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INVITED REVIEWS

Primordial black holes

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Abstract Primordial black holes (PBHs) are a profound signature of primordial cosmological structures and provide a theoretical tool to study nontrivial physics of the early Universe. The mechanisms of PBH formation are discussed and observational constraints on the PBH spectrum, or effects of PBH evaporation, are shown to restrict a wide range of particle physics models, predicting an enhancement of the ultraviolet part of the spectrum of density perturbations, early dust-like stages, first order phase transitions and stages of superheavy metastable particle dominance in the early Universe. The mechanism of closed wall contraction can lead, in the inflationary Universe, to a new approach to galaxy formation, involving primordial clouds of massive BHs created around the intermediate mass or supermassive BH and playing the role of galactic seeds.

Key words: cosmology: theory — elementary particles — black hole physics — dark matter — early universe

1 INTRODUCTION

The convergence of the frontiers of our knowledge in micro- and macro- worlds leads to the wrong circle of problems, illustrated by the mystical Uroborus (self-eating-snake). The Uroborus puzzle may be formulated as follows: *The theory of the Universe is based on the predictions of particle theory, that need cosmology for their test*. Cosmoparticle physics (Sakharov 1989; Khlopov 2001, 1999, 2004, 2005a, 2006) offers the way out of this wrong circle. It studies the fundamental basis and mutual relationship between micro-and macro-worlds in the proper combination of physical, astrophysical and cosmological signatures. Some aspects of this relationship, which arise in the astrophysical problem of Primordial Black Holes (PBH), are the subject of this review.

In particle theory, Noether's theorem relates the exact symmetry to conservation of respective charge. Extensions of the standard model imply new symmetries and new particle states. The respective symmetry breaking induces new fundamental physical scales in particle theory. If the symmetry is strict, its existence implies new conserved charge. The lightest particle, bearing this charge, is stable. It gives rise to the deep relationship between dark matter candidates and particle symmetry beyond the Standard model.

The mechanism of spontaneous breaking of particle symmetry also has cosmological impacts. Heating of condensed matter leads to restoration of its symmetry. When the heated matter cools down, phase transition to the phase of broken symmetry takes place. In the course of the phase transitions, corresponding to a given type of symmetry breaking, topological defects can form. One can directly observe formation of such defects in liquid crystals or in superfluid He. In the same manner, the mechanism of spontaneous breaking of particle symmetry implies restoration of the underlying symmetry. When temperature decreases in the course of cosmological expansion, transitions to the phase of broken symmetry can lead, depending on the symmetry breaking pattern, to formation of topological defects in the very early Universe. Defects can represent new forms of stable particles (as in the case of magnetic monopoles 't Hooft 1974; Polyakov 1974; Zeldovich & Khlopov 1978; Khlopov 1979; Preskill 1979; Khlopov 1988), or extended structures, such as cosmic strings (Zeldovich 1980; Vilenkin 1981) or cosmic walls (Zel'Dovich et al. 1974).

In the old Big Bang scenario, cosmological expansion and its initial conditions were given *a priori* (Weinberg 1972; Zeldovich & Novikov 1985). In the modern cosmology, expansion of the Universe and its initial conditions are related to inflation (Starobinsky 1980; Guth 1981; Linde 1982; Albrecht & Steinhardt 1982; Linde 1983), baryosynthesis and nonbaryonic dark matter (see review in Linde 1990; Kolb & Turner 1990). Physics, underlying inflation, baryosynthesis and dark matter, is referred to as extensions of the standard model, and the variety of such extensions makes the whole picture in general ambiguous. However, in a framework of each particular physical realization of the inflationary model with baryosynthesis and dark matter, the corresponding model dependent cosmological scenario can be specified in detail. In such scenario, main stages of cosmological evolution, structure and physical content of the Universe reflect structure of the underlying physical model. The latter should include necessary aspects of the standard model, describing properties of baryonic matter, and its extensions, which are responsible for inflation, baryosynthesis and dark matter. In no case can cosmological impact of such extensions be reduced to reproduction of only these three phenomena. A nontrivial path of cosmological evolution, specific for each particular realization of the inflationary model with baryosynthesis and nonbaryonic dark matter, always contains some additional model dependent cosmologically viable predictions, which can be confronted with astrophysical data. Here we concentrate on Primordial Black Holes as a profound signature of such phenomena.

It was probably Pierre-Simon Laplace (Laplace 1836) in the beginning of the nineteenth century who first noted that in very massive stars escape velocity can exceed the speed of light and light cannot come from such stars. This conclusion, which was made in the framework of Newtonian mechanics and the Newtonian corpuscular theory of light, has been further transformed into the notion of a "black hole" in the framework of general relativity and electromagnetic theory. Any object of mass M can become a black hole, being put within its gravitational radius $r_{\rm g} = 2GM/c^2$. At the present time, black holes (BH) can only be created by the gravitational collapse of compact objects with mass more than about three Solar masses (Oppenheimer & Snyder 1939; Zeldovich & Novikov 1971). It can be a natural end of massive stars or can result from evolution of dense stellar clusters. However, in the early Universe there were no limits on the mass of BHs. Ya. B. Zeldovich and I. D. Novikov (Zel'Dovich & Novikov 1966) noticed that if cosmological expansion stops in some region, a black hole can be formed in this region within the cosmological horizon. It corresponds to strong deviation from general expansion and reflects strong inhomogeneity in the early Universe. There are several mechanisms for such strong inhomogeneity and we will trace their links to cosmological consequences of particle theory.

Primordial Black Holes (PBHs) are a very sensitive cosmological probe for physical phenomena occurring in the early Universe. They could be formed by many different mechanisms, e.g., initial density inhomogeneities (Hawking 1971; Carr & Hawking 1974) and non-linear metric perturbations (Bullock & Primack 1997; Ivanov 1998; Bullock & Primack 1998), blue spectra of density fluctuations (Khlopov et al. 1985b; Polnarev & Khlopov 1985; Lidsey et al. 1995; Kotok & Naselsky 1998; Dubrovich et al. 2004; Sendouda et al. 2006), a softening of the equation of state (Canuto 1978; Khlopov et al. 1985b; Polnarev & Khlopov 1985), development of gravitational instability of early dust-like stages of dominance of supermassive particles and scalar fields (Khlopov & Polnarev 1980; Polnarev & Khlopov 1981, 1982; Khlopov et al. 1985a) and evolution of gravitationally bound objects formed at these stages (Kalashnikov & Khlopov 1983; Kadnikov et al. 1989), collapse of cosmic strings (Hawking 1989; Polnarev & Zembowicz 1991; Hansen et al. 2000; Cheng & Li 1996; Nagasawa 2005) and necklaces (Matsuda 2006), a double inflation scenario (Naselskii & Pelikhov 1979; Kim 2000; Yamaguchi 2001, 2002), first order phase transitions (Hawking et al. 1982; Jedamzik & Niemeyer 1999; Konoplich et al. 1999, 1998; Khlopov et al. 2000a), and a step in the power spectrum (Sakharov & Khlopov 1993; Blais et al. 2003), etc. (see Polnarev & Khlopov 1985; Khlopov 1999, 2004; Carr 2003; Khlopov & Rubin 2004 for a review).

After being formed, PBHs should be retained in the Universe and, if they survive to the present time, represent a specific form of dark matter (Khlopov & Chechetkin 1987; Ivanov et al. 1994; Khlopov 1999, 2004; Blais et al. 2002; Chavda & Chavda 2002; Afshordi et al. 2003; Khlopov & Rubin 2004; Chen 2005). The effect of PBH evaporation by S. W. Hawking (Hawking 1975) makes evaporating PBHs a source of fluxes of products of evaporation, particularly of γ radiation (Page & Hawking 1976). MiniPBHs with mass below 10^{14} g evaporate completely and do not survive to the present time. However, effects of their evaporation should cause influence on physical processes in the early Universe, thus providing a test for their existence by methods of cosmoarcheology (Khlopov 1996), studying cosmological imprints of new physics in astrophysical data. In a wide range of parameters, the predicted effect of PBHs contradicts the data and it puts restrictions on mechanisms of PBH formation and the underlying physics of the very early Universe. On the other hand, at some fixed values of parameters, PBHs or effects of their evaporation can provide a nontrivial solution for astrophysical problems.

Various aspects of PBH physics, mechanisms of their formation, evolution and effects are discussed in Carr et al. (1994); Carr & MacGibbon (1998); Liddle & Green (1998); MacGibbon et al. (1998); Wichoski et al. (1998); Chechetkin et al. (1982a); Polnarev & Khlopov (1985); Grillo & Srivastava (1981); Chapline (1975); Hayward & Pavón (1989); Yokoyama (1997); Kim & Lee (1996); Heckler (1997); MacGibbon et al. (2008); Page et al. (2008); Green & Liddle (1997); Niemeyer & Jedamzik (1998); Kribs et al. (1999); Green et al. (2004); Yokoyama (1998a,b, 1999); Bringmann et al. (2002); Dimopoulos & Axenides (2005); Nozari (2007); Lyth et al. (2006); Zaballa et al. (2007); Harada & Carr (2005a); Custódio & Horvath (2005); Bousso & Hawking (1995, 1996); Elizalde et al. (1999); Nojiri et al. (1999); Bousso & Hawking (1999); Silk (2000); Polarski (2002); Barrow & Carr (1996); Paul (2000); Paul et al. (2002); Paul & Paul (2005); Polarski & Dolgov (2001); Carr & Lidsey (1993); Yokoyama (1998c); Kaloper et al. (2005); Pelliccia (2007); Stojkovic et al. (2005); Murata et al. (2007); Ishihara & Soda (2007); Ahn (2007a); Ahn & Kim (2008); Ahn (2007b, 2006); Ahn et al. (2008); Babichev et al. (2004, 2005, 2006); Guariento et al. (2008); Kavic et al. (2008); Doroshkevich et al. (2009); Shatskiy (2008); Bisnovatyi-Kogan & Tsupko (2008); Alexeyev et al. (2002a); Khovanskaya (2002a,b); Alexeyev & Khovanskaya (2000); Alexeyev et al. (2002c,b); Bambi et al. (2009b,a, 2008); Spaans & Meijerink (2008); Gallo & Marolf (2009); Kawasaki et al. (2007); Kawaguchi et al. (2008); MacGibbon (2007); Flambaum (2009); MacGibbon (2009); Kimura et al. (2008); Takahashi & Soda (2009); Ricotti et al. (2008); Mack & Wesley (2008) particularly specifying PBH formation and effects in braneworld cosmology (Guedens et al. 2002b,a; Clancy et al. 2003; Tikhomirov & Tsalkou 2005), on inflationary preheating (Bassett & Tsujikawa 2001), on formation of PBHs in QCD phase transitions (Jedamzik 1998; Widerin & Schmid 1998a,b), properties of superhorizon BHs (Harada & Carr 2005b; Harada 2006), the role of PBHs in baryosynthesis (Grillo 1980; Barrow et al. 1991; Dolgov & Silk 1993; Turner 1979; Upadhyay et al. 1999; Bugaev et al. 2003), effects of PBH evaporation in the early Universe and in modern cosmic ray, neutrino and gamma fluxes (Miyama & Sato 1978; Fegan et al. 1978; Green 2002; Frampton & Kephart 2005; MacGibbon & Webber 1990; MacGibbon 1991; Halzen et al. 1991, 1995; Bugaev & Konishchev 2002a,b; Volkova & Dokuchaev 1994; Gibilisco 1997; Golubkov et al. 2000; He & Fang 2002; Gibilisco 1996; Custódio & Horvath 2002; Sendouda et al.

2003; Maki et al. 1996; Barrau et al. 2002, 2003c; Wells et al. 1999; Cline 1996; Xu et al. 1998; Cline 1998; Sendouda 2006; Barrau 2000; Derishev & Belyanin 1999; Tikhomirov et al. 2004; Seto & Cooray 2007; Barrau et al. 2003a,b; Barrau & Ponthieu 2004; Ukwatta et al. 2009), in creation of hypothetical particles (Bell & Volkas 1999; Lemoine 2000; Green 1999; Barrau et al. 2004a), PBH clustering and creation of supermassive BHs (Bean & Magueijo 2002; Düchting 2004; Chisholm 2006; Dokuchaev et al. 2005a; Mack et al. 2007; Rubin 2005), and effects in cosmic rays and colliders from PBHs in low scale gravity models (Barrau et al. 2005, 2004b). Here we outline the role of PBHs as a link in the cosmoarcheoLOGICAL chain, connecting cosmological predictions of particle theory with observational data. We discuss the way in which the spectrum of PBHs reflects properties of superheavy metastable particles and of phase transitions on inflationary and post-inflationary stages. We briefly review possible cosmological reflections of particle physics (Sect. 2), illustrate, in Section 3, some mechanisms of PBH formation at the stage of dominance of superheavy particles and fields (Subsect. 3.1) and from second order phase transitions at the inflationary stage. Effective mechanisms of BH formation during bubble nucleation provides a sensitive tool to probe existence of cosmological first order phase transitions by PBHs (Sect. 4). Existence of stable remnants of PBH evaporation can strongly increase the sensitivity of such probes and we demonstrate this possibility in Section 5 with an example of gravitino production in PBH evaporation. Being formed within the cosmological horizon, PBHs seem to have masses much less than the mass of stars and are constrained by the small horizon size of the very early Universe. However, if a phase transition takes place at the inflationary stage, closed walls of practically any size can be formed (Subsect. 6.2) and their successive collapse can give rise to clouds of massive black holes, which can play the role of seeds for galaxies (Sect. 6). The impact of constraints and cosmological scenarios, involving primordial black holes, is briefly discussed in Section 7.

2 PBHS AS A COSMOLOGICAL REFLECTION OF NEW PHYSICS

The simplest primordial form of new physics is a gas of new stable massive particles which originated from the early Universe. For particles with mass m, at high temperature $T > m$, the equilibrium condition, $n \cdot \sigma v \cdot t > 1$ is valid if their annihilation cross section $\sigma > 1/(m m_{\rm pl})$ is sufficiently large to establish equilibrium. At $T < m$, such particles go out of equilibrium and their relative concentration freezes out. Weakly interacting species decouple from plasma and radiation at $T > m$, when $n \cdot \sigma v \cdot t \sim 1$, i.e. at $T_{\rm dec} \sim (\sigma m_{\rm pl})^{-1}$. This is the main idea of calculating the primordial abundance for WIMP-like dark matter candidates (see e.g. Khlopov 1999, 2004, 1996 for details). The maximal temperature, which is reached in the inflationary Universe, is the reheating temperature, $T_{\rm r}$, after inflation. So, very weakly interacting particles with annihilation cross section $\sigma < 1/(T_{\rm r} m_{\rm pl})$, as well as very heavy particles with mass $m \gg T_{\rm r}$ cannot be in thermal equilibrium, and the detailed mechanism of their production should be considered to calculate their primordial abundance.

Decaying particles with lifetime τ, exceeding the age of the Universe, $t_{\rm U}$, $\tau > t_{\rm U}$, can be treated as stable. By definition, primordial stable particles survive to the present time and should be present in the modern Universe. The net effect of their existence is given by their contribution into the total cosmological density. They can dominate the total density as the dominant form of cosmological dark matter, or they can represent its subdominant fraction. In the latter case, more detailed analysis of their distribution in space, of their condensation in galaxies, of their capture by stars, Sun and Earth, as well as effects of their interaction with matter and of their annihilation provides more sensitive probes for their existence. In particular, hypothetical stable neutrinos of the fourth generation with mass about 50 GeV are predicted to form the subdominant form of modern dark matter, contributing less than 0.1 to the total density (Zeldovich et al. 1980; Fargion et al. 1995). However, direct experimental search for cosmic fluxes of weakly interacting massive particles (WIMPs) may be sensitive to existence of such components (see Bernabei et al. 2000, 2003; Abrams et al. 2002; Akerib et al. 2005 and references therein). It was shown in Golubkov et al. (1999); Fargion (2000);

Belotsky & Khlopov (2002); Belotsky et al. (2008) that annihilation of the fourth neutrinos and their antineutrinos in the Galaxy can explain the galactic gamma-background, measured by EGRET in the range above 1 GeV, and that it can give some clue to explain the cosmic positron anomaly, claimed to be found by HEAT. Fourth neutrino annihilation inside the Earth should lead to the flux of underground monochromatic neutrinos of known types, which can be traced by the analysis of the already existing and future data of underground neutrino detectors (Belotsky & Khlopov 2002; Belotsky et al. 2001, 2002a,b). Decays of heavy WIMPs with lifetime $\tau > t_{\mathrm{U}}$ can be responsible for anomalies in spectra of high energy cosmic electrons and positrons (Nardi et al. 2009; Ibarra & Tran 2009; Ibarra et al. 2010).

New particles with electric charge and/or strong interaction can form anomalous atoms and be contained in ordinary matter as anomalous isotopes. Positively charged particles play the role of nuclei of anomalous atoms, while negatively charged particles with charge -1 are bound with primordial helium in an ion with charge $+1$, being a nucleus of anomalous hydrogen (Belotsky & Khlopov 2001; Belotsky et al. 2005; Fargion & Khlopov 2005; Khlopov et al. 2006b; Khlopov 2005b; Belotsky et al. 2006). To avoid contradiction with experimental searches for anomalous isotopes, such particles cannot be stable. However, particles with charge -2 can be stable, being bound with primordial helium in neutral atom like systems that can play the role of nontrivial composite nuclear-interacting dark matter, providing new types of nuclear transformations, giving rise to formation of primordial heavy elements and offering a possibility to solve the puzzles of direct dark matter searches (Khlopov et al. 2006b; Khlopov 2005b; Khlopov & Kouvaris 2008b; Khlopov 2007; Khlopov & Mankoc-Borstnik 2007; Khlopov 2008b,c; Khlopov & Kouvaris 2008a; Khlopov 2008a; Khlopov et al. 2008; Bregar et al. 2008).

Primordial unstable particles with lifetimes less than the age of the Universe, $\tau < t_{\mathrm{U}}$, cannot survive to the present time. However, if their lifetime is sufficiently large to satisfy the condition $\tau \gg (m_{\mathrm{pl}}/m) \cdot (1/m)$, their existence in the early Universe can lead to direct or indirect traces. Cosmological flux of decay products contribute to the cosmic and gamma ray backgrounds which represent the direct trace of unstable particles. If the decay products do not survive to the present time, their interaction with matter and radiation can cause indirect trace in the light element abundance or in the fluctuations of thermal radiation.

If a particle's lifetime is much less than 1 s, multi-step indirect traces are possible, provided that particles dominate in the Universe before their decay. During the dust-like stage of their dominance, black hole formation takes place, and the spectrum of such primordial black holes traces particle properties (mass, frozen concentration, lifetime) (Polnarev & Khlopov 1985). Particle decay in the end of the dust like stage influences the baryon asymmetry of the Universe. The final result is that cosmophenomenoLOGICAL chains link the predicted properties of even unstable new particles to the effects accessible in astronomical observations. Such effects may be important in analysis of the observational data.

Parameters of new stable and metastable particles are also determined by a pattern of particle symmetry breaking. This pattern is reflected in a succession of phase transitions in the early Universe. First order phase transitions proceed through bubble nucleation, which can result in black hole formation (see e.g. Konoplich et al. 1999; Khlopov & Rubin 2004 for review and references). Phase transitions of the second order can lead to formation of topological defects, such as walls, strings or monopoles. The observational data put severe constraints on a magnetic monopole (Zeldovich & Khlopov 1978) and cosmic wall production (Zel'Dovich et al. 1974), as well as on the parameters of cosmic strings (Zeldovich 1980; Vilenkin 1981). The structure of cosmological defects can be changed through a succession of phase transitions. More complicated forms, like walls surrounded by strings, can appear. Such structures can be unstable, but their existence can leave a trace in the nonhomogeneous distribution of dark matter and can give rise to large scale structures of nonhomogeneous dark matter like *archioles* (Sakharov & Khlopov 1994b; Sakharov et al. 1996; Khlopov et al. 1999). Primordial Black Holes represent a profound signature of such structures.

3 PBHS FROM EARLY DUST-LIKE STAGES

A possibility to form a black hole is highly improbable in a homogeneous expanding Universe, since it implies metric fluctuations of order 1. For metric fluctuations distributed according to a Gaussian law with dispersion

$$\left\langle \delta^2 \right\rangle \ll 1, \tag{1}$$

the probability for fluctuation of order 1 is determined by an exponentially small tail of the high amplitude part of this distribution. This probability can be even more suppressed in case of non-Gaussian fluctuations (Bullock & Primack 1997).

In the Universe with equation of state

$$p = \gamma \epsilon, \tag{2}$$

with numerical factor γ being in the range

$$0 \leq \gamma \leq 1, \tag{3}$$

the probability to form a black hole from fluctuations within the cosmological horizon is given by (see e.g. Khlopov 1999, 2004 for review and references)

$$W_{\mathrm{PBH}} \propto \exp\left(-\frac{\gamma^2}{2\left\langle \delta^2 \right\rangle}\right). \tag{4}$$

It provides exponential sensitivity of the PBH spectrum to softening of equation of state in the early Universe ($\gamma \to 0$) or to increase the ultraviolet part of the spectrum of density fluctuations ($\left\langle \delta^2 \right\rangle \to 1$). These phenomena can appear as cosmological consequences of particle theory.

3.1 Dominance of Superheavy Particles in the Early Universe

Superheavy particles cannot be studied by accelerators directly. If they are stable, their existence can be probed by cosmological tests, but there is no direct link between astrophysical data and existence of superheavy metastable particles with lifetimes $\tau \ll 1\,\mathrm{s}$. It was first noticed in Khlopov & Polnarev (1980) that dominance of such particles in the Universe before their decay at $t \leq \tau$ can result in formation of PBHs, which are retained in the Universe after the particles decay and keep some information on particle properties in their spectrum. Although indirect, it still provided a possibility to probe the existence of such particles in astrophysical observations. Even the absence of observational evidence for PBHs is important. It puts restrictions on allowed properties of superheavy metastable particles, which might form such PBHs at the stage of particle dominance, and thus constrain parameters of models, which are useful for predicting these particles.

After reheating, at

$$T < T_0 = rm, \tag{5}$$

particles with mass m and relative abundance $r = n/n_{\mathrm{r}}$ (where n is the frozen out concentration of particles and n_{r} is the concentration of relativistic species) must dominate the Universe before their decay. Dominance of these nonrelativistic particles at $t > t_0$, where

$$t_0 = \frac{m_{\mathrm{pl}}}{T_0^2}, \tag{6}$$

corresponds to the dust like stage with equation of state $p = 0$, at which particle density fluctuations grow as

$$\delta(t) = \frac{\delta\rho}{\rho} \propto t^{2/3}, \tag{7}$$

and development of gravitational instability results in formation of gravitationally bound systems, which decouple from general cosmological expansion at

$$t \sim t_{\mathrm{f}} \approx t_{\mathrm{i}} \delta(t_{\mathrm{i}})^{-3/2}, \tag{8}$$

when $\delta(t_{\mathrm{f}}) \sim 1$ for fluctuations, entering the horizon at $t = t_{\mathrm{i}} > t_0$ with amplitude $\delta(t_{\mathrm{i}})$.

Formation of these systems can result in black hole formation either immediately after the system decouples from expansion or as the result of evolution of the initially formed nonrelativistic gravitationally bound system.

If density fluctuation is especially homogeneous and isotropic, it directly collapses to a BH as soon as the amplitude of fluctuation grows to 1 and the system decouples from expansion. The probability for direct BH formation by the collapse of such homogeneous and isotropic configurations gives a minimal estimation of the probability of BH formation at the dust-like stage.

This probability was calculated in Khlopov & Polnarev (1980) with the use of the following arguments. In the period $t \sim t_{\mathrm{f}}$, when fluctuation decouples from expansion, its configuration is defined by averaged density ρ_1, size r_1, deviation from sphericity s and by inhomogeneity u of internal density distribution within the fluctuation. Having decoupled from expansion, the configuration contracts and the minimal size to which it can contract is

$$r_{\min} \sim s r_1, \tag{9}$$

which is determined by the deviation from sphericity

$$s = \max\{|\gamma_1 - \gamma_2|, |\gamma_1 - \gamma_3|, |\gamma_3 - \gamma_2|\}, \tag{10}$$

where γ_1, γ_2 and γ_3 define a deformation of configuration along its three main orthogonal axes. It was first noticed in Khlopov & Polnarev (1980) that in order to form a black hole as the result of such contraction, it is sufficient that the configuration returns to the size

$$r_{\min} \sim r_{\mathrm{g}} \sim t_{\mathrm{i}} \sim \delta(t_{\mathrm{i}}) r_1, \tag{11}$$

which had the initial fluctuation $\delta(t_{\mathrm{i}})$ when it entered the horizon at cosmological time t_{i}. If

$$s \leq \delta(t_{\mathrm{i}}), \tag{12}$$

configuration is sufficiently isotropic to concentrate its mass in the course of collapse within its gravitational radius; such concentration also implies sufficient homogeneity of the configuration. Density gradients can result in gradients of pressure, which can prevent collapse to a BH. This effect does not take place for a contracting collisionless gas composed of weakly interacting massive particles, but due to inhomogeneity of the process of collapse of the particles. The particles have already passed the caustics and can freely stream beyond the gravitational radius before the whole mass is concentrated within it. Collapse of a nearly spherically symmetric dust configuration is described by the Tolmen solution. Its analysis (Polnarev & Khlopov 1981, 1982; Khlopov & Polnarev 1983; Polnarev & Khlopov 1985) has provided a constraint on the inhomogeneity $u = \delta\rho_1/\rho_1$ within the configuration. It was shown that both for collisionless and interacting particles, the condition

$$u < \delta(t_{\mathrm{i}})^{3/2} \tag{13}$$

is sufficient for the configuration to contract to within its gravitational radius.

The probability for direct BH formation is then determined by the product of probabilities for sufficient initial sphericity W_s and homogeneity W_u of the configuration, which is determined by the phase space for such configurations. In the calculation of W_s, one should take into account that the condition (12) implies 5 conditions for independent components of tensor of deformation before its diagonalization (2 conditions for three diagonal components to be close to each other and 3 conditions for nondiagonal components to be small). Therefore, the probability of sufficient sphericity is given by Khlopov & Polnarev (1980); Polnarev & Khlopov (1981, 1982); Khlopov & Polnarev (1983); Polnarev & Khlopov (1985)

$$W_s \sim \delta(t_{\rm i})^5, \tag{14}$$

and together with the probability for sufficient homogeneity

$$W_u \sim \delta(t_{\rm i})^{3/2}, \tag{15}$$

results in the strong power-law suppression of probability for direct BH formation

$$W_{\rm PBH} = W_s \cdot W_u \sim \delta(t_{\rm i})^{13/2}. \tag{16}$$

Though this calculation was originally done in Khlopov & Polnarev (1980); Polnarev & Khlopov (1981, 1982); Khlopov & Polnarev (1983); Polnarev & Khlopov (1985) for a Gaussian distribution of fluctuations, it does not imply a specific form of the high amplitude tail of this distribution and thus should not change significantly in the case of non-Gaussian fluctuations (Bullock & Primack 1997).

The mechanism (Khlopov & Polnarev 1980; Polnarev & Khlopov 1981, 1982; Khlopov & Polnarev 1983; Polnarev & Khlopov 1985; Khlopov 1999, 2004) is effective for formation of PBHs with mass in an interval

$$M_0 \le M \le M_{\rm bhmax}. \tag{17}$$

The minimal mass corresponds to the mass within the cosmological horizon in the period $t \sim t_0$, when particles start to dominate in the Universe and it is equal to (Khlopov & Polnarev 1980; Polnarev & Khlopov 1981, 1982; Khlopov & Polnarev 1983; Polnarev & Khlopov 1985; Khlopov 1999, 2004)

$$M_0 = \frac{4\pi}{3}\rho t_0^3 \approx m_{\rm pl}\left(\frac{m_{\rm pl}}{rm}\right)^2. \tag{18}$$

The maximal mass is indirectly determined by the condition

$$\tau = t(M_{\rm bhmax})\delta(M_{\rm bhmax})^{-3/2}, \tag{19}$$

that fluctuations on the considered scale $M_{\rm bhmax}$, entering the horizon at $t(M_{\rm bhmax})$ with an amplitude $\delta(M_{\rm bhmax})$, can manage to increase to the nonlinear stage, decouple and collapse before particles decay at $t = \tau$. For a scale invariant spectrum $\delta(M) = \delta_0$, the maximal mass is given by Khlopov & Rubin (2004)

$$M_{\rm bhmax} = m_{\rm pl}\frac{\tau}{t_{\rm Pl}}\delta_0^{3/2} = m_{\rm pl}^2\tau\delta_0^{3/2}. \tag{20}$$

The probability, given by Equation (16), is also appropriate for formation of PBHs at the dust-like preheating stage after inflation (Khlopov et al. 1985a; Khlopov 1999, 2004). The simplest example of such a stage can be given with the use of a model of homogeneous massive scalar field (Khlopov 1999, 2004). Slow rolling of the field in the period $t \ll 1/m$ (where m is the mass of the field) leads to a chaotic inflation scenario, while at $t > 1/m$ the field oscillates with period $1/m$. Coherent oscillations of the field correspond to, on average, the dust-like equation of state $p = 0$, at which gravitational instability can develop. The minimal mass in this case corresponds to the Jeans mass of the scalar field, while the maximal mass is also determined by a condition that fluctuation grows and collapses before the scalar field decays and reheats the Universe.

The probability $W_{\rm PBH}(M)$ determines the fraction of total density

$$\beta(M) = \frac{\rho_{\rm PBH}(M)}{\rho_{\rm tot}} \approx W_{\rm PBH}(M), \tag{21}$$

corresponding to PBHs with mass M. For $\delta(M) \ll 1$, this fraction, given by Equation (16), is small. It means that the bulk of particles do not collapse directly into black holes, but form gravitationally bound systems. Evolution of these systems can yield a much larger amount of PBHs, but the result strongly depends on particle properties.

Superweakly interacting particles form gravitationally bound systems of collisionless gas, which are similar to modern galaxies which consist of a collisionless gas of stars. Such a system can finally collapse to a black hole, but energy dissipation in it and, consequently its evolution, is a relatively slow process (Zel'Dovich & Podurets 1966; Khlopov 1999, 2004). The evolution of these systems is dominantly determined by evaporation of particles, which gain velocities, exceeding the parabolic velocity of the system. In the case of binary collisions, the evolution timescale can be roughly estimated (Zel'Dovich & Podurets 1966; Khlopov 1999, 2004) as

$$t_{\rm ev} = \frac{N}{\ln N} t_{\rm ff} \tag{22}$$

for a gravitationally bound system of N particles, where the free fall time $t_{\rm ff}$ for the system with density ρ is $t_{\rm ff} \approx (4\pi G\rho)^{-1/2}$. This time scale can be shorter due to collective effects in collisionless gas (Gurzadian & Savvidy 1986) and, at large N, be of the order of

$$t_{\rm ev} \sim N^{2/3} t_{\rm ff}. \tag{23}$$

However, since the free fall time scale for the gravitationally bound systems of collisionless gas is of the order of cosmological time $t_{\rm f}$ for the period, when these systems are formed, even in the latter case the particles should have very long lifetimes of $\tau \gg t_{\rm f}$ to form black holes in a slow evolutional process.

The evolutional time scale is much smaller for gravitationally bound systems of superheavy particles, interacting with light relativistic particles and radiation. Such systems have an analogy with stars, in which evolution time scale is defined by energy loss by radiation. An example of such a particle is the superheavy color octet fermions of the asymptotically free SU(5) model (Kalashnikov & Khlopov 1983) or magnetic monopoles of the GUT models. Having decoupled from expansion, frozen out particles and antiparticles can annihilate in gravitationally bound systems, but detailed numerical simulation (Kadnikov et al. 1989) has shown that annihilation cannot prevent collapse of most of the mass and the timescale of collapse does not exceed the cosmological time of the period, when the systems are formed.

3.2 Spikes from Phase Transitions in the Inflationary Stage

A scale variant spectrum of fluctuations, in which the amplitude of small scale fluctuations is enhanced, can be another factor which increases the probability of PBH formation. The simplest functional form of such a spectrum is represented by a blue spectrum with a power law dispersion

$$\left\langle \delta^2(M) \right\rangle \propto M^{-k}, \tag{24}$$

with amplitude of fluctuations growing at $k > 0$ for small M. A realistic account of the existence of other scalar fields together with the field of inflaton in the period of inflation can give rise to spectra with distinguished scales, determined by parameters of considered fields and their interaction.

In the chaotic inflation scenario, interaction of a Higgs field ϕ with inflaton η can give rise to phase transitions at the inflationary stage, if this interaction induces a positive mass term $+\frac{\nu^2}{2}\eta^2\phi^2$. During the course of slow rolling, the amplitude of the field of inflaton decreases below a certain critical value $\eta_c = m_\phi/\nu$, the mass term in Higgs potential

$$V(\phi, \eta) = -\frac{m_\phi^2}{2}\phi^2 + \frac{\lambda_\phi}{4}\phi^4 + \frac{\nu^2}{2}\eta^2\phi^2 \tag{25}$$

changes sign and a phase transition takes place. Such phase transitions at the inflationary stage lead to the appearance of characteristic spikes in the spectrum of initial density perturbations. These spike–like perturbations re-enter the horizon during the radiation or dust like era and could, in principle, collapse to form primordial black holes. The possibility of such spikes in the chaotic inflation scenario was first pointed out in Kofman & Linde (1987) and developed in Sakharov & Khlopov (1993) as a mechanism of PBH formation for the model of horizontal unification (Berezhiani & Khlopov 1990a,b,c; Sakharov & Khlopov 1994a).

For the vacuum expectation value of a Higgs field

$$\langle \phi \rangle = \frac{m}{\lambda} = v, \tag{26}$$

and $\lambda \sim 10^{-3}$, the amplitude δ of a spike in the spectrum of density fluctuations, generated by the phase transition at the inflationary stage is given by Sakharov & Khlopov (1993)

$$\delta \approx \frac{4}{9s} \tag{27}$$

with

$$s = \sqrt{\frac{4}{9} + \kappa 10^5 \left(\frac{v}{m_{\rm pl}} \right)^2} - \frac{3}{2}, \tag{28}$$

where $\kappa \sim 1$.

If a phase transition takes place at e–folding N before the end of inflation and the spike re-enters the horizon during the radiation dominated (RD) stage, it forms a black hole of mass

$$M \approx \frac{m_{\rm Pl}^2}{H_0} \exp\{2N\}, \tag{29}$$

where H_0 is the Hubble constant during the period of inflation.

If the spike re-enters the horizon during the matter dominated (MD) stage, it should form black holes of mass

$$M \approx \frac{m_{\rm Pl}^2}{H_0} \exp\{3N\}. \tag{30}$$

4 FIRST ORDER PHASE TRANSITIONS AS A SOURCE OF BLACK HOLES IN THE EARLY UNIVERSE

First order phase transitions go through bubble nucleation. This is similar to the common example of boiling water. The simplest way to describe first order phase transitions with bubble creation in the early Universe is based on a scalar field theory with two non degenerated vacuum states. Being stable at a classical level, the false vacuum state decays due to quantum effects, leading to a nucleation of bubbles of the true vacuum and their subsequent expansion (Watkins & Widrow 1992). Effects of the cosmological expansion on the bubble nucleation rate are discussed in Metaxas (2008). The potential energy of the false vacuum is converted into a kinetic energy of bubble walls thus making them highly relativistic in a short time. The bubble expands till it collides with another one. As was shown in Hawking et al. (1982); Moss (1994), a black hole may be created in a collision of several bubbles. The probability for collision of two bubbles is much higher. The notion of the absence of BHs in such processes was based on strict conservation of the original O(2,1) symmetry. As was shown in Konoplich et al. (1999, 1998); Khlopov et al. (2000a), there are ways to break it. Firstly, radiation of scalar waves indicates the entropy is increasing and hence there is a permanent breaking of the symmetry during the bubble collision. Secondly, the vacuum decay due to thermal fluctuation does not possess this symmetry from the beginning. The investigations (Konoplich et al. 1999, 1998;

Khlopov et al. 2000a) have shown that BHs can be created as well with a probability of order unity in collisions of only two bubbles. This process initiates an enormous production of BHs that leads to essential cosmological consequences discussed below.

In Subsection 4.1 the evolution of the field configuration in the collisions of bubbles is discussed. The BH mass distribution is obtained in Subsection 4.2. In Subsection 4.3 cosmological consequences of BH production in bubble collisions at the end of inflation are considered.

4.1 Evolution of Field Configuration in Collisions of Vacuum Bubbles

Consider a theory where the probability of a false vacuum decay equals Γ and the difference of energy density between the false and true vacuum outside equals ρ_v. Initially, bubbles are produced at rest, however, the walls of the bubbles quickly increase their velocity up to the speed of light $v = c = 1$ because a conversion of the false vacuum energy into the corresponding kinetic one is energetically favorable.

Let us discuss dynamics of the collision of two true vacuum bubbles that have been nucleated in points $(\boldsymbol{r}_1, t_1), (\boldsymbol{r}_2, t_2)$ and which are expanding into a false vacuum. Following papers (Hawking et al. 1982; Coleman 1977) let us assume for simplicity that the horizon size is much greater than the distance between the bubbles. Just after collision, mutual penetration of the walls up to a distance comparable with its width is accompanied by a significant potential energy increase (Konoplich 1980). Then the walls reflect and accelerate backwards. The space between them is filled by the field in the false vacuum state converting the kinetic energy of the wall back to the energy of the false vacuum state and slows down the velocity of the walls. Meanwhile, the outer area of the false vacuum is absorbed by the outer wall, which expands and accelerates outwards. Evidently, there is an instant when the central region of the false vacuum is separated. Let us note that this false vacuum bag (FVB) does not possess spherical symmetry at the moment of its separation from outer walls but wall tension restores the symmetry during the first oscillation of the FVB. As was shown in Coleman (1977), further evolution of the FVB consists of several stages:

1) FVB grows to the definite size D_M until the kinetic energy of its wall becomes zero;

2) After this moment, the FVB begins to shrink to a minimal size D^*;

3) Secondary oscillation of the FVB occurs.

The process of periodical expansions and contractions leads to energy losses of the FVB in the form of quanta of scalar fields. It has been shown in Coleman (1977); Belova & Kudryavtsev (1988) that only a few oscillations take place. On the other hand, an important note is that the secondary oscillations might only occur if the minimal size of the FVB would be larger than its gravitational radius, $D^* > r_{\rm g}$. Then, oscillating solutions of "quasilumps" can happen (Dymnikova et al. 2000). The opposite case ($D^* < r_{\rm g}$) leads to a BH being created with mass of about the mass of the FVB. As was shown in Konoplich et al. (1999, 1998); Khlopov et al. (2000a), the probability of BH formation is almost unity in a wide range of parameters of theories with first order phase transitions.

4.2 Gravitational Collapse of FVB and BH Creation

Consider following Konoplich et al. (1999, 1998); Khlopov et al. (2000a); Khlopov & Rubin (2004); Khlopov (2004) in more detail about the conditions of converting a FVB into a BH. The mass M of a FVB can be calculated in a framework of a specific theory and can be estimated in a coordinate system K' where the colliding bubbles are nucleated simultaneously. The radius of each bubble b' in this system equals to half of their initial coordinate distance at the first moment of collision. Apparently, the maximum size $D_{\rm M}$ of the FVB is of the same order as the size of the bubble, since this is the only necessary parameter of dimension on such a scale: $D_{\rm M} = 2b'C$. The parameter $C \simeq 1$ is obtained by numerical calculations in the framework of each theory, but its exact numerical value does not significantly affect the conclusions.

One can find the mass of a FVB that arises at the collision of two bubbles of radius:

$$M = \frac{4\pi}{3}\left(Cb'\right)^3 \rho_v. \tag{31}$$

This mass is contained in the shrinking area of the false vacuum. Suppose for estimations that the minimum size of a FVB is of order of wall width Δ. The BH is created if the minimal size of a FVB is smaller than its gravitational radius. This means that, at least at the condition,

$$\Delta < r_{\rm g} = 2GM, \tag{32}$$

the FVB can be converted into a BH (where G is the gravitational constant).

As an example, consider a simple model with Lagrangian

$$L = \frac{1}{2}\left(\partial_\mu \Phi\right)^2 - \frac{\lambda}{8}\left(\Phi^2 - \Phi_0^2\right)^2 - \epsilon\Phi_0^3\left(\Phi + \Phi_0\right). \tag{33}$$

In the thin wall approximation, the width of the bubble wall can be expressed as $\Delta = 2\left(\sqrt{\lambda}\Phi_0\right)^{-1}$. Using Equation (32), one can easily derive that a FVB with mass of at least

$$M > \frac{1}{\sqrt{\lambda}\Phi_0 G} \tag{34}$$

should be converted into a BH of mass M. The last condition is valid only in the case when the FVB is completely contained within the cosmological horizon, namely $M_{\rm H} > 1/\sqrt{\lambda}\Phi_0 G$ where the mass of the cosmological horizon at the moment of phase transition is given by $M_{\rm H} \cong m_{\rm pl}^3/\Phi_0^2$. Thus for the potential (33) at the condition $\lambda > (\Phi_0/m_{\rm pl})^2$, a BH is formed. This condition is valid for any realistic set of parameters of the theory.

The mass and velocity distribution of FVBs, supposing its mass is large enough to satisfy the inequality (32), has been found in Konoplich et al. (1999, 1998); Khlopov et al. (2000a). This distribution can be written in terms of dimensionless mass $\mu \equiv \left(\frac{\pi}{3}\Gamma\right)^{1/4}\left(\frac{M}{C\rho_v}\right)^{1/3}$:

$$\begin{aligned} \frac{dP}{\Gamma^{-3/4} V dv d\mu} &= 64\pi\left(\tfrac{\pi}{3}\right)^{1/4}\mu^3 e^{\mu^4}\gamma^3 J(\mu, v), \\ J(\mu, v) &= \int_{\tau}^{\infty} d\tau e^{-\tau^4}, \tau_- = \mu\left[1 + \gamma^2\left(1 + v\right)\right]. \end{aligned} \tag{35}$$

The numerical integration of (35) revealed that the distribution is rather narrow. For example, the number of BHs with mass 30 times greater than the average one is suppressed by factor 10^5. The average value of the non dimensional mass is equal to $\mu = 0.32$. This allows us to relate the average mass of a BH and the volume containing the BH at the moment of the phase transition:

$$\langle M_{\rm BH}\rangle = \frac{C}{4}\mu^3\rho_v\langle V_{\rm BH}\rangle \simeq 0.012\rho_v\langle V_{\rm BH}\rangle. \tag{36}$$

4.3 First Order Phase Transitions in the Early Universe

Inflation models ending by a first order phase transition hold a dignified position in modern cosmology of the early Universe (see for example La & Steinhardt 1989; Holman et al. 1990, 1991; Adams & Freese 1991; Copeland et al. 1994; Occhionero & Amendola 1994; Amendola et al. 1996). The interest in these models is due to the fact that such models are able to generate the observed large-scale voids as remnants of the primordial bubbles for which the characteristic wavelengths are several tens of Mpc (Occhionero & Amendola 1994; Amendola et al. 1996). A detailed analysis of a first order phase transition in the context of extended inflation can be found in Turner et al. (1992). Hereafter, we will only be interested in the final stage of inflation when the phase transition is completed. We should remind ourselves that a first order phase transition is believed to happen immediately after establishing the true vacuum percolation regime. Such regime is established approximately when at least one bubble per unit Hubble volume is nucleated. Accurate computation (Turner et al. 1992) shows that a first order phase transition is successful if the following condition is valid:

$$Q \equiv \frac{4\pi}{9}\left(\frac{\Gamma}{H^4}\right)_{t_{\rm end}} = 1, \tag{37}$$

where Γ is the bubble nucleation rate. In the framework of first order inflation models, the filling of all space by a true vacuum takes place due to bubble collisions, nucleated at the final moment that exponential expansion occurs. The collisions between such bubbles occur when they have comoving spatial dimension less or equal to the effective Hubble horizon $H_{\rm end}^{-1}$ at the transition epoch. If we take $H_0 = 100h\,{\rm km\ s^{-1}\ Mpc^{-1}}$ and $\Omega = 1$ as parameters in the Universe, the comoving size of these bubbles is approximately $10^{-21}h^{-1}$ Mpc. In the standard approach, it is believed that such bubbles are rapidly thermalized without leaving a trace in the distribution of matter and radiation. However, in the previous subsection it has been shown that for any realistic parameters of the theory, the collision between only two bubbles leads to BH creation with probability close to 100% . The mass of this BH is given by (see Eq. (36))

$$M_{\rm BH} = \gamma_1 M_{\rm bub}, \tag{38}$$

where $\gamma_1 \simeq 10^{-2}$ and $M_{\rm bub}$ is the mass that could be contained in the bubble volume at the epoch of collision in the condition of a full thermalization of bubbles. The discovered mechanism leads to a new direct possibility of PBH creation at the epoch of reheating in first order inflation models. In the standard picture, PBHs are formed in the early Universe if density perturbations are sufficiently large, and the probability of PBHs forming from small post- inflation initial perturbations is suppressed (see Sect. 3). A completely different situation takes place at the final epoch of the first order inflation stage, namely collisions between bubbles of Hubble size in the percolation regime leads to copious PBH formation with masses

$$M_0 = \gamma_1 M_{\rm end}^{\rm hor} = \frac{\gamma_1}{2}\frac{m_{\rm pl}^2}{H_{\rm end}}, \tag{39}$$

where $M_{\rm end}^{\rm hor}$ is the mass of the Hubble horizon at the end of inflation. According to Equation (36), the initial mass fraction of these PBHs is given by $\beta_0 \approx \gamma_1/e \approx 6\times10^{-3}$. For example, for a typical value of $H_{\rm end} \approx 4\times10^{-6}m_{\rm pl}$ the initial mass fraction β is contained in PBHs with mass $M_0 \approx 1$ g.

In general, the Hawking evaporation of mini BHs could give rise to a variety possible end states. It is generally assumed that evaporation proceeds until the PBH vanishes completely (Hawking 1974), but there are various arguments against this proposal (see e.g. Barrow et al. 1992; Carr et al. 1994; Alexeyev & Pomazanov 1997; Dymnikova 1996). If one supposes that BH evaporation leaves a stable relic, then it is natural to assume that it has a mass of order $m_{\rm rel} = km_{\rm pl}$, where $k \simeq 1/10^2$. We can investigate the consequences of a PBH forming at the percolation epoch after first order inflation, supposing that the stable relic is a result of its evaporation. Following the above consideration, the PBHs are preferentially formed with a typical mass M_0 at a single time t_1. Hence, the total density ρ at this time is

$$\rho(t_1) = \rho_\gamma(t_1) + \rho_{\rm PBH}(t_1) = \frac{3(1-\beta_0)}{32\pi t_1^2}m_{\rm pl}^2 + \frac{3\beta_0}{32\pi t_1^2}m_{\rm pl}^2, \tag{40}$$

where β_0 denotes the fraction of the total density corresponding to PBHs in the period of their formation t_1. The evaporation time scale can be written in the following form

$$\tau_{\rm BH} = \frac{M_0^3}{g_* m_{\rm pl}^4}, \tag{41}$$

where g_* is the number of effective massless degrees of freedom.

Let us derive the density of PBH relics. There are two distinct possibilities to consider.

The Universe is still radiation dominated (RD) at $\tau_{\rm BH}$. This situation will hold if the following condition is valid: $\rho_{\rm BH}(\tau_{\rm BH}) < \rho_\gamma(\tau_{\rm BH})$. It is possible to rewrite this condition in terms of the Hubble constant at the end of inflation

$$\frac{H_{\rm end}}{m_{\rm pl}} > \beta_0^{5/2} g_*^{-1/2} \simeq 10^{-6}. \tag{42}$$

Taking the present radiation density fraction of the Universe to be $\Omega_{\gamma_0} = 2.5 \times 10^{-5} h^{-2}$ (h is the Hubble constant in units of 100 km s^{-1} Mpc^{-1}), and using the standard values for the present time and time when the density of matter and radiation become equal, we find the current density fraction of relics to be

$$\Omega_{\rm rel} \approx 10^{26} h^{-2} k \left(\frac{H_{\rm end}}{m_{\rm pl}}\right)^{3/2}. \tag{43}$$

It is easy to see that relics overclose the Universe ($\Omega_{\rm rel} \gg 1$) for any reasonable k and $H_{\rm end} > 10^{-6} m_{\rm pl}$.

The second case takes place if the Universe becomes PBH dominated at period $t_1 < t_2 < \tau_{\rm BH}$. This situation occurs under the condition $\rho_{\rm BH}(t_2) > \rho_\gamma(t_2)$, which can be rewritten in the form

$$\frac{H_{\rm end}}{m_{\rm pl}} < 10^{-6}. \tag{44}$$

The present day density fraction of relics takes the form

$$\Omega_{\rm rel} \approx 10^{28} h^{-2} k \left(\frac{H_{\rm end}}{m_{\rm pl}}\right)^{3/2}. \tag{45}$$

Thus, the Universe is not overclosed by relics only if the following condition is valid

$$\frac{H_{\rm end}}{m_{\rm pl}} \le 2 \cdot 10^{-19} h^{4/3} k^{-2/3}. \tag{46}$$

This condition implies that the masses of PBHs created at the end of inflation have to be larger than

$$M_0 \ge 10^{11} g \cdot h^{-4/3} \cdot k^{2/3}. \tag{47}$$

On the other hand, there are a number of well–known cosmological and astrophysical limits (Zeldovich & Starobinskii 1976; Miyama & Sato 1978; Naselskii 1978; Lindley 1980; Zeldovich et al. 1977; Rothman & Matzner 1981; MacGibbon & Carr 1991) which prohibit the creation of PBHs in the mass range (47) with initial fraction of mass density close to $\beta_0 \approx 10^{-2}$.

One has to conclude that the effect of the false vacuum bag mechanism of PBH formation prevents the coexistence of stable remnants of PBH evaporation with the first order phase transitions at the end of inflation.

5 GRAVITINO PRODUCTION BY PBH EVAPORATION AND CONSTRAINTS ON THE INHOMOGENEITY OF THE EARLY UNIVERSE

Presently, there is no observational evidence proving the existence of PBHs. However, even the absence of PBHs provides a very sensitive theoretical tool to study physics of the early Universe. PBHs represent a nonrelativistic form of matter and their density decreases with scale factor a as

$\propto a^{-3} \propto T^3$, while the total density is $\propto a^{-4} \propto T^4$ in the period of radiation dominance (RD). Being formed within the horizon, a PBH of mass M cannot be formed earlier than time

$$t(M) = \frac{M}{m_{\rm pl}} t_{\rm pl} = \frac{M}{m_{\rm pl}^2}. \tag{48}$$

If they are formed at the RD stage, the smaller the masses of PBHs are, the larger their relative contribution is to the total density of the modern MD stage. Therefore, even the modest constraint for PBHs of mass M on their density

$$\Omega_{\rm PBH}(M) = \frac{\rho_{\rm PBH}(M)}{\rho_c}, \tag{49}$$

in units of critical density $\rho_c = 3H^2/(8\pi G)$ from the condition that their contribution $\alpha(M)$ to the the total density $\rho_{\rm tot} = \rho_c$,

$$\alpha(M) \equiv \frac{\rho_{\rm PBH}(M)}{\rho_{\rm tot}} = \Omega_{\rm PBH}(M) \tag{50}$$

does not exceed the density of dark matter

$$\alpha(M) = \Omega_{\rm PBH}(M) \leq \Omega_{\rm DM} = 0.23, \tag{51}$$

becomes a severe constraint on this contribution

$$\beta \equiv \frac{\rho_{\rm PBH}(M, t_{\rm f})}{\rho_{\rm tot}(t_{\rm f})}, \tag{52}$$

in the period $t_{\rm f}$ of their formation. If formed during the RD stage at $t_f = t(M)$, given by Equation (48), which corresponds to the temperature $T_{\rm f} = m_{\rm pl}\sqrt{m_{\rm pl}/M}$, PBHs contribute to the total density at the end of the RD stage when $t = t_{\rm eq}$, corresponding to $T_{\rm eq} \approx 1\,{\rm eV}$, by factor $a(t_{\rm eq})/a(t_{\rm f}) = T_{\rm f}/T_{\rm eq} = m_{\rm pl}/T_{\rm eq}\sqrt{m_{\rm pl}/M}$ larger than in the period of their formation. The constraint on $\beta(M)$, following from Equation (51), is then given by

$$\beta(M) = \alpha(M)\frac{T_{\rm eq}}{m_{\rm pl}}\sqrt{\frac{M}{m_{\rm pl}}} \leq 0.23\frac{T_{\rm eq}}{m_{\rm pl}}\sqrt{\frac{M}{m_{\rm pl}}}. \tag{53}$$

The possibility of PBH evaporation, revealed by S. Hawking (Hawking 1975), strongly influences effects of PBHs. In the strong gravitational field near gravitational radius $r_{\rm g}$ of a PBH, the quantum effect of creation of particles with momentum $p \sim 1/r_{\rm g}$ is possible. Due to this effect, a PBH turns into a black body source of particles with temperature (in the units $\hbar = c = k = 1$)

$$T = \frac{1}{8\pi G M} \approx 10^{13}{\rm GeV}\frac{1{\rm g}}{M}. \tag{54}$$

The evaporation timescale for the BH is $\tau_{\rm BH} \sim M^3/m_{\rm pl}^4$ (see Eq. (41) and discussion in previous section) and at $M \leq 10^{14}$ g is less than the age of the Universe. Such PBHs cannot survive to the present time and the magnitude, Equation (51), for them should be re-defined and has the meaning of contributing to the total density at the moment of PBH evaporation. For PBHs formed during the RD stage and which evaporated at the RD stage at $t < t_{\rm eq}$, the relationship of Equation (53) between $\beta(M)$ and $\alpha(M)$ is given by Novikov et al. (1979); Polnarev & Khlopov (1985)

$$\beta(M) = \alpha(M)\frac{m_{\rm pl}}{M}. \tag{55}$$

The relationship between $\beta(M)$ and $\alpha(M)$ has a more complicated form if PBHs are formed in the early dust-like stages (Polnarev & Khlopov 1982, 1985; Chechetkin et al. 1982a; Khlopov 1999), or such stages take place after PBH formation (Chechetkin et al. 1982a; Khlopov 1999). Relative contribution of PBHs to total density does not grow during the dust-like stage and the relationship between $\beta(M)$ and $\alpha(M)$ depends on details of the considered model. The minimum model independent factor $\alpha(M)/\beta(M)$ follows from the account for enhancement, taking place only during the RD stage between the first second of expansion and the end of the RD stage at t_{eq}, since radiation dominance in this period is supported by observations of light element abundance and the spectrum of the CMB (Polnarev & Khlopov 1982, 1985; Chechetkin et al. 1982a; Khlopov 1999).

Effects of PBH evaporation make astrophysical data much more sensitive to the existence of PBHs. Constraining the abundance of primordial black holes can lead to invaluable information on cosmological processes, particularly as they are probably the only viable probe for the power spectrum on very small scales which remain far from the sensitivity ranges of the Cosmological Microwave Background (CMB) and Large Scale Structures (LSS). To date, only PBHs with initial masses between $\sim 10^9$ g and $\sim 10^{16}$ g have led to stringent limits (see e.g. Carr et al. 1994; Carr & MacGibbon 1998; Liddle & Green 1998; Polnarev & Khlopov 1985) from consideration of the entropy per baryon, the deuterium destruction, the ^{4}He destruction and the cosmic-rays currently emitted by the Hawking process (Hawking 1975). The existence of light PBHs should lead to important observable constraints, either through the direct effects of the evaporated particles (for initial masses between 10^{14} g and 10^{16} g) or through the indirect effects of their interaction with matter and radiation in the early Universe (for PBH masses between 10^9 g and 10^{14} g). Under these constraints, the effects taken into account are those related to known particles. However, since the evaporation products are created by the gravitational field, any quantum with a mass lower than the black hole temperature should be emitted, independent of the strength of its interaction. This could provide a copious production of superweakly interacting particles that cannot be in equilibrium with the hot plasma of the very early Universe. It makes evaporating PBHs a unique source of all the species which can exist in the Universe.

Following Khlopov (1999, 2004); Chechetkin et al. (1982a); Khlopov & Chechetkin (1987); Lemoine (2000); Green (1999) (but in a different framework and using more stringent constraints), limits on the mass fraction of black holes at the time of their formation ($\beta \equiv \rho_{\mathrm{PBH}}/\rho_{\mathrm{tot}}$) were derived in Khlopov et al. (2006a) using the production of gravitinos during the evaporation process. Depending on whether gravitinos are expected to be stable or metastable, the limits are obtained using the requirement that they do not overclose the Universe and that the formation of light nuclei by the interactions of ^{4}He nuclei with nonequilibrium flux of D, T, ^{3}He and ^{4}He does not contradict the observations. This approach is more constraining than the usual study of photo-dissociation induced by photons-photino pairs emitted by decaying gravitinos. It opened a new window for the upper limits on β below 10^9 g. The cosmological consequences of the limits, obtained in Khlopov et al. (2006a), are briefly reviewed in the framework of three different scenarios: a blue power spectrum, a step in the power spectrum and first order phase transitions.

5.1 Limits on the PBH Density

Several constraints on the density of PBHs have been derived in different mass ranges assuming the evaporation of only standard model particles: for 10^9 g $< M < 10^{13}$ g the entropy per baryon at nucleosynthesis was used (Miyama & Sato 1978) to obtain $\beta < (10^9\ \mathrm{g}/M)$, for 10^9 g $< M < 10^{11}$ g the production of $n\bar{n}$ pairs at nucleosynthesis was used (Zeldovich et al. 1977) to obtain $\beta < 3 \times 10^{-17}(10^9\ \mathrm{g}/M)^{1/2}$, for 10^{10} g $< M < 10^{11}$ g deuterium destruction was used (Lindley 1980) to obtain $\beta < 3 \times 10^{-22}(M/10^{10}\ \mathrm{g})^{1/2}$, for 10^{11} g $< M < 10^{13}$ g spallation of ^{4}He was used (Vainer et al. 1978; Chechetkin et al. 1982a) to obtain $\beta < 3 \times 10^{-21}(M/10^9\ \mathrm{g})^{5/2}$, for $M \approx 5 \times 10^{14}$ g the gamma-rays and cosmic-rays were used (MacGibbon & Carr 1991; Barrau et al. 2003a) to obtain $\beta < 10^{-28}$. Slightly more stringent limits were obtained in Kohri & Yokoyama (2000), leading to $\beta < 10^{-20}$ for masses between 10^9 g and 10^{10} g and in Barrau et al. (2003b), leading to $\beta < 10^{-28}$ for $M = 5 \times 10^{11}$ g. Gamma-rays and antiprotons were also recently reanalyzed in Barrau et al. (2002); Custódio & Horvath (2002), improving the previous estimates a little. Such constraints, related to phenomena occurring after the nucleosynthesis, apply only for black holes with initial masses above $\sim 10^9$ g. Below this value, the only limits are the very weak entropy constraint (related to the photon-to-baryon ratio) and the constraint which assumes that stable remnants of black holes form at the end of the evaporation mechanism as described in the previous section.

To derive a limit in the initial mass range $m_{\rm pl} < M < 10^{11}$ g, gravitinos emitted by black holes were considered in Khlopov et al. (2006a). Gravitinos are expected to be present in all local supersymmetric models, which are regarded as the more natural extensions of the standard model of high energy physics (see, e.g., Olive 2000 for an introductory review). In the framework of minimal Supergravity (mSUGRA), the gravitino mass is, by construction, expected to lie around the electroweak scale, i.e. in the 100 GeV range. In this case, the gravitino is *metastable* and decays after nucleosynthesis, leading to important modifications of the nucleosynthesis paradigm. Instead of using the usual photon-photino decay channel, the study of Khlopov et al. (2006a) relied on the more sensitive gluon-gluino channel. Based on Khlopov & Linde (1984); Balestra et al. (1884); Levitan et al. (1988); Khlopov et al. (1994); Sedel'Nikov et al. (1995), the antiprotons produced by the fragmentation of gluons emitted by decaying gravitinos were considered as a source of nonequilibrium light nuclei resulting from collisions of those antiprotons with equilibrium nuclei. Then, ^{6}Li, ^{7}Li and ^{7}Be nuclei production by the interactions of the nonequilibrium nuclear flux with ^{4}He equilibrium nuclei was taken into account and compared with data (this approach is supported by several recent analyses, such as Jedamzik 2004; Kawasaki et al. 2005, which lead to similar results). The resulting Monte-Carlo estimates (Khlopov et al. 1994) lead to the following constraint on the concentration of gravitinos: $n_{3/2} < 1.1 \times 10^{-13} m_{3/2}^{-1/4}$, where $m_{3/2}$ is the gravitino mass in GeV. This constraint has been successfully used to derive an upper limit on the reheating temperature of the order (Khlopov et al. 1994): $T_R < 3.8 \times 10^6$ GeV. The consequences of this limit on cosmic-rays emitted by PBHs was considered, e.g., in Barrau & Ponthieu (2004). In the approach of Khlopov et al. (2006a), this stringent constraint on the gravitino abundance was related to the density of PBHs through direct gravitino emission. The usual Hawking formula (Hawking 1975) was used for the number of particles of type i emitted per unit of time t and per unit of energy Q. Introducing the temperature defined by Equation (54), $T = hc^3/(16\pi^2 kGM) \approx (10^{13}\text{g})/M$ GeV, taking the relativistic approximation for Γ_s, and integrating over time and energy, the total number of quanta of type i can be estimated as:

$$N_i^{\rm TOT} = \frac{27 \times 10^{24}}{64\pi^3 \alpha_{\rm SUGRA}} \int_{T_{\rm i}}^{T_{\rm Pl}} \frac{dT}{T^3} \int_{m/T}^{x} \frac{x^2 dx}{e^x - (-1)^s}, \tag{56}$$

where T is in GeV, $m_{\rm pl} \approx 10^{-5}$ g, $x \equiv Q/T$, m is the particle mass and $\alpha_{\rm SUGRA}$ accounts for the number of degrees of freedom through $M^2 dM = -\alpha_{\rm SUGRA} dt$ where M is the black hole mass. Once the PBH temperature is higher than the gravitino mass, gravitinos will be emitted with a weight related to their number of degrees of freedom. Computing the number of emitted gravitinos as a function of the PBH initial mass and matching it with the limit on the gravitino density imposed by nonequilibrium nucleosynthesis of light elements leads to an upper limit on the PBH number density. If PBHs are formed during the RD stage, this limit can easily be converted into an upper limit on β by evaluating the energy density of the radiation at the formation epoch. The resulting limit is shown in Figure 1 and leads to an important improvement over previous limits, nearly independent of the gravitino mass in the interesting range. This opens a new window on the very small scales in the early Universe.

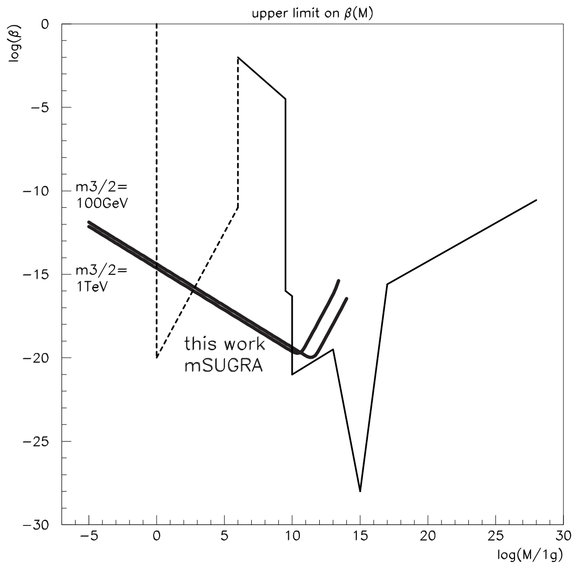

Fig. 1 Constraints of Khlopov et al. (2006a) on the fraction of the Universe going into PBHs (adapted from Carr et al. 1994; Carr & MacGibbon 1998; Liddle & Green 1998; Polnarev & Khlopov 1985). The two curves obtained with gravitino emission in mSUGRA correspond to $m_{3/2}$ = 100 GeV (*lower curve in the high mass range*) and $m_{3/2}$ = 1 TeV (*upper curve in the high mass range*).

It is also possible to consider limits arising in Gauge Mediated Susy Breaking (GMSB) models (Kolda 1998). Those alternative scenarios, incorporating a natural suppression of the rate of flavor-changing neutral-current due to the low energy scale, predict the gravitino to be the Lightest Supersymmetric Particle (LSP). The LSP is stable if R-parity is conserved. In this case, the limit was obtained (Khlopov et al. 2006a) by requiring $\Omega_{3/2,0} < \Omega_{M,0}$, i.e. by requiring that the current gravitino density does not exceed the matter density. It can easily be derived from the previous method by taking into account the dilution of gravitinos in the period of PBH evaporation and conservation of gravitinos with respect to specific entropy ratio, that is (Khlopov et al. 2006a):

$$\beta \leq \frac{\Omega_{M,0}}{N_{3/2}\frac{m_{3/2}}{M}\left(\frac{t_{\rm eq}}{t_{\rm f}}\right)^{\frac{1}{2}}}, \tag{57}$$

where $N_{3/2}$ is the total number of gravitinos emitted by a PBH with initial mass M, $t_{\rm eq}$ is the end of the RD stage and $t_{\rm f} = \max(t_{\rm form}, t_{\rm end})$ when a non-trivial equation of state for the period of PBH formation is considered, e.g. a dust-like phase which ends at $t_{\rm end}$ (Polnarev & Khlopov 1982). The limit (57) does not imply thermal equilibrium of relativistic plasma in the period before PBH evaporation and is valid even for low reheating temperatures provided that the equation of state for the preheating stage is close to relativistic. With the present matter density $\Omega_{M,0} \approx 0.27$ (Spergel et al. 2003) this leads to the limit shown in Figure 2 for $m_{3/2} = 10$ GeV. Following (57), this limit scales with gravitino mass as $\propto m_{3/2}^{-1}$. Models of gravitino dark matter with $\Omega_{3/2,0} = \Omega_{\rm CDM,0}$, corresponding to the case of equality in the above formula, were recently considered in Jedamzik et al. (2006); Lemoine et al. (2007).

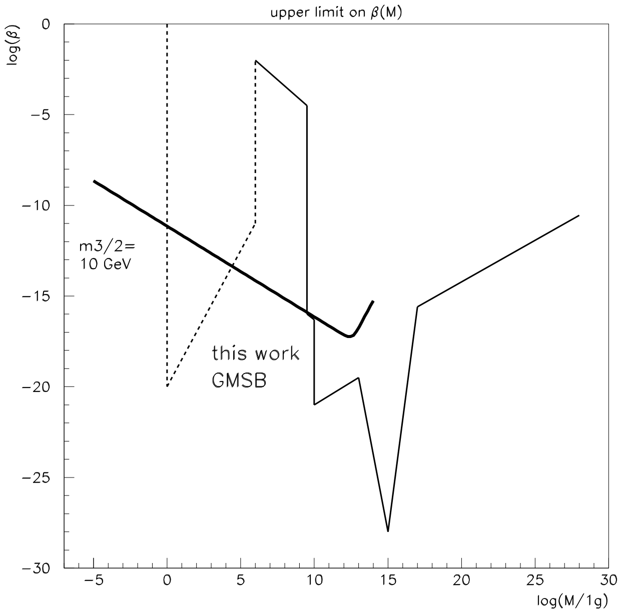

Fig. 2 Constraints of Khlopov et al. (2006a) on the fraction of the Universe going into PBHs (adapted from Carr et al. 1994; Carr & MacGibbon 1998; Liddle & Green 1998; Polnarev & Khlopov 1985). The curve obtained with gravitino emission in GMSB corresponds to $m_{3/2} = 10\,\mathrm{GeV}$ and scales with gravitino mass as $\propto m_{3/2}^{-1}$.

5.2 Cosmological Consequences

Upper limits on the fraction of the Universe in primordial black holes can be converted into cosmological constraints on models with significant power on small scales (Khlopov et al. 2006a).

The easiest way to illustrate the importance of such limits is to consider a blue power spectrum and to derive a related upper value on the spectral index n of scalar fluctuations ($P(k) \propto k^n$). It has recently been shown by WMAP (Spergel et al. 2003) that the spectrum is nearly of the Harrison-Zel'dovich type, i.e. scale invariant with $n \approx 1$. However, this measure was obtained for scales between 10^{45} and 10^{60} times larger that those probed by PBHs and it remains very important to probe the power available on small scales. The limit on n given in Khlopov et al. (2006a) must therefore be understood as a way to constrain $P(k)$ at small scales rather than a way to measure its derivative at large scales; it is complementary to CMB measurements. Using the usual relations between the mass variance at the PBH formation time $\sigma_{\mathrm{H}}(t_{\mathrm{form}})$ and the same quantity today $\sigma_{\mathrm{H}}(t_0)$ (Green & Liddle 1997),

$$\sigma_{\mathrm{H}}(t_{\mathrm{form}}) = \sigma_{\mathrm{H}}(t_0) \left[\frac{M_{\mathrm{H}}(t_0)}{M_{\mathrm{H}}(t_{\mathrm{eq}})}\right]^{\frac{n-1}{6}} \left[\frac{M_{\mathrm{H}}(t_{\mathrm{eq}})}{M_H(t_{\mathrm{form}})}\right]^{\frac{n-1}{4}}, \tag{58}$$

where $M_{\mathrm{H}}(t)$ is the Hubble mass at time t and t_{eq} is the equilibrium time; it is possible to set an upper value on β which can be expressed as

$$\beta \approx \frac{\sigma_H(t_{\mathrm{form}})}{\sqrt{2\pi}\delta_{\mathrm{min}}} e^{-\frac{\delta_{\mathrm{min}}^2}{2\sigma_{\mathrm{H}}^2(t_{\mathrm{form}})}}, \tag{59}$$

where $\delta_{\min} \approx 0.3$ is the minimum density contrast required to form a PBH. The limit derived in the previous subsection leads to $n < 1.20$ in the mSUGRA case whereas the usually derived limits range between 1.23 and 1.31 (Green & Liddle 1997; Bringmann et al. 2002; Kim et al. 1999). In the GMSB case, it remains at the same level for $m_{3/2} \sim 10$ GeV and is slightly relaxed for smaller gravitino masses. This improvement is due to the much more important range of masses probed by the method (Khlopov et al. 2006a).

In the standard cosmological paradigm of inflation, the primordial power spectrum is expected to be nearly, but not exactly, scale invariant (Liddle & Lyth 2000). The sign of the running can, in principle, be either positive or negative. It has been recently shown that models with a positive running α_s, defined as

$$P(k) = P(k_0)\left(\frac{k}{k_0}\right)^{n_s(k_0)+\frac{1}{2}\alpha_s \ln\left(\frac{k}{k_0}\right)}, \qquad (60)$$

are very promising in the framework of supergravity inflation (see, e.g., Kawasaki et al. 2003). The analysis (Khlopov et al. 2006a) strongly limits a positive running, setting the upper bound at a tiny value $\alpha_s < 2 \times 10^{-3}$. This result is more stringent than the upper limit obtained through a combined analysis of the Lyα forest, SDSS and WMAP data (Seljak et al. 2005), $-0.013 < \alpha_s < 0.007$, as it deals with scales very far from those probed by usual cosmological observations. The order of magnitude of the running naturally expected in most models, either inflationary ones (see, e.g., Peiris et al. 2003) or alternative ones (see, e.g., Khoury et al. 2003), of a few times 10^{-3} for our upper bound, should help to distinguish between different scenarios.

In the case of an early dust-like stage in cosmological evolution (Khlopov & Polnarev 1980; Polnarev & Khlopov 1985; Khlopov 1999, 2004), the PBH formation probability is increased to $\beta > \delta^{13/2}$ where δ is the density contrast for the considered small scales (see Subsect. 3.1). The associated limit on n is strengthened to $n < 1.19$.

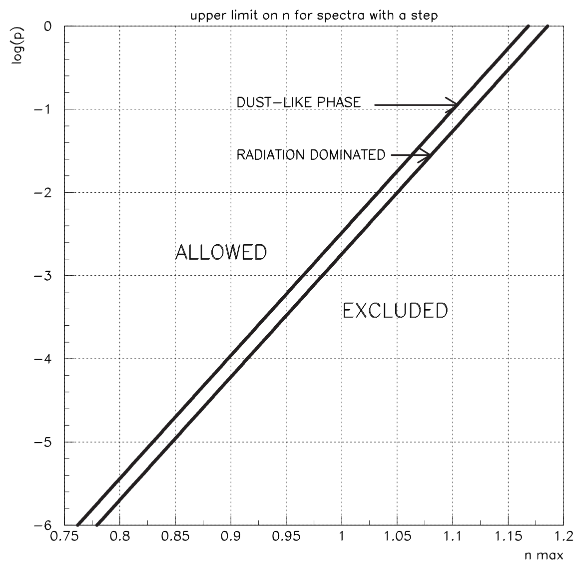

Fig. 3 Upper limit from Khlopov et al. (2006a) on the spectral index of the power spectrum as a function of the amplitude of the step.

Following Green & Liddle (1997), it is also interesting to consider primordial density perturbation spectra with both a tilt and a step. Such a feature can arise from underlying physical processes (Starobinskij 1992) and allows investigation of a wider class of inflation potentials. If the amplitude of the step is defined so that the power on small scales is p^{-2} times higher than the power on large scales, the maximum allowed value for the spectral index can be computed as a function of p. Figure 3, taken from Khlopov et al. (2006a), shows those limits, which become extremely stringent when p is small enough, for both the radiation-dominated and dust-like cases.

Another important consequence of limits that Khlopov et al. (2006a) discusses concerns PBH relics which form dark matter (see also discussion in Subsec. 4.3). The idea, introduced in MacGibbon (1987), that relics possibly formed at the end of the evaporation process could account for the cold dark matter has been extensively studied. The amplitude of the power boost required on small scales has been derived, e.g., in Barrau et al. (2004a) as a function of the relic mass and of the expected density. The main point was that the "step" (or whatever structure is in the power spectrum) should occur at low masses to avoid the constraints available between 10^9 g and 10^{15} g. The limit on β derived in Khlopov et al. (2006a) closes this dark matter issue except within a small window below 10^3 g.

This result can be re-formulated in a more general way. If the nature of cosmological dark matter is related with superweakly interacting particles, which cannot be present in equilibrium in the early Universe and for which nonequilibrium processes of production, e.g. in reheating, are suppressed, the early Universe should be sufficiently homogeneous on small scales to exclude copious creation of these species in miniPBH vaporation.

Finally, the limits (Khlopov et al. 2006a) also completely exclude the possibility of a copious PBH formation process in bubble wall collisions (Konoplich et al. 1999, 1998; Khlopov et al. 2000a), considered in the previous section. This has important consequences for the related constraints on first order phase transitions in the early Universe and on the symmetry breaking pattern of particle theory.

6 MASSIVE PRIMORDIAL BLACK HOLES FROM THE COLLAPSE OF CLOSED WALLS

A wide class of particle models possesses a symmetry breaking pattern, which can be effectively described by a pseudo-Nambu–Goldstone (PNG) field and which corresponds to the formation of unstable topological defect structures in the early Universe (see Khlopov & Rubin 2004 for review and references). The Nambu–Goldstone nature in such an effective description reflects the spontaneous breaking of global U(1) symmetry, resulting in continuous degeneracy of vacua. The explicit symmetry breaking at smaller energy scales changes this continuous degeneracy by discrete vacuum degeneracy. The character of formed structures is different for phase transitions, taking place at post-inflationary and inflationary stages.

6.1 Structures from Succession of U(1) Phase Transitions

At high temperatures, such a symmetry breaking pattern implies the succession of second order phase transitions. In the first transition, continuous degeneracy of vacua leads, at scales exceeding the correlation length, to the formation of topological defects in the form of a string network; in the second phase transition, continuous transitions in space between degenerated vacua form surfaces: domain walls surrounded by strings. This last structure is unstable, but, as was shown in the example of the invisible axion (Sakharov & Khlopov 1994b; Sakharov et al. 1996; Khlopov et al. 1999), it is reflected in the large scale inhomogeneity of the distribution of energy density of coherent PNG (axion) field oscillations. This energy density is proportional to the initial value of phase, which acquires dynamical meaning of the amplitude of the axion field, when axion mass is switched on as the result of the second phase transition.

The value of phase changes by 2π around a string. This strong inhomogeneity of phase, leading to corresponding inhomogeneity of energy density of coherent PNG (axion) field oscillations, is usually considered (see e.g. Kim 1987; Sikivie 2008 and references therein) only on scales corresponding to the mean distance between strings. This distance is small, being of the order of the scale of the cosmological horizon in the period when PNG field oscillations start. However, since the inhomogeneity of phase follows the pattern of the axion string network, this argument misses large scale correlations in the distribution of oscillations' energy density.

Indeed, numerical analysis of the string network (see review in Vilenkin & Shellard 1994) indicates that large string loops are strongly suppressed and the fraction of about 80% of string lengths, corresponding to long loops, remains virtually the same in all large scales. This property is the other side of the well known scale invariant character of the string network. Therefore, the correlations of energy density should persist on large scales, as was revealed in Sakharov & Khlopov (1994b); Sakharov et al. (1996); Khlopov et al. (1999).

The large scale correlations in topological defects and their imprints in primordial inhomogeneities is the indirect effect of inflation, if phase transitions take place after reheating of the Universe. In this case, inflation provides equal conditions for phase transition, taking place in causally disconnected regions.

If phase transitions take place at the inflationary stage, new forms of primordial large scale correlations appear. The value of phase after the first phase transition is inflated over the region corresponding to the period of inflation, while fluctuations of this phase change in the course of inflation with its initial value staying within the regions of smaller size. Owing to such fluctuations, for the fixed value of θ_{60} in the period of inflation with *e-folding* $N = 60$ corresponding to the part of the Universe within the modern cosmological horizon, strong deviations from this value appear at smaller scales, corresponding to later periods of inflation with $N < 60$. If $\theta_{60} < \pi$, the fluctuations can move the value of θ_N to $\theta_N > \pi$ in some regions of the Universe. After reheating as the result of the second phase transition, these regions correspond to a vacuum with $\theta_{\rm vac} = 2\pi$, being surrounded by the bulk of the volume with vacuum $\theta_{\rm vac} = 0$. As a result, massive walls are formed at the border between the two vacua. Since regions with $\theta_{\rm vac} = 2\pi$ are confined, the domain walls are closed. After their size equals the horizon, closed walls can collapse into black holes.

This mechanism can lead to formation of primordial black holes of a whatever large mass (up to the mass of AGNs (Rubin et al. 2001), see for latest review Dokuchaev et al. 2007). Such black holes appear in the form of primordial black hole clusters, exhibiting fractal distributions in space (Rubin et al. 2000; Khlopov et al. 2005; Khlopov & Rubin 2004). This can shed new light on the problem of galaxy formation (Khlopov & Rubin 2004; Dokuchaev et al. 2005b, 2004).

6.2 Formation of Closed Walls in the Inflationary Universe

To describe a mechanism for the appearance of massive walls of a size essentially greater than the horizon at the end of inflation, let us consider a complex scalar field with the potential (Rubin et al. 2001, 2000; Khlopov et al. 2005; Khlopov & Rubin 2004)

$$V(\varphi) = \lambda(|\varphi|^2 - f^2/2)^2 + \delta V(\theta), \tag{61}$$

where $\varphi = re^{i\theta}$. This field coexists with an inflation field which drives the Hubble constant H during the inflationary stage. The term

$$\delta V(\theta) = \Lambda^4 \left(1 - \cos\theta\right), \tag{62}$$

reflecting the contribution of instanton effects to the Lagrangian renormalization (see for example Adams et al. 1993), is negligible at the inflationary stage and during some period in the FRW expansion. The omitted term (62) becomes significant when the temperature falls to the values $T \sim \Lambda$. The mass of the radial field component r is assumed to be sufficiently large with respect to H, which means that the complex field is in the ground state even before the end of inflation. Since the term (62) is negligible during inflation, the field has the form $\varphi \approx f/\sqrt{2} \cdot e^{i\theta}$; the quantity $f\theta$ acquires the meaning of a massless field.

At the same time, the well established behavior of quantum field fluctuations on the de Sitter background (Starobinsky 1980) implies that the wavelength of a vacuum fluctuation of every scalar field grows exponentially with a fixed amplitude. Namely, when the wavelength of a particular fluctuation, in the inflating Universe, becomes greater than H^{-1}, the average amplitude of this fluctuation freezes out at some non-zero value because of the large friction term in the equation of motion of the scalar field, whereas its wavelength grows exponentially. Such a frozen fluctuation is equivalent to the appearance of a classical field that does not vanish after averaging over macroscopic space intervals. Because the vacuum must contain fluctuations of every wavelength, inflation leads to the creation of more and more new regions containing a classical field of different amplitudes with scale greater than H^{-1}. In the case of an effectively massless Nambu–Goldstone field considered here, the averaged amplitude of phase fluctuations generated during each e-fold (time interval H^{-1}) is given by

$$\delta\theta = H/2\pi f. \tag{63}$$

Let us assume that the part of the Universe observed inside the contemporary horizon $H_0^{-1} = 3000h^{-1}$Mpc was inflating, with over $N_U \simeq 60$ e-folds, out of a single causally connected domain of size H^{-1}, which contains some average value of phase θ_0 over it. When inflation begins in this region, after one e-fold, the volume of the Universe increases by a factor e^3. The typical wavelength of the fluctuation $\delta\theta$ generated during every e-fold is equal to H^{-1}. Thus, the whole domain H^{-1}, containing θ_0, after the first e-fold, effectively becomes divided into e^3 separate, causally disconnected domains of size H^{-1}. Each domain contains almost homogeneous phase value $\theta_0 \pm \delta\theta$. Thereby, more and more domains appear with time, in which the phase differs significantly from the initial value θ_0. A principally important point is the appearance of domains with phase $\theta > \pi$. Appearing only after a certain period of time during which the Universe exhibited exponential expansion, these domains turn out to be surrounded by a space with phase $\theta < \pi$. The coexistence of domains with phases $\theta < \pi$ and $\theta > \pi$ leads, in the following, to formation of a large-scale structure of topological defects.

The potential (61) possesses a $U(1)$ symmetry, which is spontaneously broken, at least, after some period of inflation. Note that the phase fluctuations during the first e-folds may, generally speaking, transform eventually into fluctuations of the cosmic microwave radiation, which will lead to imposing restrictions on the scaling parameter f. This difficulty can be avoided by taking into account the interaction of the field φ with the inflation field (i.e. by making parameter f a variable, Khlopov & Rubin 2004). This spontaneous breakdown is supported by the condition on the radial mass, $m_{\mathrm{r}} = \sqrt{\lambda\varphi} > H$. At the same time, the condition

$$m_\theta = \frac{2f^2}{\Lambda} \ll H \tag{64}$$

on the angular mass provides the freezing out of the phase distribution until some moment of the FRW epoch. After the violation of condition (64), the term (62) contributes significantly to the potential (61) and explicitly breaks the continuous symmetry along the angular direction. Thus, potential (61) eventually has a number of discrete degenerate minima in the angular direction at the points $\theta_{\min} = 0,\ \pm 2\pi,\ \pm 4\pi,\ \ldots$.

As soon as the angular mass m_θ is of the order of the Hubble rate, the phase starts oscillating about the potential minimum, with initial values being different in various space domains. Moreover, in the domains with the initial phase $\pi < \theta < 2\pi$, the oscillations proceed around the potential minimum at $\theta_{\min} = 2\pi$, whereas the phase in the surrounding space tends to a minimum at the point $\theta_{\min} = 0$. Upon ceasing of the decaying phase oscillations, the system contains domains characterized by the phase $\theta_{\min} = 2\pi$ surrounded by space with $\theta_{\min} = 0$. Apparently, on moving in any direction from inside to outside of the domain, we will unavoidably pass through a point where $\theta = \pi$ because the phase varies continuously. This implies that a closed surface characterized by the phase $\theta_{\text{wall}} = \pi$ must exist. The size of this surface depends on the moment of domain formation in the inflation period, while the shape of the surface may be arbitrary. The principal point for the subsequent considerations is that the surface is closed. After reheating of the Universe, the evolution of domains with the phase $\theta > \pi$ proceeds on the background of the Friedman expansion and is described by the relativistic equation of state. When the temperature falls to $T^* \sim \Lambda$, an equilibrium state between the "vacuum" phase $\theta_{\text{vac}} = 2\pi$ inside the domain and the $\theta_{\text{vac}} = 0$ phase outside it is established. Since the equation of motion corresponding to potential (62) admits a kink-like solution (see Vilenkin & Shellard 1994 and references therein), which interpolates between two adjacent vacua $\theta_{\text{vac}} = 0$ and $\theta_{\text{vac}} = 2\pi$, a closed wall corresponding to the transition region at $\theta = \pi$ is formed. The surface energy density of a wall of width $\sim 1/m \sim f/\Lambda^2$ is of the order of $\sim f\Lambda^2$ [1].

Note that if the coherent phase oscillations do not decay for a long time, their energy density can play the role of CDM. This is the case, for example, in the cosmology of the invisible axion (see Kim 1987 and references therein).

It is clear that immediately after the end of inflation, the size of domains which contain a phase $\theta_{\text{vac}} > 2\pi$ essentially exceeds the horizon size. This situation is replicated in the size distribution of vacuum walls, which appear at the temperature $T^* \sim \Lambda$ when the angular mass m_θ starts to build up. Those walls, which are larger than the cosmological horizon, still follow the general FRW expansion until the moment when they get causally connected as a whole; this happens as soon as the size of a wall becomes equal to the horizon size R_{h}. Evidently, internal stresses which developed in the wall after crossing the horizon initiate processes tending to minimize the wall surface. This implies that the wall tends, first, to acquire a spherical shape and, second, to contract toward the center. For simplicity, we will consider below the motion of closed spherical walls [2].

The wall energy is proportional to its area at the instant of crossing the horizon. At the moment of maximum contraction, this energy is almost completely converted into kinetic energy (Rubin 1999). Should the wall at the same moment be localized within the gravitational radius, a PBH is formed.

Detailed consideration of BH formation was performed in Rubin et al. (2001). The results of these calculations are sensitive to changes in the parameter Λ and the initial phase θ_U. As the Λ value decreases to $\approx 1\,\text{GeV}$, still greater PBHs appear with masses of up to $\sim 10^{40}$ g. A change in the initial phase leads to sharp variations in the total number of black holes. As was shown above, each domain generates a family of subdomains in the close vicinity. The total mass of such a cluster is only 1.5–2 times that of the largest initial black hole in this space region. Thus, the calculations confirm the possibility of formation of clusters of massive PBHs ($\sim 100M_\odot$ and above) in the pregalactic stages of the evolution of the Universe. These clusters represent stable energy density fluctuations around which increased baryonic (and cold dark matter) density may concentrate in the subsequent stages, followed by the evolution into galaxies.

It should be noted that additional energy density is supplied by closed walls of small sizes. Indeed, because of the smallness of their gravitational radius, they do not collapse into BHs. After several oscillations such walls disappear, leaving coherent fluctuations of the PNG field. These fluctuations contribute to a local energy density excess, thus facilitating the formation of galaxies.

The mass range of formed BHs is constrained by fundamental parameters of the model's f and Λ. The maximal BH mass is determined by the condition that the wall does not dominate locally before it enters the cosmological horizon. Otherwise, local wall dominance leads to a superluminal $a \propto t^2$ expansion for the corresponding region, separating it from the other part of the Universe.

[1] The existence of such domain walls in the theory of the invisible axion was first pointed out in Sikivie (1982).

[2] The motion of closed vacuum walls has been derived analytically in Berezin et al. (1983); Ipser & Sikivie (1984).

This condition corresponds to the mass (Khlopov & Rubin 2004)

$$M_{\max} = \frac{m_{\rm pl}}{f} m_{\rm pl} \left(\frac{m_{\rm pl}}{\Lambda}\right)^2 . \qquad (65)$$

The minimal mass follows from the condition that the gravitational radius of the BH exceeds the width of the wall and it is equal to (Rubin et al. 2000; Khlopov & Rubin 2004)

$$M_{\min} = f \left(\frac{m_{\rm pl}}{\Lambda}\right)^2 . \qquad (66)$$

Closed wall collapse leads to the primordial gravitational wave spectrum, which is peaked at

$$\nu_0 = 3 \times 10^{11} (\Lambda/f)\,\mathrm{Hz} \qquad (67)$$

with energy density up to

$$\Omega_{\rm GW} \approx 10^{-4} (f/m_{\rm pl}). \qquad (68)$$

At $f \sim 10^{14}$GeV this primordial gravitational wave background can reach $\Omega_{\rm GW} \approx 10^{-9}$. For the physically reasonable values of

$$1 < \Lambda < 10^8\,\mathrm{GeV}, \qquad (69)$$

the maximum of the spectrum corresponds to

$$3 \times 10^{-3} < \nu_0 < 3 \times 10^5\,\mathrm{Hz}. \qquad (70)$$

Another profound signature of the considered scenario is gravitational wave signals from the merging of BHs in the PBH cluster. These effects can provide a test of the considered approach in the LISA experiment.

7 DISCUSSION

For a long time, scenarios with Primordial Black Holes belonged dominantly to cosmological *anti-Utopias*, to “fantasies,” which provided restrictions on physics of the very early Universe from contradiction of their predictions with observational data. Even this “negative” type of information makes PBHs an important theoretical tool. Being formed in the very early Universe as an initially nonrelativistic form of matter, PBHs should have increased their contribution to the total density during the RD stage of expansion, while effects of PBH evaporation should have strongly increased the sensitivity of astrophysical data to their presence. It links astrophysical constraints on hypothetical sources of cosmic rays or gamma background, on hypothetical factors, causing influence on light element abundance and the spectrum of the CMB, to restrictions on superheavy particles in the early Universe and on first and second order phase transitions, thus becoming a sensitive astrophysical probe to particle symmetry structure and patterns of its breaking at superhigh energy scales.

The gravitational mechanism of particle creation in PBH evaporation makes an evaporating PBH a unique source of any species of particles, which can exist in our space-time. At least theoretically, PBHs can be treated as the source of such particles, which are strongly suppressed in any other astrophysical mechanism of particle production, either due to a very large mass of these species, or owing to their superweak interactions with ordinary matter.

By construction, the astrophysical constraint excludes effects, which are predicted to be larger than observed. At the edge, such a constraint is converted into an alternative mechanism for the observed phenomenon. At some fixed values of parameters, the PBH spectrum can play a positive role and shed new light on old astrophysical problems.

Our common sense is to think that PBHs should have small sub-stellar masses. Formation of PBHs within the cosmological horizon, which was very small in the very early Universe, seems to argue for this viewpoint. However, phase transitions at the inflationary stage can provide spikes in the spectrum of fluctuations at any scale, or provide formations of closed massive domain walls of any size.

In the latter case, primordial clouds of massive black holes around an intermediate mass or supermassive black hole is possible. Such clouds have a fractal spatial distribution. A development of this approach gives ground for a principally new scenario of galaxy formation in the model of the Big Bang Universe. Traditionally, the Big Bang model assumes a homogeneous distribution of matter on all scales, whereas the appearance of observed inhomogeneities is related to the growth of small initial density perturbations. However, the analysis of the cosmological consequences of the particle theory indicates the possible existence of strongly inhomogeneous primordial structures in the distribution of both dark matter and baryons. These primordial structures represent a new factor in galaxy formation theory. Topological defects, such as cosmological walls and filaments, primordial black holes, archioles in the models of axionic CDM, and essentially inhomogeneous baryosynthesis (leading to the formation of antimatter domains in the baryon-asymmetric Universe (Stecker & Puget 1972; Steigman 1976; Cohen et al. 1998; Kinney et al. 1997; Khlopov 2000, 1998; Chechetkin et al. 1982b; Khlopov et al. 2000b; Cohen & Kaplan 1987; Dolgov et al. 1997; Golubkov & Khlopov 2001; Belotsky et al. 2000; Fargion & Khlopov 2003; Khlopov 1999; Khlopov & Rubin 2004; Khlopov 2004)) offer by no means a complete list of possible primary inhomogeneities inferred from the existing elementary particle models.

Observational cosmology offers strong evidences favoring the existence of processes, determined by new physics, and the experimental physics approaches to their investigation. Cosmoparticle physics (Sakharov 1989; Khlopov 2001, 1999, 2004), which studies the physical, astrophysical and cosmological impact of new laws of Nature, explores the new forms of matter and their physical properties. Its development offers a great challenge for theoretical and experimental research. The physics of Primordial Black Holes can play an important role in this process.

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