about 24 arcsec in SHARP-O. Two sources can be clearly identified.

construction algorithm, which is much smaller than the pinhole diffraction limit of 870 arcsec and also beyond the PSF width of $\Delta_i = 37$ arcsec in the limit of geometrical optics. Therefore, SHARP has been verified experimentally with SHARP-O. More measurements will be done in the near future.

5 CONCLUSIONS

In this work, we prove that SHARP can be considered as a generalized coded mask imaging method with a coding pattern comprised of diffraction-interference fringes of the mask pattern. The angular resolution of SHARP is further improved beyond the limit of geometrical optics due to detection of a more precise diffraction-interference fringe with a positional resolution finer than the mask element size. The nonlinear Lucy-Richardson method applied to SHARP is also briefly discussed. We can dramatically decrease the SHARP-X mast length from 50 m to 30 m in the C30-2 configuration but maintain the same angular resolution as that in the C50-1 configuration with mast length of 50 m by improving the detector's spatial resolution to 25 μm and applying the Lucy-Richardson method. More detailed studies on SHARP as well as SHARP-X configurations are still in development.

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