Research in Astron. Astrophys. **2010** Vol. **10** No. **8**, 797–804
http://www.raa-journal.org *http://www.iop.org/journals/raa*

Research in Astronomy and Astrophysics

An optimization of the shape of FAST reflector panels *

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Received 2010 January 7; accepted 2010 April 23

Abstract The Five-hundred-meter Aperture Spherical radio Telescope (FAST) will be the largest radio telescope in the world. The surface tolerance of the main reflector is one of the most important parameters for evaluating the performance of the telescope. The relationship between the reflector's surface tolerance and the curvature of FAST reflector panels is analyzed and discussed. According to the calculation of reflector tolerance and antenna gain, an optimized panel shape for minimum surface tolerance and maximum gain is derived. The far field pattern of the FAST telescope is presented while the optimized shape is utilized for reflector panels. The results show that FAST could be operated at a frequency band of 8 GHz or even higher with an acceptable efficiency.

Key words: telescopes — reflector panel — antenna tolerance

1 INTRODUCTION

FAST (Five-hundred-meter Aperture Spherical radio Telescope) will be the largest single dish radio telescope in the world. It is currently under construction (Nan 2006, 2008). FAST's construction consists of the following sub-systems: site exploration, active main reflector, focus cabin suspension, measurement and control, and receivers. The active main reflector is one of the major outstanding features enabling correction of spherical aberration from the ground. The reflector is divided into thousands of triangular panels. By changing the spatial position of the panels, the main reflector can be actively deformed. While observing a celestial source, the illuminated part of the spherical reflector is deformed into an instantaneous paraboloid. The active reflector is the principal part of the telescope, and it consumes the biggest portion of the construction budget.

The operational frequency band of the FAST telescope is from 70 MHz to 3 GHz, and it will be expanded to 8 GHz in a future upgrade. To provide acceptable reflectivity at 3 GHz, the rms surface tolerance should be less than 5 mm (Skolnik 1990). Choosing a proper panel shape could help minimize the total surface tolerance. The panel shape has been investigated from the very early stage of FAST's feasibility study. A hexagonal panel design was originally made by Qiu (1998). From the calculations and discussions (Qiu 1998), many specifications of the FAST reflector were chosen, such as reflector aperture, opening angle, illuminated area and focal length.

An innovative cable-mesh reflector was investigated starting from 2003 (Nan 2008). In the new design, the main reflector is a spherical cap composed of about 4600 panels which have the shapes

* Supported by the National Natural Science Foundation of China.

of triangles with a side length of about 11 m. There are two types of panels in the reflector design, which are defined as Type-I and Type-II.

The Type-I panel is divided into a number of small triangular planes whose vertexes are attached to a spherical cap. The number of small planes should be the square of a natural number, such as 9, 16, 25, 64 etc. A Type I panel consists of nine planes which is shown on the left of Figure 1. In order to reduce the surface tolerance, the number of planes should be large enough, but the manufacturing expenses and difficulties with assembly could increase with number of planes. The advantage of the Type-I panel is the relatively simple production process. The surface of a Type-II panel is part of a sphere, so it is also referred to as the spherical panel (right of Fig. 1). It is difficult to make this kind of panel due to its small curvature. As far as the reflector tolerance is concerned, the Type-II panel is superior to the Type-I.

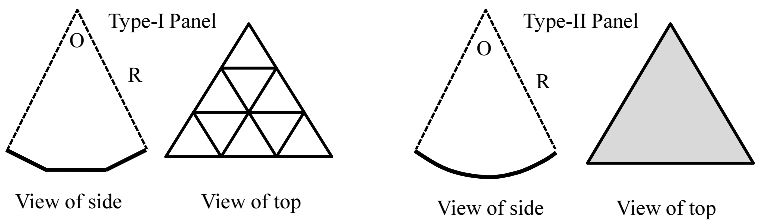

Fig. 1 Two types of panels in the FAST reflector.

2 THE PANEL SHAPE OF THE FAST REFLECTOR

According to the conceptual design of FAST, the main reflector is a spherical bowl composed of about 4600 triangular panels. The panels, especially those near the center portion of the reflector, can be positioned in different parts of the paraboloid when observations of radio sources in different directions takes place. A usual approach, therefore, is that the panels have the same shape to insure that a paraboloid shape is maintained for sources in all directions. The deviation of the reflector from an ideal paraboloid, therefore, is unavoidable when using the same shaped panel for fitting the paraboloid. An optimized shape of panels should be investigated to minimize the reflector tolerance. In the following, the optimized shape of the panels is derived by taking the average of the different portions of the paraboloid.

We will now introduce an algorithm for calculating the best shape of the panels in two dimensions (Fig. 2). The distances from the panel to the line or plane of the panel vertexes are represented by $\boldsymbol{S}$ $(s_1, s_2, s_3, \cdots, s_n)$. The distances between the paraboloid and the line or plane of the panel vertexes are $\boldsymbol{A}(a_1, a_2, a_3, \cdots, a_n)$ (see Fig. 2). For a different segment, they are $\boldsymbol{S}$ and $\boldsymbol{B}$ $(b_1, b_2, b_3, \cdots, b_n)$. Due to the identical shape, the distances $\boldsymbol{S}$ for all the panels are the same.

So, the rms deviations of panels A and B are

$$\mathrm{RMS_A} = \sqrt{\frac{\sum_{i=1}^{n} (a_i - s_i)^2}{N}}, \tag{1}$$

$$\mathrm{RMS_B} = \sqrt{\frac{\sum_{i=1}^{n} (b_i - s_i)^2}{N}}. \tag{2}$$

For the a general panel M, the rms deviation is

$$\mathrm{RMS_M} = \sqrt{\frac{\sum_{i=1}^{n} (m_i - s_i)^2}{N}}. \tag{3}$$

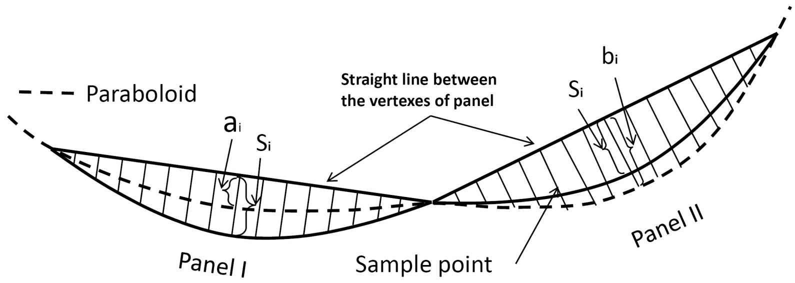

Fig. 2 Schematic diagram of the panel and the paraboloid.

The rms deviation of the total reflector is

$$\mathrm{RMS}_{\mathrm{Total}} = \sqrt{\frac{\mathrm{RMS_A}^2 + \mathrm{RMS_B}^2 + \cdots + \mathrm{RMS_M}^2}{M}}\,. \tag{4}$$

Substituting $\mathrm{RMS_A}$, $\mathrm{RMS_B}$ and $\mathrm{RMS_M}$ by formulas (1), (2) and (3) respectively, then

$$\mathrm{RMS}_{\mathrm{Total}} = \sqrt{\frac{\sum_{i=1}^{n}(a_i - s_i)^2 + \sum_{i=1}^{n}(b_i - s_i)^2 + \cdots + \sum_{i=1}^{n}(m_i - s_i)^2}{N \times M}}\,. \tag{5}$$

In order to minimize the total RMS, the following statements all need to be satisfied

$$(a_1 - s_1)^2 + (b_1 - s_1)^2 + \cdots + (m_1 - s_1)^2\,, \tag{6}$$
$$(a_2 - s_2)^2 + (b_2 - s_2)^2 + \cdots + (m_2 - s_2)^2\,, \tag{7}$$
$$\cdots$$
$$(a_n - s_n)^2 + (b_n - s_n)^2 + \cdots + (m_n - s_n)^2\,. \tag{8}$$

Then we have

$$s_1 = \frac{a_1 + b_1 + \cdots + m_1}{M}, \tag{9}$$
$$s_2 = \frac{a_2 + b_2 + \cdots + m_2}{M}, \tag{10}$$
$$\cdots$$
$$s_n = \frac{a_n + b_n + \cdots + m_n}{M}. \tag{11}$$

As seen above, $\boldsymbol{S}$ is the average of $\boldsymbol{A}$, $\boldsymbol{B}$, $\cdots$, and $\boldsymbol{M}$. We may further modify the average shape by weighting the contribution from different portions of the paraboloid according to the illumination function. This algorithm is directly adoptable to the three dimensional case. The average shape and four cross sections of this shape are shown in the top left of Figure 3. Due to the different scales and contour outlines of FAST's panels, the average shape looks like a polygon in the top view. In order to check the curvature of the shapes, comparisons of the respective cross sections and circular arcs are made and shown in the top right and bottom of Figure 3. It could be concluded that the optimized shape is almost identically curved in different directions. The deviation of the optimized shape from a sphere with a radius of 318 m is less than 0.1 mm (bottom of Fig. 3). Regarding the surface tolerance of the FAST reflector, the spherical panel is finally considered to be the optimized panel shape.

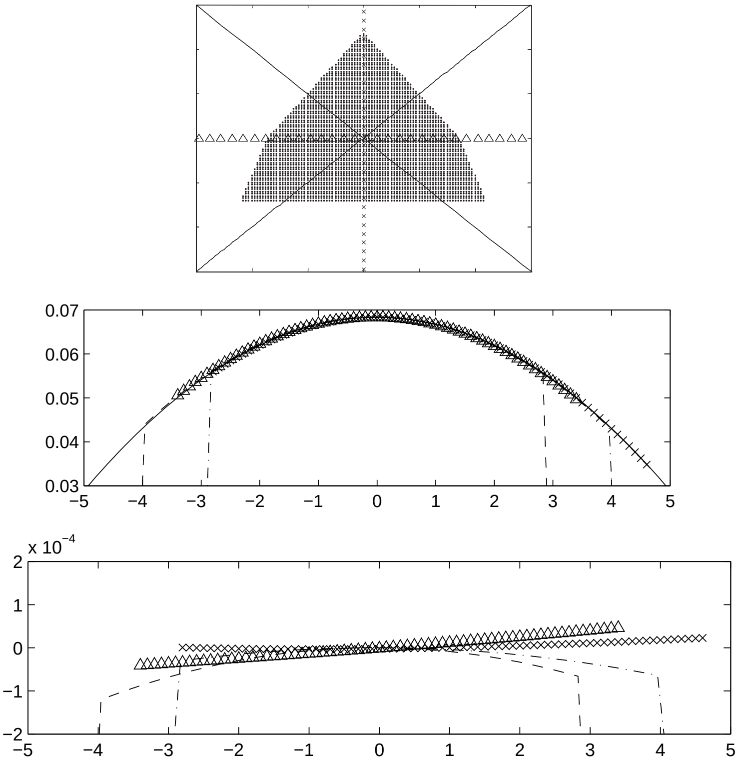

Fig. 3 Average shape (*upper*), comparison of average shape and circular arc (*middle*) and the differences between average shape and circular arc (*bottom*). Units: meter. Line of triangle- horizontal cross section; Line of crosses- vertical cross section; Dashed line- cross section of bottom left to top right; Dot-dashed line-cross section of top left to bottom right; Solid line-spherical curve.

3 CALCULATION AND OPTIMIZATION OF FAST REFLECTOR'S TOLERANCE

During observations, the telescope reflector's surface, with a diameter of 300 m, is deformed into a paraboloid with f/D of 0.4665 by adjusting the spatial positions of the panels. Any deviation of the reflector panels from an ideal paraboloid would reduce the antenna's gain. It is, therefore, necessary to calculate and analyze the components of deviation, and try to reduce the deviation of the reflector as much as possible. The deviation of FAST's reflector surface can be calculated in three steps as listed in the following:

1. Create a model of the FAST reflector. There is only a 300 m aperture of the FAST reflector utilized in normal observations. The reflector is composed of thousands of triangular panels.
2. Sample the reflector's surface. The coordinates of the respective vertexes of the panels are already known from step 1. In order to sample all the panels uniformly, an equally-distributed array of points representing each panel is selected. The coordinates of these sampling points are saved in a database.
3. Calculate the total rms deviation of the reflector. The deviation is the total distance from each sample point to the ideal paraboloid in the direction normal to its surface.

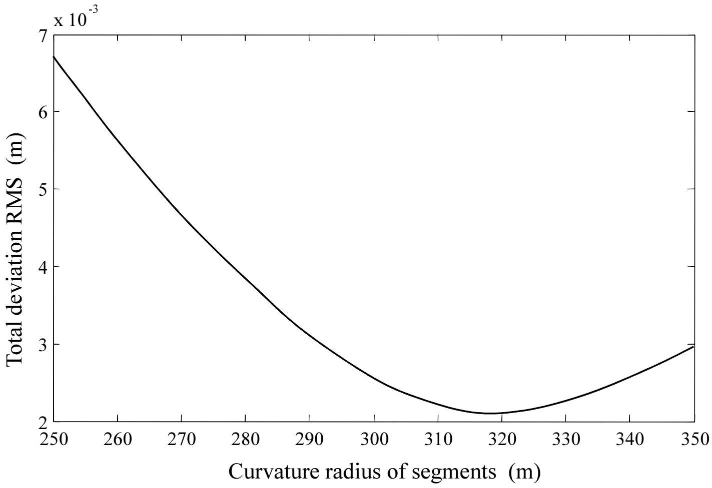

Fig. 4 Variation of reflector deviation as a function of the curvature of the panels.

To reduce the rms deviation, we mainly focus on the curvature and position optimization of the panels. In the calculation, the curvature radius and spatial positions of each panel have been set as variables. For example, the rms deviation will vary with the curvature radius of the panels if the curvature radius is set as a variable (Fig. 4). This method is referred to as curvature optimization. The position optimization is where only the spatial position of a panel is set as a variable in the calculation.

4 RESULTS AND DISCUSSIONS OF THE DEVIATION CALCULATION

The minimum rms deviation of the FAST reflector is 2.3 mm if the Type-I panel is applied. In order to achieve this small number, the Type-I panel should be divided into 64 planes with side lengths of about 1.3 m. The radius of the circumscribing sphere of plane vertexes is 327 m (Nan 2006). The construction cost of this panel would be too high for such a large number of small planes. In order to reduce the expense, the number of planes should be reduced. The minimum rms deviation will be increased to 4 mm if the panel is divided into 9 planes. In order to ensure an acceptable aperture efficiency, the reflector tolerance should be less than 1/20 of a wavelength. For the case of 4 mm rms deviation, the maximum operating frequency will be 3.7 GHz.

The results of the spherical panel are shown in Figure 4. The minimum RMS deviation is 2.1 mm when the curvature radius of the spherical panel is 318 m. The reflector tolerance of a spherical panel is improved by $\sim 10\%$ compared with the Type-I panel. If the average shape is employed, the minimum RMS deviation is also 2.1 mm. The spherical panel, therefore, is the first option for the FAST reflector in terms of a trade-off between the reflector tolerance and manufacturing cost.

There are three main sources of error in the calculation. The first is from the segmentation of the FAST reflector. It is impossible to divide the spherical reflector into identical triangles. In the calculation, the segmentation is based on the principle of geodesics. The number of types of panels in the calculation is larger than that in the detailed design of FAST because the optimization of the mechanics of the panel's side length was not carried out.

In the calculation, the panels' areas will be expanded or shrunk as the reflector's shape is transformed from the initial sphere to a paraboloid. The maximum displacement between the initial sphere and the resulting paraboloid is about 67 cm, within the 300 m utilized area. As a result, the side length of the panels will be shortened or lengthened by 0.2%. The fluctuation of space between sam-

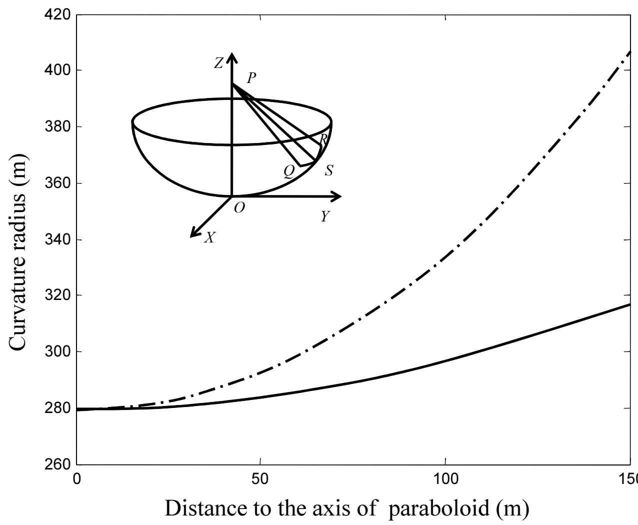

Fig. 5 Curvature of the paraboloid with a focal length of 139.95 m. Solid-curvature in the plane of ZOY; Dot-dashed-curvature in plane of $PQSR$.

ple points will be about 0.2% which is the second source of error. The third source of error is the overlapped sample points at the vertexes of the panels. The ratio of the number of overlapped sample points to the total number of sample points is about 0.03%. The overlapped sample points will make very little contribution to the total error due to their small proportion.

The calculation of reflector tolerance is in three dimensions which is closer to reality than the two dimensional simulation. In two dimensions, only the curvature of the parabolic line is considered, which is equal to the curvature of the paraboloid in the plane of ZOY (Fig. 5). The curvature radius in the plane of ZOY ranges from 280 to 320 m. The curvature radius of the best fitting panel will be 300 m in two dimensions. As a curved surface, the paraboloid could be approximately described by two curvature radii in two orthogonal planes, the plane of ZOY and the plane of $PQSR$ in Figure 5. They are quite different in the two planes. In this calculation, it has been found that these two curvatures of the paraboloid have an effect on the rms deviation. The reason is that the curvature radius of the best fit panels should be larger than 300 m in three dimensions.

5 CALCULATION OF THE FAR FIELD PATTERN OF THE FAST REFLECTOR WITH DEVIATION

In order to estimate the antenna gain of the radio telescope, in general, it is sufficient to calculate the ratio of the antenna gain with surface tolerance to the no-error gain if the antenna tolerance is known. This ratio is the so-called aperture efficiency, which can be derived by the Ruze formula (Ruze 1966; Ruze 1952)

$$\eta_{\rm A} = e^{-(\frac{4\pi\delta}{\lambda})^2}, \tag{12}$$

where δ is the RMS deviation of the reflector surface, λ the wavelength of operation frequency, and $\eta_{\rm A}$ the aperture efficiency. There is an assumption in utilizing the Ruze formula that the distribution of the deviation on the reflector is uniform.

From the above, it could be noted that the tolerance distribution on the FAST reflector is related to the distance from the axis to the illuminated area. The curvature of FAST's reflector panels is uniform and close to the curvature of the paraboloid at about 80 m from the axis. The surface tolerance would be from large to small and then to large again following the distance from the axis of the illuminated area. The tolerance distribution on the FAST reflector is not uniform (Nan 2003). There will therefore be some errors in estimating the antenna gain of FAST by using the Ruze formula.

The far field pattern of FAST could be derived from integrations of radiation from currents on the aperture. The no-error gain is given by

$$G(l,m) = \frac{4\pi}{\lambda^2} \frac{|F_{\mathrm{e}}(l,m) \iint g(x,y) e^{j2\pi(xl+ym)} dxdy|^2}{\iint g^2(x,y) dxdy}\,, \tag{13}$$

where $g(x, y)$ is the illuminating function on the aperture, null beyond the aperture and $F_{\mathrm{e}}(l, m)$ for the far field pattern of integrated elements on the aperture. Then the far field pattern of a reflector with a tolerance $\delta(x, y)$ could be written as

$$G(l,m) = \frac{4\pi}{\lambda^2} \frac{|F_{\mathrm{e}}(l,m) \iint g(x,y) e^{j2\pi(xl+ym)} e^{j2\pi \frac{2\delta(x,y)}{\lambda}} dxdy|^2}{\iint g^2(x,y) dxdy}\,, \tag{14}$$

where $e^{j2\pi \frac{2\delta(x,y)}{\lambda}}$ is phase aberration caused by the reflector tolerance.

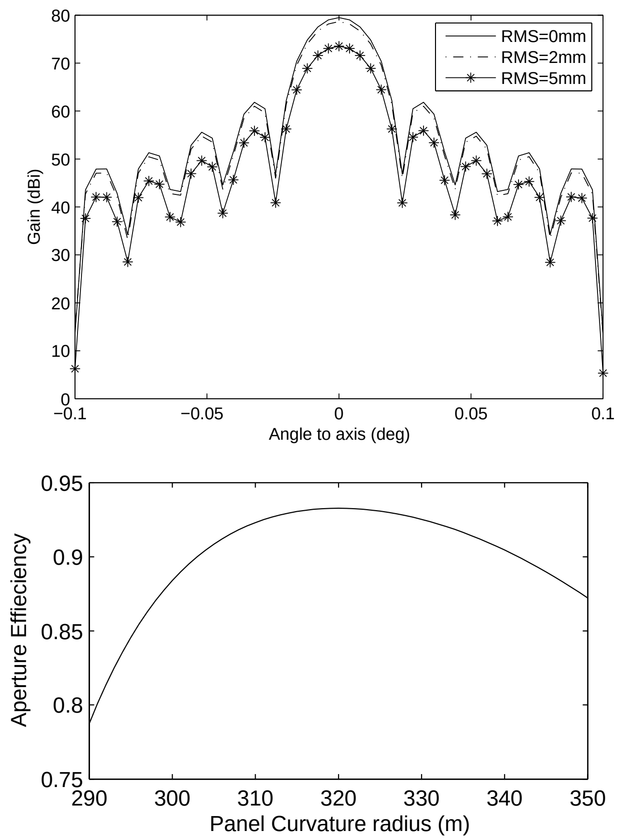

Fig. 6 Far field pattern of FAST at 3 GHz (*upper*) and FAST aperture efficiency varies with curvature radius of the reflector panel (*bottom*).

The far field pattern of the FAST reflector with tolerances of 0 mm, 2.1 mm and 5.0 mm is calculated and shown in the upper of Figure 6. The antenna gain of the telescope is improved by optimizing the panel shape which can be seen from the figure. Bottom of Figure 6 shows the FAST aperture efficiency as a function of curvature radius of the panel when the shape of the reflector panel is spherical. It can be noted that the maximum aperture efficiency is achieved when the curvature radius of the panel is 318 m. The maximum aperture efficiency indexes the maximum antenna gain and the minimum surface tolerance. The results could be extended to whatever the operation frequency is. The aperture efficiency within the entire FAST operation frequency band is shown in Table 1. The aperture efficiency is improved, especially in the high frequency band, by reducing the reflector tolerance. From the calculation, FAST will be able to operate at a frequency of 8 GHz without refining the panels if the gain loss is bearable (aperture efficiency is 60%).

Table 1 Aperture Efficiency of FAST with Two Surface Tolerances

RMS	100 MHz	500 MHz	1 GHz	2 GHz	3 GHz	5 GHz	8 GHz
2.1 mm	0.999923	0.998067	0.992292	0.969523	0.93273	0.824116	0.609438
5.0 mm	0.999561	0.989094	0.957083	0.839071	0.673825	0.333997	0.060365

6 CONCLUSIONS

Based on the calculation and discussion of the FAST reflector tolerance and antenna gain, it could be derived that the spherical panel is a good candidate of panel shape. The minimum RMS deviation of 2.1 mm is achieved when the curvature radius of the spherical panel is set at 318 m. The maximum gain of FAST at the given operation frequency is also reached. From the calculation of the antenna gain of FAST, the aperture efficiency could be higher than 90% at a lower frequency band (70–1.5 GHz). Regarding the higher frequency band (above 3 GHz), the aperture efficiency is improved by applying an optimized panel configuration, typically from 40% to 80% at 5 GHz.

Acknowledgements The authors would like to thank Richard Strom and Jaap Bregman of ASTRON for their many useful suggestions and discussions. This work was supported by the Chinese Academy of Sciences and the National Natural Science Foundation of China (Grant No. 10433020).

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