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Forecast of the key parameters of the 24-th solar cycle

Oleg Vasilievich Chumak and Tatiana Viktorovna Matveychuk

Sternberg Astronomical Institute, Moscow State University, Universitetsky pr. 13, Moscow 119992, Russia; *matvey@sai.msu.ru; chumak@sai.msu.ru*

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Abstract To predict the key parameters of the solar cycle, a new method is proposed based on the empirical law describing the correlation between the maximum height of the preceding solar cycle and the entropy of the forthcoming one. The entropy of the forthcoming cycle may be estimated using this empirical law, if the maximum height of the current cycle is known. The cycle entropy is shown to correlate well with the cycle's maximum height and, as a consequence, the height of the forthcoming maximum can be estimated. In turn, the correlation found between the height of the maximum and the duration of the ascending branch (the Waldmeier rule) allows the epoch of the maximum, Tmax, to be estimated, if the date of the minimum is known. Moreover, using the law discovered, one can find out the analogous cycles which are similar to the cycle being forecasted, and hence, obtain the synoptic forecast of all main features of the forthcoming cycle. The estimates have shown the accuracy level of this technique to be 86%. The new regularities discovered are also interesting because they are fundamental in the theory of solar cycles and may provide new empirical data. The main parameters of the future solar cycle 24 are as follows: the height of the maximum is Wmax = 95$\pm$20, the duration of the ascending branch is Ta = 4.5$\pm$0.5 yr, the total cycle duration according to the synoptic forecast is 11.3 yr.

Key words: Sun: solar activity — Sun: forecasts

1 INTRODUCTION

Global indices of solar activity make up a stochastic non-stationary time series, and their forecasting is a challenging problem. The stochastic behavior of indices is accounted for by the spatial and temporal stochasticity of magnetohydrodynamic (MHD) processes in the upper layers of the Sun which manifest themselves as solar activity phenomena. It is the same reason why MHD theories of the solar cycle are unlikely to be able to considerably enhance the forecast accuracy, with stochasticity and a rather short "memory" of the solar MHD processes being their specific features. Thus, Bushby & Tobias (2007) showed that the forecasting capability of the currently available MHD theories of the solar cycle approaches zero. In addition, a number of empirical regularities are revealed in solar activity phenomena, in Wolf numbers particularly, which enable us to foresee the evolution of future events to some extent. First and foremost, the well-known 11-year cycles are surprisingly persistent. Douglass (1919) discovered them in growth rings of ancient trees. Dergachev & Veksler (1991) showed that the 11-year cycle is distinctly retraced in radiocarbon data covering the past millennia. The cycle was disclosed in sediments which refer to the geological epoch dating back a few hundred million years (Cram 1983). Ruel (1991) observed reasonably that "the systems which admit such ‘eternal recovery’ are, in general, moderately complex ones.” Actually, owing to the eternal recovery of the solar cycle, it may be said that the next cycle of solar activity is most likely to occur, and, therefore, the goal is how best to forecast the main parameters of the new cycle. Mandelbrod & Wallis (1968) showed that the strong persistence of the averaged monthly Wolf numbers (with the corresponding value of the Hurst parameter H = 0.93) arises mainly from the 11-year cycle. The rescaled range (R/S) analysis carried out using a larger volume of data than that available in the cited work allowed us to reveal the persistent and anti-persistent regions in the sequence of the Wolf number data (Chumak 2005). The mean value of the Hurst parameter turned out to be 0.63 in an 8.4–18.7 yr interval and this argues for certain persistence being present in this interval. A moderate anti-persistence (H = 0.39) is present in an interval of about 25–47 yr. Over larger intervals, up to 80 yr, the Hurst parameter decreases, that is, the anti-persistence increases. Time intervals of more than 80 yr cannot be analyzed with confidence because of limited observational data. The anti-persistent series are also called the ergodic ones, since the “regression to the mean” is their peculiar property; namely, if the series demonstrates a certain tendency at a given moment in time, it is likely to re-orient to the opposite situation later: the lesser the values of H, the greater the probability of this process. Since the solar activity processes have a stochastic nature and a rather “short memory,” the most successful prognostication methods are those which refer to a relatively short timescale covering only adjacent cycles with a relatively strong persistence. One example of such a method is using the well-known Gnevyshev–Ohl correlation between the Wolf numbers summarized over a cycle when considering odd and even solar cycles separately (see, for instance, Vitinskij et al. 1986). Forecasts on the 1–3 yr timescale may also be useful to some extent. It is over this timescale where the prominent 1.5–2.5 yr quasi-periodic phenomena were discovered (Vitinskij et al. 1986). As for time intervals of more than 50 yr, only mean parameters may be reasonable enough to be considered.

2 PROGNOSTICATION METHOD

The Gnevyshev–Ohl rule is only known to be applicable for forecasting odd cycles. Below, we present a forecast based on the rule which can be employed on the same time interval (a pair of adjacent cycles), but this new rule may apply to both odd and even cycles. A definite correlation has been discovered between the height of the preceding cycle maximum and the Shannon entropy of the subsequent one. The Shannon entropy (in bits) was calculated using the formula:

$$\mathrm{ES} = -\sum_{i=1}^{n} p_i \log_2 p_i = -\sum_{i=1}^{n} p_i \log_{10} p_i / \log_{10} 2,$$

where $p_i = W_i/\mathrm{SW}$, W_i are the averaged monthly Wolf numbers, $\mathrm{SW} = \sum_{i=0}^{N} W_i$, N is the number of months in the cycle, and $n = 300$ corresponds to the maximum monthly Wolf number registered in observations (the maximum of cycle 19). The minimum value of H = 0 is achieved when all W_i are equal to each other (one W-scale level is filled up). The maximum value of H = 8.229 corresponds to the uniform filling of all W-scale levels. The entropy quantities of various cycles are enclosed between these two values. The forecast is based on analyzing the table. This is a well-known NOAA table containing the key cycle parameters of the Wolf numbers obtained from the averaged monthly data (*ftp://ftp.ngdc.noaa.gov*) supplemented by two columns: the sum of all averaged monthly Wolf numbers over the cycle (SW) and the cycle entropy (ES) calculated using the formula given above. Line 24 contains the predicted parameters of the forthcoming cycle 24 obtained using the technique described below. Let us look at the Gnevyshev–Ohl rule (Fig. 1). The straight line is drawn using the least squares method applied to all the latest cycles from the table. Reliability in terms of Pearson’s approximation is found to be very low $(R^2 = 0.16)$. Therefore, it seems evident that the rule is not applicable to all cycles. The rule is fulfilled considerably better if the pair of cycles in question (4, 5)

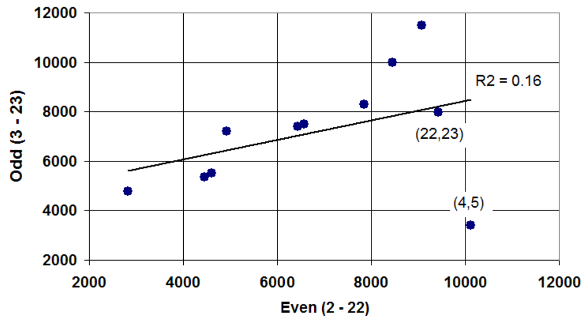

Fig. 1 Gnevyshev–Ohl rule applied to all cycles.

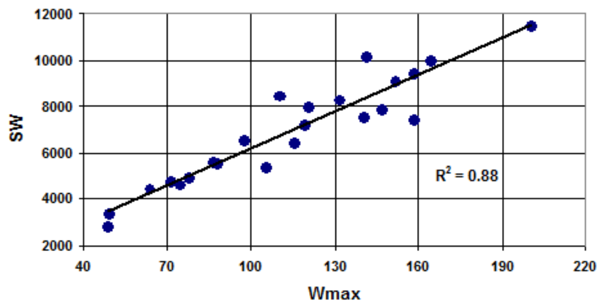

Fig. 2 Correlation between power and maximum value.

is excluded (and this was done by Gnevyshev and Ohl). Higher correlation coefficients, up to 0.9 and more, can be found elsewhere (see, for example, Vitinskij et al. 1986). These estimates are obtained when the pair of the latest cycles (22, 23), as well as several pairs before cycles 10 and 11, are excluded; various authors take into account different considerations when doing this procedure. One way or another, all the authors agree that the rule is not seriously violated for the cycles subsequent to 10 and 11. Significant violation of the rule occurred only in the latest pair of cycles (22, 23).

In this context, this pair of cycles is likely to be anomalous; maybe, this indicates that the activity cycle is undergoing considerable changes in the current epoch. The question is how the Gnevyshev–Ohl rule will be fulfilled in the cycles to come: it is now unclear whether it is able to return or if this regularity will be lost completely. It should be noted that there are many reasons to exclude the Wolf numbers of cycles 4, 5, and 6 from the statistical analysis of the sequence. Usoskin et al. (2003) believe that a whole solar cycle was lost in the 1790s, with the minimum of 1792–1793 and maximum of 1794–1795. Vitinskij et al. (1986; and references therein) consider the Wolf numbers observed before cycle 10 (before the 1850s) to be statistically insignificant because of numerous random and systematic errors. Whatever the case, most researchers justify excluding cycles 4 and 5 from consideration and believe that the subsequent data, starting from cycle 8, may be relied upon due to their higher statistical weights compared to the data from cycles 1–7.

The Gnevyshev–Ohl rule may be referred to as a “resolving rule.” The name means that the prognostication problem may be resolved, that is, the key parameters of the forthcoming cycle may be found in terms of the rule, once an essential parameter of the cycle, its “power” (SW) is estimated. In doing so, such prominent relationships as “power vs. the height of the maximum” and “the height of the maximum vs. the length of the ascending branch” should be taken into account.

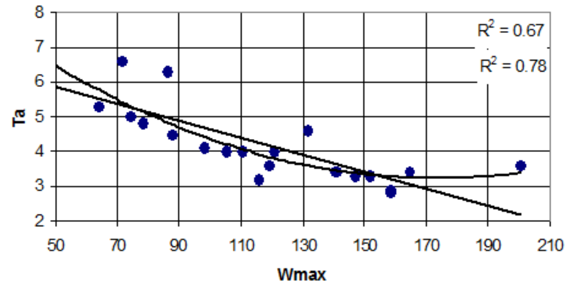

Fig. 3 Height of maximum vs. length of the ascending branch of the cycle (Waldmeier rule).

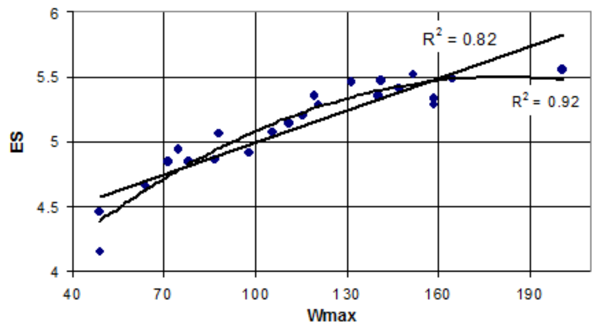

Fig. 4 Correlation between the height of the maximum and the Shannon entropy.

Figure 2 shows the "power vs. maximum" relationship to be fulfilled rather well for the Wolf numbers in all cycles, without any significant exclusions; the reliability of the linear approximation is $R^2 = 0.88$. The "height of the maximum vs. the length of the ascending branch" relationship (the Waldmeier rule, Fig. 3) is also well fulfilled for the whole series of the Wolf number data. The reliability of the linear approximation is $R^2 = 0.67$. When approximating the data by a second degree polynomial, the reliability proves to be $R^2 = 0.78$. Excluding cycles from 1 to 7 from the consideration yields an even more accurate fitting of the Waldmeier rule to the remaining cycles, $R^2 = 0.80$.

Figure 4 shows "the height of the maximum vs. the Shannon entropy" diagram. The reliability of the linear approximation is R^2 = 0.82, and that of the square approximation is R^2= 0.92, which is more than in the case of the "power vs. maximum" relationship or the Waldmeier rule. It should be noted that such high approximation reliability is achieved for the whole Wolf series without excluding any cycles. When excluding cycles 4 and 5 from the consideration (two leftmost points in Fig. 4), even the linear approximation yields R^2 = 0.90. When shifting all the cycles relative to each other by one cycle, a noticeable effect in the relationship between the height of the preceding cycle maximum and the entropy of the subsequent cycle is revealed (Fig. 5): the points corresponding to even cycles lie predominantly above the line shown in the plot while those corresponding to odd cycles lie below it. On average, the straight line is equidistant from even and odd cycles. Cycles 4 and 22 disobey this rule; these are the same "wrong" cycles which do not match the Gnevyshev–Ohl rule. Cycles 1, 7, and 17 are also exceptional. Thus, three exceptions belong to cycles from 1 to 7, and two exceptions fall within cycles from 8 to 22. In general, the relative number of exceptions over the Wolf series as a whole is 22.7%, and the portion of "correct" cycles is 77.3%. Therefore, if the Wolf series is considered en masse, the given rule is fulfilled considerably better than the

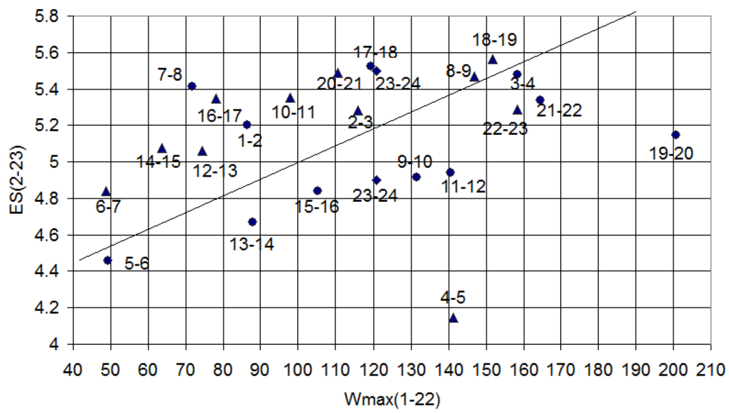

Fig. 5 Height of maximum value (Wmax) of the preceding cycle vs. entropy (ES) of the subsequent cycle (Wmax – ES rule).

Gnevyshev–Ohl rule. Starting from cycle 8, there are only two exceptions to our rule, namely, cycles 17 and 22. For cycles from 8 to 23, the rule is fulfilled with 86% probability which is also better than the result demonstrated by the Gnevyshev–Ohl rule in the same interval. In addition, in contrast to the latter case, our rule is valid for both even and odd cycles.

3 PREDICTING THE KEY PARAMETERS OF CYCLE 24

To predict the parameters of the forthcoming solar cycle 24, let us use the Wmax–ES rule (Fig. 5). The height of the maximum of cycle 23 is 120.8; therefore, if the forthcoming cycle 24 is “normal” (86% “for” and 14% “against”) rather than “anomalous” (14% “for” and 86% “against,” respectively), its representative point, having 120.8 as its abscissa, should fall into the region between cycles 15 and 9. The Shannon entropy of the forthcoming cycle 24 is likely to be between 4.8 and 5.2, that is, ES = 5.0$\pm$0.2 or lower, nearly 4.9 (see the lower rhombus-point (23–24) in Fig. 5). Using the “height of the maximum value vs. Shannon entropy” relationship (Fig. 4), one can find the height of the maximum of the forthcoming cycle 24 to be equal to Wmax = 95$\pm$20. The Waldmeier rule (Fig. 3) yields the duration of the ascending branch of cycle 24, Ta = 4.5$\pm$0.5 yr (Ta=5 yr for ES=4.9), and the “power vs. maximum” relationship (Fig. 2) enables us to estimate the power of cycle 24: SW = 6000$\pm$500. A probable epoch of the maximum of cycle 24 may be obtained by adding the value of Ta to the epoch of the minimum of cycle 23. If, according to Li (2009), 2008 September–October is taken as the epoch of minimum, the epoch of maximum of cycle 24 is likely to occur in the year of Tmax = 2013.3$\pm$0.5. Adding Ta and the mean value of the duration of the descending branch, Td, we can estimate the duration of cycle 24: T = 11.1$\pm$1.2 yr. All these parameters are shown in Table 1 (line 24). As was mentioned above, unlike the forecasts based on the Gnevyshev–Ohl rule, the given forecast algorithm can be applied both for the even and odd cycles; it is based on approximations having higher reliability levels.

4 THE SYNOPTIC FORECAST OF CYCLE 24

The results obtained allow a simple synoptic forecast procedure to be performed. Let us apply once more the Wmax–ES rule (Fig. 5). With the value of height obtained for the maximum of cycle 24 (Wmax = 95$\pm$20), we find out that the nearest-ordinate point on the even side of the diagram (Fig. 5) corresponds to cycle 10. The point of cycle 20 is more distant but, nevertheless, within the Wmax

Table 1 Basic Parameters of Wolf's Cycles

No	Tmin	Wmin	Tmax	Wmax	Ta	Td	T	SW	ES
1	1755.2	8.4	1761.5	86.5	6.3	5.0	11.3	5610.8	4.864
2	1766.5	11.2	1769.7	115.8	3.2	5.8	9.0	6426.0	5.202
3	1775.5	7.2	1778.4	158.5	2.9	6.3	9.2	7403.9	5.284
4	1784.7	9.2	1788.1	141.2	3.4	10.2	13.6	10121.0	5.477
5	1798.3	3.2	1805.2	49.2	6.9	5.4	12.3	3392.2	4.145
6	1810.6	0.0	1816.4	48.7	5.8	6.9	12.7	2821.0	4.459
7	1823.6	0.1	1829.9	71.7	6.6	4.0	10.6	4783.4	4.841
8	1833.9	7.3	1837.2	146.9	3.3	6.3	9.6	7838.1	5.415
9	1843.5	10.5	1848.1	131.6	4.6	7.9	12.5	8293.7	5.468
10	18563	2.0	1860.1	97.9	4.1	7.1	11.2	6558.4	4.914
11	1867.2	5.2	1870.6	140.5	3.4	8.3	11.7	7494.2	5.354
12	1878.9	2.2	1883.9	74.6	5.0	5.7	10.7	4604.0	4.940
13	1889.6	5.6	1894.1	87.9	4.5	7.6	12.1	5530.0	5.060
14	1901.7	2.8	1907.0	63.8	5.3	6.6	11.9	4465.3	4.668
15	1913.6	1.5	1917.6	105.4	4.0	6.0	10.0	5363.9	5.074
16	1923.6	5.6	1928.4	78.1	4.8	5.4	10.2	4927.2	4.842
17	1933.8	3.4	1937.4	119.2	3.6	6.8	10.4	7204. 6	5.350
18	1944.5	7.7	1947.5	151.8	3.3	6.8	10.1	9078.7	5.524
19	1954.5	3.4	1957.9	200.8	3.6	7.0	10.6	11480.0	5.563
20	1964.8	9.6	1968.9	110.6	4.0	7.6	11.6	8446.2	5.145
21	1976.5	12.2	1979.9	164.5	3.4	6.9	10.3	9983.1	5.490
22	1986.8	12.3	1989.6	158.5	2.8	6.8	9.7	9420.1	5.335
23	1996.4	8.0	2000.3	120.8	4.0	6.8	10.8	7979.8	5.287
24	2008.8	5.0	2013.3	95.0	4.5	6.8	11.3	6000.0	5.000

forecast error. Thus, the forthcoming cycle 24 may be expected to be similar to cycle 10, with some characteristics of cycle 20. The point corresponding to cycle 10 is approximately four times nearer to the ordinate of Wmax = 95 than that corresponding to cycle 20. Superposition of the onsets of these two cycles (Fig. 6) yields the forecasted monthly averaged parameters of cycle 24 as a weighted mean of the two cycles which are analogous to each other, where the values of cycle 10 are taken with weight 4, and those of cycle 20 are taken with weight 1. Cycle 24, reconstructed by means of such a procedure, is shown in Figure 7 (two upper curves: three-month and half-year smoothing). According to this forecast, the total duration of cycle 24 is expected to be 11.3 yr. The length of the ascending branch and the height of maximum correspond to the values given above. In the case of Wmax=85, cycle 14 becomes the nearest cycle-analog, according to Figure 5. The lower curve in Figure 7 (three-month smoothing) corresponds to this forecast.

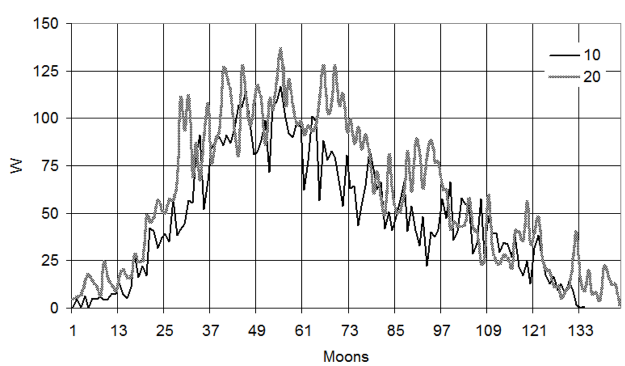

Fig. 6 Cycles analogous to the forthcoming cycle 24.

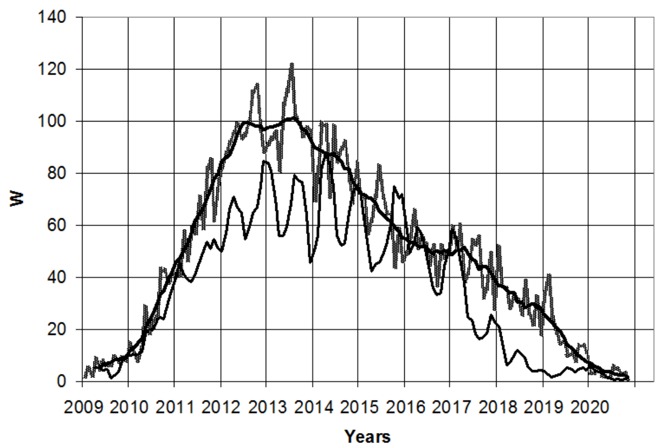

Fig. 7 Synoptic forecast of cycle 24. Two upper curves correspond to ES=5.0, and the low one to ES=4.9.

5 DISCUSSION

Introduction of a new integral parameter of solar cycles, the Shannon entropy, yielded new regularities disclosed in the Wolf sequence: the dependence between the height of the cycle maximum and the Shannon entropy (Fig. 4) with the reliability level of $R^2 = 0.92$, and the separation between regions of even and odd cycles in Figure 5. These relations are satisfied over the whole Wolf series, and their reliability level is higher than those of the Waldmeier and Gnevyshev–Ohl rules. The new properties discovered in the solar cycles are important on their own, since they provide additional empirical information which is essential for creating a physical theory of the solar cycle. On the other hand, they allowed us to develop the forecast algorithm which may be useful in predicting the key parameters of the forthcoming solar cycles regardless of whether they are even or odd. It should be noted, however, that the forecast algorithm is not based on a large volume of data in a statistical sense and, in general, cannot be considered to be statistically significant enough. This remark, however, may equally (if not to a greater extent) be referring as well to the Gnevyshev–Ohl algorithm, the Waldmeier rule, and other empirical laws concerning the Wolf sequence which have been discussed elsewhere. Moreover, statistical significance of such rules in terms of classical stationary statistics is hardly correct at all, since the Wolf series is non-stationary and anti-persistent over long timescales. In addition, when considering empirical rules in general, they are obviously useful for solving forecasting problems.

In conclusion, it is worthwhile noticing that more than a dozen forecasts of cycle 24 have been published for the past two decades. Their detailed discussion may be the subject of another review. Now, we are going to indicate the most cited articles and specify only one – in our opinion, the most significant factor. The main parameter of a cycle is the height of its maximum. According to different authors and their forecasts, the spread of this value expressed in terms of the international averaged monthly Wolf numbers is found to be within a wide interval. Thus, Li et al. (2005) suggested that cycle 24 may be predicted in two alternative versions, with the anomalously high maximum (Wmax = 190$\pm$16) or the relatively low one (Wmax = 108$\pm$29). A high value of Wmax was accepted by Dikpati et al. (2006) who argued for Wmax = 168$\pm$12. A more moderate estimate, Wmax = 115$\pm$30, was given in Rabin (2007). Schatten (2005) believed that the value is even lower: Wmax = 80$\pm$30. Hamid & Galal (2006), as a matter of fact, confirmed this estimate: they obtained Wmax = 90.7$\pm$9.2. Recently, Pesnell & Schatten (2007) gave new arguments in favor of the low estimate, Wmax = 80$\pm$35, based on the Babcock-Leighton model of the solar cycle and on the analysis of radio flux variations. Our estimate, Wmax = 95±20, seems to be in good agreement with the results of the four last-mentioned works.

The final remark concerns the fact that, as was mentioned above, solar cycle 23 was not in agreement with the forecast provided by the Gnevyshev–Ohl rule. In this regard, the cycle which elapsed was anomalous. The present forecast is made under the presumption that the anomaly of cycle 23 is an exception rather than a reflection of some fundamental changes in the physical processes responsible for the solar cyclicity. If this forecast is not verified, it will indicate that the processes of solar cyclicity undergo serious and, possibly, far-reaching changes. In this case, according to Figure 5, we may expect the value ES=5.5 (see the upper rhomb-point (23–24) in Fig. 5) and, respectively, Wmax=160 and above. The probability of such development of events, as was noticed before, may be near 14%.

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