# SWIFT J1749.4-2807: A neutron or quark star? * 

Jun-Wei Yu and Ren-Xin Xu<br>School of Physics and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, China; J.W.Yu@pku.edu.cn; r.x.xu@pku.edu.cn<br>Received 2010 June 10; accepted 2010 July 19


#### Abstract

We investigate a unique accreting millisecond pulsar with X-ray eclipses, SWIFT J1749.4-2807 (hereafter J1749), and try to set limits on the binary system by various methods including use of the Roche lobe, the mass-radius relations of both main sequence (MS) and white dwarf (WD) companion stars, as well as the measured mass function of the pulsar. The calculations are based on the assumption that the radius of the companion star has reached its Roche radius (or is at $90 \%$ ), but the pulsar's mass has not been assumed to be a certain value. Our results are as follows. The companion star should be an MS one. For the case that the radius equals its Roche one, we have a companion star with mass $M \simeq 0.51 M_{\odot}$ and radius $R_{\mathrm{c}} \simeq 0.52 R_{\odot}$, and the inclination angle is $i \simeq 76.5^{\circ}$; for the case that the radius reaches $90 \%$ of its Roche one, we have $M \simeq 0.43 M_{\odot}, R_{\mathrm{c}} \simeq 0.44 R_{\odot}$ and $i \simeq 75.7^{\circ}$. We also obtain the mass of J1749, $M_{\mathrm{p}} \simeq 1 M_{\odot}$, and conclude that the pulsar could be a quark star if the ratio of the critical frequency of rotation-mode instability to the Keplerian one is higher than $\sim 0.3$. The relatively low pulsar mass (about $\sim M_{\odot}$ ) may also challenge the conventional recycling scenario for the origin and evolution of millisecond pulsars. The results presented in this paper are expected to be tested by future observations.


Key words: X-rays: binaries — binaries: eclipsing — pulsars: general — pulsars: individual: SWIFT J1749.4-2807

## 1 INTRODUCTION

One of the daunting challenges nowadays is to understand the fundamental strong interaction between quarks in the low-energy limit, i.e., the non-perturbative QCD (quantum chromo-dynamics). This unsolved problem results in an uncertainty regarding the nature of pulsar-like compact stars which are either normal neutron stars or quark stars (e.g., Lattimer \& Prakash 2004; Xu 2009). Nevertheless, compact stars may provide astrophysical laboratories to understand the nonperturbative QCD and, in turn, the pulsar mass and radius distribution would have important implications for the nature of pulsars and for the states of matter at supra-nuclear density. Certainly, it is a very difficult task to precisely determine the masses of pulsars even in binary systems because of unknown inclination angles, except for general relativistic binaries in which many observed post-Keplerian parameters can be applied.

An amazing system, SWIFT J1749.0-2807, provides us with a perfect opportunity for measuring the mass of a pulsar. J1749 is in a binary system, and the pulsar is in the phase of accreting matter and

[^0]Table 1 Timing Parameters of J1749 to be Used in Our Calculations (taken from table 1 of Altamirano et al. 2010)

| Parameter | Value |
| :--- | :---: |
| Orbital Period, $P_{\text {orb }}(\mathrm{d})$ | $0.3673696(2)$ |
| Projected semi major axis, $a_{\mathrm{p}} \sin i(\mathrm{lt-s})$ | $1.89953(2)$ |
| Eccentricity, $e(95 \%$ c.l.) | $<2 \times 10^{-5}$ |
| Spin frequency 1st overtone, $\nu_{0}(\mathrm{~Hz})$ | $1035.8400279(1)$ |
| Pulsar mass function, $f_{\mathrm{p}}\left(M_{\odot}\right)$ | $0.0545278(13)$ |

is also eclipsed by its companion star. It was discovered in 2006 June 2 (Schady et al. 2006). J1749 is the first eclipsing accretion-powered millisecond X-ray pulsar (AMXP) system. Observations set an upper limit on the distance to this system of $6.7 \pm 1.3 \mathrm{kpc}$ (Wijnands et al. 2009). The rotation period of the pulsar is $\simeq 1.93 \mathrm{~ms}$ and the eclipse by the companion star is centered at the orbital phase of superior conjunction of the pulsar (Markwardt \& Strohmayer 2010). The eclipse duration is $\simeq 2065 \mathrm{~s}$ (corresponding to an eclipse half-angle of $\simeq 11.7^{\circ}$ ) (Altamirano et al. 2010). Unfortunately, no optical counterpart has been identified yet, with a $3 \sigma$ upper limit in the $I$-band of $>19.6$ (Yang et al. 2010).

In this Letter, we are investigating the properties of the companion star in the case of both a main sequence (MS) star and white dwarf (WD), while also trying to obtain the pulsar mass. The calculation details are presented in Sections 2 and 3, and the paper is summarized in Section 4. Through detailed calculations, we find that the companion star could be a main-sequence one but cannot be a white dwarf.

## 2 TO UNDERSTAND THE NATURE OF J1749 BY OBSERVATIONS

It is known from observations that J1749 is similar to other low-mass X-ray binary systems driven by disk accretion (Markwardt \& Strohmayer 2010), and the observational facts of J1749 are summarized in Table 1 taken from Altamirano et al. (2010).

We assume that the companion star should reach its Roche lobe which is approximated by (Eggleton 1983)

$$
\begin{equation*}
R_{\mathrm{L}}=a \times \frac{0.49 \cdot q^{2 / 3}}{0.69 \cdot q^{2 / 3}+\ln \left(1+q^{1 / 3}\right)}, \tag{1}
\end{equation*}
$$

where $a$ is the semi-major axis of the system and $q=M_{\mathrm{c}} / M_{\mathrm{p}}$ is the ratio between the companion star and the pulsar masses. Importantly, J1749 is a unique system showing ongoing eclipses. In such an eclipsing system, from geometrical considerations, $R_{\mathrm{L}}$ is also related to the inclination $i$ and the eclipse half-angle $\phi \simeq 11.7^{\circ}$ by (Chakrabarty et al. 1993, Altamirano et al. 2010)

$$
\begin{equation*}
R_{\mathrm{L}}=a \times \sqrt{\cos ^{2} i+\sin ^{2} i \cdot \sin ^{2} \phi} \tag{2}
\end{equation*}
$$

where the eccentricity of the system is chosen to be zero. With the above two equations, we can obtain a rough lower limit of this system according to $\ln \left(1+q^{1 / 3}\right)>0$, which is about $i>46^{\circ}$, and we will then consider only large inclination angles in this paper.

As is known, the information of the orbital inclination of a binary system is crucial for understanding the properties of its member stars. Determining the inclination is a fantastically difficult task. Fortunately, for the case of the eclipsing J1749 system, with the assumption that the radius of the companion star has reached its Roche lobe, we can investigate the relation between the mass of the pulsar, $M_{\mathrm{p}}$ and the radius of the companion star, $R_{\mathrm{c}}$ (namely, $R_{\mathrm{L}}$ ). This relation can be obtained as follows. According to Equations (1) and (2), with different fixed $i$, there will be different
q. Additionally, Kepler's third law is

$$
\begin{equation*}
a=\left[\frac{G\left(M_{\mathrm{p}}+M_{\mathrm{c}}\right) P_{\mathrm{orb}}^{2}}{4 \pi^{2}}\right]^{\frac{1}{3}}=1.47716[x(1+q)]^{\frac{1}{3}} R_{\odot}, \tag{3}
\end{equation*}
$$

where $P_{\text {orb }}=8.82 \mathrm{hr}$ is the orbital period of the J1749 system, $x=M_{\mathrm{p}} / M_{\odot}$ is the mass of the pulsar in units of solar masses and $R_{\odot}$ is the radius of the Sun. With certain $i$ and Equations (2) and (3), the relation between the mass of the pulsar and the radius of the companion star is obtained.

Another consideration comes from the mass-radius relation of the companion star, which is empirically given by ZAMSs (zero-age main-sequences) as (Schmidt-Kaler 1982)

$$
\begin{align*}
\log \left(\frac{R}{R_{\odot}}\right) & =0.640 \log \left(\frac{M}{M_{\odot}}\right)+0.011 \quad\left(0.12<\log \left(\frac{M}{M_{\odot}}\right)<1.3\right)  \tag{4}\\
\log \left(\frac{R}{R_{\odot}}\right) & =0.917 \log \left(\frac{M}{M_{\odot}}\right)-0.020 \quad\left(-1.0<\log \left(\frac{M}{M_{\odot}}\right)<0.12\right) \tag{5}
\end{align*}
$$

The above mass-radius relation for ZAMS is consistent with the statistical results presented by Demircan \& Kahraman (1991).

What we are interested in is the intersection points of $R_{\mathrm{L}}$ and the radius determined by the above mass-radius relation with fixed inclination angle. It is very meaningful to find out all of these intersection points which have available inclination angles in order to constrain properties of both the pulsar and the companion star. The result is shown in Figure 1.

The third constraint for the companion star comes from the measured mass function of the pulsar. The mass function for a binary system is (Altamirano et al. 2010)

$$
\begin{equation*}
f_{\mathrm{p}}=\frac{\left(M_{\mathrm{c}} \sin i\right)^{3}}{\left(M_{\mathrm{p}}+M_{\mathrm{c}}\right)^{2}}=0.0545278 M_{\odot} \tag{6}
\end{equation*}
$$

Because of the unknown orbital inclination, the mass function only sets the minimum companion mass. Nonetheless, if we cover all of the available inclination angles, the measured mass function could give another constraint on the companion star. Combining this constraint with that from previous geometrical considerations, we may obtain a unique mass ratio $q$ and thus the companion mass, as well as the inclination angle. The resulting curve is also plotted in Figure 1 for MS stars. We can see in Figure 1 that only one intersection point exists between the mass function curve and the Roche curve (i.e., the intersection between dotted and dashed lines in Fig. 1), which means that the companion star could be an MS one with a mass of $M_{\mathrm{c}} \simeq 0.512 M_{\odot}$. Consequently, the mass of the pulsar is $M_{\mathrm{p}} \simeq 0.989 M_{\odot}$ and the orbital inclination angle is $i \simeq 76.5^{\circ}$.

The above calculation is based on the assumption that the radius of the companion star equals its Roche radius. Keep in mind that for an evolving MS star, its radius will be a little bit larger than that of a ZAMS at the same age. This means that the actual radius of the companion star is a little bit larger than the Roche lobe, $R_{\mathrm{L}}$. Because of this, it is necessary that calculation of the radius of the ZAMS companion star should be slightly smaller than its Roche lobe. In our calculation, we chose the radius of the companion star to be about $90 \%$ of its Roche one, namely $R_{\mathrm{c}}=0.9 R_{\mathrm{L}}$. The corresponding curve is also plotted in Figure 1. In this case, we have $M_{\mathrm{c}} \simeq 0.43 M_{\odot}, i \simeq 75.7^{\circ}$ and $M_{\mathrm{p}} \simeq 0.715 M_{\odot}$.

We also did the same calculation for the case of a WD companion, whose mass-radius relation is (Shapiro \& Teukolsky 1983)

$$
\begin{equation*}
M=0.7011\left(\frac{R}{10^{4} \mathrm{~km}}\right)^{-3}\left(\frac{\mu_{\mathrm{e}}}{2}\right)^{-5} M_{\odot} \tag{7}
\end{equation*}
$$

where $\mu_{\mathrm{e}}=A / Z$ is the mean molecular weight per electron. For He-WDs and CO-WDs, $\mu_{\mathrm{e}}$ is about 2. Rewriting Equation (7), we have

$$
\begin{equation*}
R_{\mathrm{WD}}=0.0161846 R_{\odot}\left(\frac{M_{\mathrm{WD}}}{M_{\odot}}\right)^{-1 / 3}=0.0161846(q x)^{-1 / 3} R_{\odot} \tag{8}
\end{equation*}
$$



Fig. 1 Masses of both pulsar (solid lines) and its companion (dashed lines) as a function of orbital inclination angle. The constraint from the mass function is also drawn (dotted line). Two cases where the companion reaches its Roche lobe ( $R_{\mathrm{MS}}=R_{\mathrm{L}}$ ) or not $\left(R_{\mathrm{MS}}=0.9 R_{\mathrm{L}}\right)$ are considered. In this plot, the companion star is assumed to be an MS.


Fig. 2 Same as in Fig. 1, but for a white dwarf companion.

The result is shown in Figure 2. From Figure 2, we can see that there is no intersection between the mass function curve and the Roche curve of the WD (i.e., no intersection between dotted and dashed lines in Fig. 2), which means that the WD can be ruled out as a candidate for the companion star.

## 3 CONSTRAINT ON THE EQUATION OF STATE OF COMPACT STARS

Once the mass information of the pulsar J1749 has been obtained, one may investigate the nature of the pulsar, namely whether it is a neutron or quark star. A different equation of state (EoS) can result in different mass-radius relations, and we may calculate the Keplerian frequencies of J1749 in various EoS models. In Newtonian gravity, the Keplerian frequency of the pulsar is given by

$$
\begin{equation*}
\Omega_{\mathrm{K}}=\sqrt{\frac{G M_{\mathrm{p}}}{R^{3}}}=11549.9 \sqrt{\frac{x}{r^{3}}} \mathrm{~s}^{-1} \tag{9}
\end{equation*}
$$

where $G$ is the gravitational constant, $x=M_{\mathrm{p}} / M_{\odot}$ and $r=R /\left(10^{6} \mathrm{~cm}\right)$. The mass-radius relations resulting from typical EoSs are shown in Lattimer (2007, fig. 2 there), and the calculated Keplerian frequencies in typical neutron star models are listed in Table 2.

According to general relativity, the rotation-mode (i.e., $r$-mode) instability could be excited in fast relativistic stars and they may effectively spin down through radiating gravitational waves (Andersson et al. 2009). Certainly, it depends on the detailed EoS and the associated micro-physics about viscosity to determine the $r$-mode instability window. Nevertheless, one could calculate a critical angular frequency as a function of temperature, $\Omega_{c}(T)$, in certain star models. The lower limit of the critical frequency could be $<0.1 \Omega_{\mathrm{K}}$ (Andersson et al. 2009), and the pulsar J1749 could be a quark star if the critical frequency is higher than $\sim 0.3 \Omega_{\mathrm{K}}$ according to Table 2 , since quark stars can sustain faster spins than neutron stars.

## 4 CONCLUSIONS AND DISCUSSIONS

According to our calculations, we address the question of whether the companion star could be an MS with $\sim 0.5 M_{\odot}$ and an inclination angle of about $76^{\circ}$. Assuming a distance of 7 kpc , the peak

Table 2 Keplerian frequency $\Omega_{\mathrm{K}}$ of the pulsar J1749 under the assumption that both $R_{\mathrm{c}}=R_{\mathrm{L}}$ and $R_{\mathrm{c}}=0.9 R_{\mathrm{L}}$ for the MS companion. The angular frequency is fixed to be $\Omega=3254.06 \mathrm{~s}^{-1}$. The radii of pulsars for different EoSs are taken from Lattimer (2007).

|  | $R_{\mathrm{C}}=R_{\mathrm{L}}, M_{\mathrm{p}}=0.989 M_{\odot}$ |  | $R_{\mathrm{C}}=0.9 R_{\mathrm{L}}, M_{\mathrm{p}}=0.715 M_{\odot}$ |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| EoS | $R /(10 \mathrm{~km})$ | $\Omega_{\mathrm{K}}\left(\mathrm{s}^{-1}\right)$ | $\Omega / \Omega_{\mathrm{K}}$ | $R /(10 \mathrm{~km})$ | $\Omega_{\mathrm{K}}\left(\mathrm{s}^{-1}\right)$ | $\Omega / \Omega_{\mathrm{K}}$ |
| MS0 | 1.471 | 6438.42 | 0.505413 | 1.457 | 5554.85 | 0.585805 |
| MS2 | 1.432 | 6703.22 | 0.485447 | 1.417 | 5791.71 | 0.561848 |
| PAL1 | 1.4 | 6934.35 | 0.469267 | 1.403 | 5878.61 | 0.553542 |
| FSU | 1.325 | 7531.37 | 0.432067 | 1.368 | 6105.66 | 0.532958 |
| PAL6 | 1.225 | 8472.15 | 0.384089 | 1.3 | 6590.93 | 0.493718 |
| MPA1 | 1.225 | 8472.15 | 0.384089 | 1.218 | 7267.59 | 0.447749 |
| AP3 | 1.2 | 8738.28 | 0.372391 | 1.189 | 7535.09 | 0.431854 |
| AP4 | 1.139 | 9449.58 | 0.34436 | 1.143 | 7994.52 | 0.407037 |
| SQM1 | 0.986 | 11732.3 | 0.277359 | 0.889 | 11654.9 | 0.279201 |
| SQM3 | 0.829 | 15218.3 | 0.213825 | 0.736 | 15471.9 | 0.21032 |

Table 3 Deduced Parameters of the J1749 System

|  | $R_{\mathrm{c}}=R_{\mathrm{L}}$ | $R_{\mathrm{c}}=0.9 R_{\mathrm{L}}$ |
| :--- | :---: | :---: |
| $i$ | $76.5^{\circ}$ | $75.7^{\circ}$ |
| $R_{\mathrm{C}}\left(R_{\odot}\right)$ | 0.517 | 0.440 |
| $M_{\mathrm{c}}\left(M_{\odot}\right)$ | 0.512 | 0.430 |
| $M_{\mathrm{p}}\left(M_{\odot}\right)$ | 0.989 | 0.715 |
| $B(\mathrm{G})$ | $\sim 10^{7}$ | $\sim 10^{7}$ |

outburst luminosity was about $1.8 \times 10^{36} \mathrm{erg} \mathrm{s}^{-1}$ (Ferrigno et al. 2010). The pulsar accretes matter from its MS companion star, and we can estimate the timescale of this accreting system to be on the order of $10^{9} \mathrm{yr}$.

Under the assumption that the companion star has reached its Roche lobe, with the observed mass function and the mass-radius relations of both a MS and WD, we investigate all of the possible solutions of available inclination angles as well as the corresponding companion star and pulsar masses. All the results are presented in Table 3. The pulsar mass is $\simeq 1.0 M_{\odot}$ which is about the same as the lowest one determined to date, SMC X-1, which is $1.06_{-0.10}^{+0.11} M_{\odot}$ (van der Meer et al. 2007). According to the mass of the MS companion, we predict the luminosity of the binary system to be $\sim 0.09 L_{\odot}$ in the optical band, and we expect to detect the companion in the followup observations.

From Garcia et al. (2001, fig. 1), the minimum luminosity of J1749 should be on the order of $10^{-5} L_{\text {Edd }}$, namely $10^{33} \mathrm{erg} \mathrm{s}^{-1}$. One possible reason that J 1749 becomes undetectable by Chandra ${ }^{1}$ could be that the pulsar is now in a propeller phase, and we could then estimate the magnetic field of the pulsar. The pulsar may manifest itself as a transient X-ray source if the magnetospheric radius is approaching the corotation radius. The magnetospheric radius is

$$
\begin{equation*}
r_{\mathrm{m}}=\left(\frac{B^{2} R^{6}}{\dot{M} \sqrt{2 G M_{\mathrm{p}}}}\right)^{2 / 7} \simeq 1.3 \times 10^{11} \mu_{30}^{4 / 7}\left(\dot{M}_{10}\right)^{-2 / 7}\left(\frac{M_{\mathrm{p}}}{M_{\odot}}\right)^{-1 / 7} \mathrm{~cm} \tag{10}
\end{equation*}
$$

where $\mu_{30}=\mu /\left(10^{30} \mathrm{G} \mathrm{cm}^{3}\right)$ is the magnetic momentum $\left(\mu=B R^{3} / 2\right)$ in units of $10^{30} \mathrm{G} \mathrm{cm}^{3}$, and $\dot{M}_{10}=\dot{M} /\left(10^{10} \mathrm{~g} \mathrm{~s}^{-1}\right)$ is the accretion rate in units of $10^{10} \mathrm{~g} \mathrm{~s}^{-1}$. The corotation radius is given by

$$
\begin{equation*}
r_{\mathrm{co}}=\left(\frac{G M_{\mathrm{p}}}{4 \pi^{2}}\right)^{1 / 3} P^{2 / 3} \simeq 1.5 \times 10^{8}\left(\frac{M_{\mathrm{p}}}{M_{\odot}}\right)^{1 / 3} P^{2 / 3} \mathrm{~cm} . \tag{11}
\end{equation*}
$$

[^1]From $r_{\mathrm{m}} \sim r_{\mathrm{co}}$, we could estimate the magnetic field of J 1749 to be

$$
\begin{equation*}
B \sim 1.45 \times 10^{7}\left(\frac{M_{\mathrm{p}}}{M_{\odot}}\right)^{5 / 6} \dot{M}_{10}^{1 / 2} P^{7 / 6} R_{6}^{-3} \sim 1 \times 10^{7} R_{6}^{-3} \mathrm{G} \tag{12}
\end{equation*}
$$

where $R_{6}=R /\left(10^{6} \mathrm{~cm}\right)$. Therefore, J 1749 may have a relatively weak surface magnetic field.
An accretion-driven millisecond pulsar with $\sim 1 M_{\odot}$ mass would have profound implications on neutron star physics. Millisecond pulsars are supposed to be more massive than normal pulsars in the conventional recycling scenario. However, the mass of J1749 is much lower than the generally accepted pulsar mass $\sim 1.4 M_{\odot}$. A similar case of another accreting and relatively low mass compact star, EXO 0748-676, was also discussed in Xu (2003). These facts may also challenge the standard model for the origin and evolution of millisecond pulsars, in addition to the arguments from population synthesis (Lorimer et al. 2007). Furthermore, considering the $r$-mode instability, we also estimate the ratio of the real angular frequency to the critical angular frequency, and find that neutron stars could hardly spin so fast (spin period $\sim 2 \mathrm{~ms}$ ) but quark stars can do this due to self-confinement and solidification (Xu 2005).

Acknowledgements We are grateful to Dr. C. B. Markwardt and Dr. D. Altamirano for communicating the observations of the companion star, and we also acknowledge that Prof. Shuangnan Zhang suggested discussing the magnetic field of J1749. We thank the members at the PKU pulsar group for helpful discussions. This work is supported by the National Natural Science Foundation of China (Grant Nos. 10778611 and 10973002), and the National Basic Research Program of China (Grant No. 2009CB824800)

## References

Altamirano, D., Cavecchi, Y., et al. 2010, arxiv:1005.3527
Andersson, N., Ferrari, V., Jones, D. I., et al. 2009, Gen. Rel. Grav., in press (arXiv:0912.0384)
Chakrabarty, D., Grunsfeld, J. M., et al. 1993, ApJ, 430, L33
Demircan, O., \& Kahraman, G. 1991, ApSS, 181, 313
Eggleton, P. P. 1983, ApJ, 268, 368
Ferrigno, C., Bozzo, E., Falanga, M., Stella, L., Campana, S., Belloni, T., Israel, G., Pavan, L., Kuulkers, E., \& Papitto, A. 2010, arXiv:1005.4554
Garcia, M. R., McClintock, J. E., Narayan, R., Callanan, P., Barret, D., \& Murray, S. S. 2001, ApJ, 553, L47
Lattimer, J. M. 2007, ApSS, 308, 371
Lattimer, J. M., \& Prakash, M. 2004, Sci., 304, 536
Lorimer, D. R., McLaughlin, M. A., Champion, D. J., \& Stairs, I. H. 2007, MNRAS, 379, 282
Markwardt, C. B., \& Strohmayer, T. E. 2010, ApJ, 717, L149
Schady, P., Beardmore, A. P., et al. 2006, GRB Coordinates Network, 5200, 1
Schmidt-Kaler, Th. 1982, Landolt-Bornstein: Numerical Data and Functional Relationships in Science and Technology, eds. K. Schaifers, \& H. H. Voigt (Berlin: Springer-Verlog), VI/2b
Shapiro S. L., \& Teukolsky S. A. 1983, Black Holes, White Dwarfs and Neutron Stars: The Physics of Compact Objects (Jhon-Wiley \& Sons)
van der Meer, A., Kaper, L., van Kerkwijk, M. H., Heemskerk, M. H. M., \& van den Heuvel, E. P. J. 2007, A\&A, 473, 523
Wijnands, R., Rol, E., Cackett, E., Starling, R. L., \& Remillard, R. A. 2009, MNRAS, 393, 126
Xu, R. X. 2003, ChJAA (Chin. J. Astron. Astrophys.), 3, 33
Xu, R. X. 2005, MNRAS, 356, 359
Xu, R. X. 2009, J. Phys. G: Nucl. Part. Phys., 36, 064010
Yang, Y. J., Russell, D. M., et al. 2010, The Astronomer's Telegram, 2579, 1


[^0]:    * Supported by the National Natural Science Foundation of China.

[^1]:    ${ }^{1}$ Private communication with D. Altamirano.

