Capturing near earth objects

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Abstract Recently, Near Earth Objects (NEOs) have been attracting great attention, and thousands of NEOs have been found to date. This paper examines the NEOs' orbital dynamics using the framework of an accurate solar system model and a Sun-Earth-NEO three-body system when the NEOs are close to Earth to search for NEOs with low-energy orbits. It is possible for such an NEO to be temporarily captured by Earth; its orbit would thereby be changed and it would become an Earth-orbiting object after a small increase in its velocity. From the point of view of the Sun-Earth-NEO restricted three-body system, it is possible for an NEO whose Jacobian constant is slightly lower than C_1 but higher than C_3 to be temporarily captured by Earth. When such an NEO approaches Earth, it is possible to change its orbital energy to nearly the zero velocity surface of the three-body system at point L_1 and make the NEO become a small satellite of the Earth. Some such NEOs were found; the best example only required a $410 \, \mathrm{m \, s^{-1}}$ increase in velocity.

Key words: celestial mechanics — methods: numerical — minor planets

1 INTRODUCTION

Some Jovian comets, such as Oterma, are sometimes temporarily captured by Jupiter, making the transition from heliocentric orbits outside the orbit of Jupiter to heliocentric orbits inside the orbit of Jupiter. During this transition, Jupiter frequently captures the comet temporarily for one to several orbits (Koon et al. 2000). This is because the Jacobian constant of the comet is slightly lower than C_1 (Jacobian constant at the Sun-Jupiter Lagrange point L_1) and higher than C_3 (Jacobian constant at the Sun-Jupiter Lagrange point L_3; see Figs. 1 or 2, case $C_2 > C > C_3$). Therefore, when the comet enters Jupiter's region of influence, it can only travel through L_1 and L_2, two necks of the zero velocity surfaces. Inside Jupiter's region of influence, it reflects at the zero velocity surfaces to become a temporary satellite of Jupiter. Might this also be the case with certain objects and Earth?

To date, researchers have found thousands of Near Earth Objects and continue to find more every week. Although these NEOs do pose a threat to impact on Earth, they also provide us with great opportunities. A 2 km-size metallic NEO, for example, may contain rich metals and materials worth more than 25 trillion dollars (Hartmann & Sokolov 1994). The concept of mining NEOs is not new (Gaffey & McCord 1977), but there is still no proper practical way to do it. If approaching NEOs could be temporarily captured by Earth, exerting a small velocity change in the capture phase to bring them into orbit around Earth and finding a low-cost trajectory to sample a large amount of material would be one of the best ways to mine the NEOs. To deflect NEOs which are hazardous to
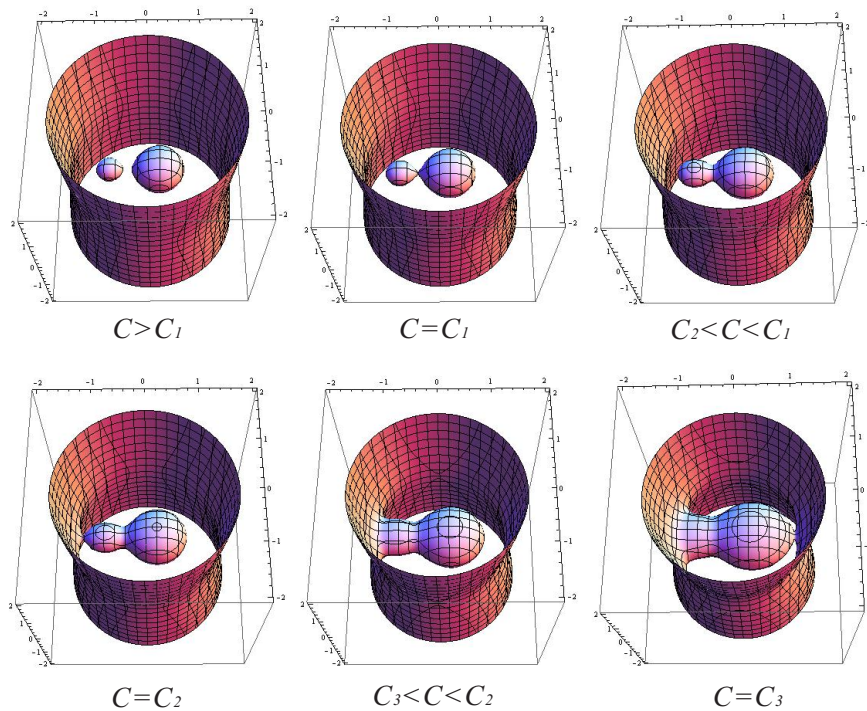
Fig. 1 Zero velocity surfaces for different values of C.

Earth, different schemes have been presented, such as direct impact, mass driver, nuclear explosion, thrusting manoeuvres, and solar radiation (Ahrens & Harris 1992; McInnes 2004; Chapman 2004; Ivashkin & Smirnov 1995). All these schemes can be used for changing the orbital elements of the NEOs, but these schemes have only been examined with regard to NEOs of less than one meter in size; most Near Earth Objects are quite large and are in high-energy orbits.

Many authors have studied the gravitational capture phenomenon by using different models of celestial mechanics. Brunini et al. (1996) examined the conditions of capture in the restricted three-body problem. Murison (1989) studied the connections between gravitational capture and chaotic motions. Makó & Szenkovits (2004) gave some necessary conditions of not being captured by using the Hill-regions in the spatial elliptic restricted three-body problem.

In this paper, we studied the necessary conditions of capture and identified how these conditions could lead to certain kinds of NEOs being captured by the Earth. Like Oterma being captured by Jupiter, it is possible for low-energy NEOs to be temporarily captured by the Earth; moreover, it is possible for them to become Earth-orbiting objects after the exertion of a small velocity increment. In our discussions of the orbital dynamics of NEOs, we used the framework of an accurate solar system model when the NEOs were far from any major celestial body. The Sun-Earth-NEO three-body system was utilized when the NEOs were approaching the Earth. From the perspective of the restricted three-body system, it is possible for an NEO whose Jacobian constant is slightly lower than the Sun-Earth C_1 but higher than C_3 to be temporarily captured by the Earth. When such an NEO approaches the Earth, it is possible to change its orbital energy to nearly the zero velocity surfaces at point L_1 (see Fig. 1 or 2, case $C=C_1$; it will remain inside the smaller ball) and make it become
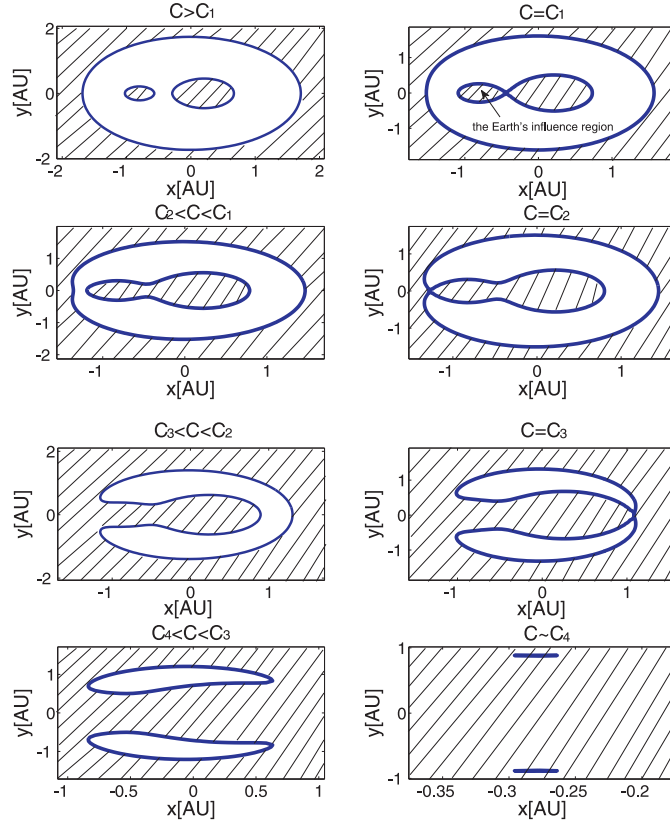
Fig. 2 Allowable regions of motion for different values of C ($\mu = 0.2$).

a small satellite of the Earth. However, based on current technology, this may only be possible with very small NEOs, as larger ones are too heavy for their orbital energy to be significantly changed. Fortunately, a practical advantage of such low-energy NEOs is that mining them requires less fuel and time than larger NEOs.

2 SYSTEM MODEL

2.1 Orbit Prediction

Because of the perturbations, especially the resonance from the planets, the dynamical model only consists of the Sun-Earth-NEO three body system that cannot accurately predict close approaches of the near Earth objects. Fortunately there is some software that can be used for predicting NEO orbits very accurately in an appropriate time scale. Milani et al. (2001) have developed a very accurate solar system model, in which the gravitational forces of all bigger celestial bodies (including the Earth and the Moon, which are treated as individual bodies instead of only the barycenter of the Earth-Moon system) of the solar system, and even the bigger NEOs themselves, have been taken into account. That model is implemented in their free software package OrbFit and its source codes can also be freely downloaded from their website. This paper will use this software to calculate the NEO orbits and to predict their close approaches.

2.2 The Elliptical Restricted Three Body Model

When we consider the orbital change of an NEO as it closely approaches the Earth, the system can be regarded as the Sun-Earth-NEO elliptical 3-dimensional three-body system. In such a system, the Sun and the Earth revolve around their common mass center in a Keplerian elliptical orbit under the influence of their mutual gravitational attraction. The NEO, of infinitesimal mass, moves in the 3-dimensional space under their gravitational influences.

To succinctly describe the elliptic restricted three-body problem, a non-uniformly rotating and pulsating coordinate system is used here. In this system, the origin of the coordinates is in the common mass center of the two massive primaries, the x axis is directed towards m_1 (the Earth), and the xy plane is the orbital plane of the two massive primaries. Such a pulsating or oscillating coordinate system might be transformed to dimensionless coordinates by using the variable distance between the primaries as the length unit and the reciprocal of the variable angular velocity of the Earth as the time unit.

$$[L] = r = \frac{a(1 - e^2)}{1 + e \cos f},$$

$$[T] = \frac{1}{f} = \frac{R^2}{\sqrt{G(m_1 + m_2)a(1 - e^2)}},$$

where R is the mutual distance, a and e are the semimajor axis and eccentricity of the elliptic orbit, and f is the true anomaly.

In the non-uniformly rotating and pulsating coordinate system, the two massive primaries are always in fixed locations on the x axis, and the dimensionless angular velocity of the two primaries is 1.

Consider first the equations of motion in the inertial coordinate system. Using dimensional quantities and variables, the equations of motion can be given as (Szebehely 1967)

$$\frac{d^2 X}{dt^2} = -Gm_1 \frac{X - X_1}{R_1^3} - Gm_2 \frac{X - X_2}{R_2^3}, \tag{1a}$$

$$\frac{d^2 Y}{dt^2} = -Gm_1 \frac{Y - Y_1}{R_1^3} - Gm_2 \frac{Y - Y_2}{R_2^3}, \tag{1b}$$

$$\frac{d^2 Z}{dt^2} = -Gm_1 \frac{Z - Z_1}{R_1^3} - Gm_2 \frac{Z - Z_2}{R_2^3}, \tag{1c}$$

where t is the dimensional time, and X, Y and Z are the dimensional coordinates of the third body in an inertial coordinate system,

$$R_i^2 = (X - X_i)^2 + (Y - Y_i)^2, \tag{1d}$$

$i = 1, 2$, and X_i, Y_i are the dimensional coordinates of the two massive primaries.

The dimensional rotating coordinates $(\tilde{x}, \tilde{y}, \tilde{z})$ and the inertial coordinates (X, Y, Z) satisfy

$$\begin{pmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} \end{pmatrix} = \begin{pmatrix} \cos f & \sin f & 0 \\ -\sin f & \cos f & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}. \tag{2}$$

The relationship between the dimensionless coordinates (x, y, z) and the dimensional coordinates $(\tilde{x}, \tilde{y}, \tilde{z})$ can be given as

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} \end{pmatrix} / [L] = \begin{pmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} \end{pmatrix} / \left[\frac{a(1 - e^2)}{1 + e \cos f} \right]. \tag{3}$$

The true anomaly as the independent variable may be introduced by the equation

$$\frac{d}{dt} = \frac{d}{df}\frac{df}{dt} = \frac{d}{df} \cdot \dot{f}. \tag{4}$$

Substitute Equations (2), (3), and (4) into the Equations (1a), (1b) and (1c) and let

$$\mu = \frac{m_2}{m_1 + m_2}, \tag{5a}$$

$$1 - \mu = \frac{m_1}{m_1 + m_2}, \tag{5b}$$

where μ is the dimensionless mass of the Earth, and 1-μ then becomes the dimensionless mass of the Sun. In addition, the dimensionless x coordinate of the Sun is μ and that of the Earth is 1-μ.

In the rotating frame, the two primaries are fixed and dimensionless equations of NEO motion in a non-uniformly rotating and pulsating coordinate system can be obtained as

$$\ddot{x} - 2\dot{y} = \frac{\partial \Omega}{\partial x}, \tag{6a}$$

$$\ddot{y} + 2\dot{x} = \frac{\partial \Omega}{\partial y}, \tag{6b}$$

$$\ddot{z} + z = \frac{\partial \Omega}{\partial z}, \tag{6c}$$

where the dots imply derivatives with respect to the true anomaly f of the Earth, and

$$\Omega = (1 + e\cos f)^{-1}\left(\frac{x^2 + y^2 + z^2}{2} + \frac{1 - \mu}{r_1} + \frac{\mu}{r_2}\right), \tag{7a}$$

$$r_1^2 = (x + \mu)^2 + y^2 + z^2, \tag{7b}$$

$$r_2^2 = (x - 1 + \mu)^2 + y^2 + z^2. \tag{7c}$$

Similar to the circular restricted three-body problem, differential Equation (6) possesses some important properties that are its local five equilibrium points, three collinear points and two equilateral points and the zero velocity surfaces.

2.3 The Local Jacobian Constant and the Local Zero Velocity Surfaces

The invariant quantity, the well-known Jacobian integral, which exists in the circular case, does not exist globally in this case. Here, however, let us define the local Jacobian constant and the local zero velocity surfaces. Multiplying Equation (6a) by $\dot{x}$, (6b) by $\dot{y}$ and (6c) by $\dot{z}$, adding them, and integrating, yields

$$\dot{x}^2 + \dot{y}^2 + \dot{z}^2 = 2\Omega - 2e\int \frac{\sin f}{1 + e\cos f}\Omega df - C - z^2. \tag{8}$$

If one considers a short time arc of a near circular orbit, in Equation (8) the integral term can be negligible, therefore the local Jacobian constant can be defined as

$$C = 2\Omega - v^2 - z^2. \tag{9}$$

The Equation (8) implies that the 3-dimensional elliptical three-body problem does not have global zero velocity surfaces. Therefore the local zero velocity surfaces can be defined as

$$C = 2\Omega - z^2. \tag{10}$$

The shape of the zero velocity surfaces, depending on the value of constant C, are shown in Figure 1.

The zero velocity surfaces form the boundaries between forbidden and allowed regions of motion. The planar ($z = 0$) allowable regions of motion (Hill-zone) for different values of the Jacobian constant are shown in Figure 2 (shaded). Here we define the Earth's influence region as the allowed regions of motion (Hill-zone) surrounding the Earth when $C=C_1$ and illustrate it in Figure 2(a) (to show clearly, here Fig. 2 took $\mu = 0.2$ rather than μ of the Sun-Earth system).

2.4 Necessary Conditions of the Capture

These zero velocity surfaces can provide a necessary condition of temporal capture (Because the NEOs are also affected by the gravitational pull of other celestial bodies besides the Sun and the Earth, so only the condition of temporary capture can be given). If an NEO is inside the zero velocity surfaces surrounding the Earth, and they satisfy the condition $C>C_1$, then it would not penetrate the zero velocity surfaces if they become a small satellite of the Earth.

We use this property to search for the possible temporarily captured NEOs and to estimate the velocity requirement for changing an NEO orbit into the orbit around the Earth when an NEO enters the Earth's influence region.

We added some codes, including a sequence control function, to the software ORBFIT and examined all the close approaching NEOs (we considered the Earth approaching distance to be closer than 0.008 AU and the allowed time is up to year 2060) from over six thousand NEOs[1]. The local Jacobian constants of the close approaching NEOs are listed in Table 1. From Table 1, we can conclude that there are no close approaching NEOs that seem to be naturally temporarily captured by the Earth. However, the Jacobian constants of some NEOs are close to C_1, such as 2008EA9 and 2009BD, while others are quite different from it, but we cannot deny that we may find some such kind of NEOs in the future.

3 ORBITAL CHANGE OF THE CLOSE APPROACHING NEOS

3.1 Orbital Change of an NEO

Here we consider giving a velocity increment to change an NEO's orbital energy to have it captured by the Earth. In the close approaching phase, the orbital dynamics is considered to be the Sun-Earth-NEO elliptical three-body system, so using the local Jacobian integral, it can be written as

$$v^2 = 2\Omega - C + z^2. \tag{11}$$

If a velocity increment is applied in a short time, the velocity difference can be written as

$$v_1^2 - v_0^2 = 2\Delta\Omega - \Delta C + \Delta z. \tag{12}$$

The maneuver is assumed to be implemented in a short time, so $\Delta\Omega = 0$ and $\Delta z = 0$ can be applied, therefore

$$\Delta v = \sqrt{v_0^2 - \Delta C} - v_0. \tag{13}$$

For engineering applications, this velocity increment is not only a criterion for local capture, but also a useful value for NEOs sampling. For a large fraction of the NEO sample, for example, choosing a small velocity increment, NEO can cut the fuel cost of the spacecraft greatly. Table 1 shows that most of the close-approaching NEOs are in high energy orbits, see the column of C and ΔV. It is very difficult to change these high energy orbit NEOs to have them captured by the Earth, but there are some NEOs whose orbital energy is easily changed, such as the last two in the table.

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[1] For the orbital data of these asteroids, refer to *http://neo.jpl.nasa.gov/cgi-bin/neo_elem*

Table 1 Jacobian Constant of the Close Approaching NEOs

Name	Abs Mag	Dia (m)	Approach (AU) (Year/MM)	Local C	Δv (km s^{-1})
137108	17.9	800	0.002607 (2027/08)	2.25105	−24.82
2008KO	17.4	30–80	0.00753075 (2052/06)	2.41202	−22.66
2002NY40	19.0	280	0.00730030 (2038/02)	2.47592	−20.37
2007EH26	24.2	40–90	0.00498466 (2049/09)	2.57581	−18.87
2007PF2	24.4	30–80	0.00512391 (2024/08)	2.62260	−18.14
221455	19.4	370–840	0.00783329 (2052/03)	2.62078	−17.55
2009SU104	25.6	20–50	0.00147771 (2057/02)	2.66291	−14.74
2000LF3	21.6	140–320	0.0079286 (2046/06)	2.78643	−14.42
2004VZ14	25.3	20–50	0.00596776 (2042/11)	2.73614	−14.06
2008YF	20.9	190–440	0.00362470 (2035/12)	2.70749	−13.96
1998HH49	21.3	280	0.0078544 (2023/10)	2.76691	−13.46
2007VX83	27.8	0–10	0.00555639 (2030/11)	2.76124	−13.09
2000YA	23.7	700	0.00735641 (2011/12)	2.74495	−13.07
2005EQ95	23.4	60–140	0.00708734 (2045/10)	2.78439	−12.92
1997XF11	16.7	1300–2800	0.00621084 (2028/10)	2.77705	−12.87
2009RR	25.6	20–50	0.00713957 (2014/09)	2.81182	−12.74
2001TB	24.8	30–80	0.00482637 (2010/10)	2.80398	−12.19
2005YU55	22.0	120–280	0.00362956 (2011/11)	2.78383	−12.10
2007VL3	26.0	10–40	0.00740713 (2014/10)	2.82102	−11.34
2009WV25	24.0	40–100	0.00644959 (2053/04)	2.86257	−10.71
2008GY21	27.6	0–20	0.00444996 (2018/04)	2.85320	−10.67
2007TL16	26.2	10–30	0.006224 (2037/10)	2.86095	−10.29
2006BX147	25.8	150–350	0.00205264 (2013/01)	2.89963	−9.79
2004VZ	24.5	30–80	0.00414594 (2056/11)	2.83855	−9.96
2008VM	30.2	0–0	0.00474005 (2051/11)	2.84631	−9.86
2006DM63	26.7	10–30	0.00524844 (2053/02)	2.84449	−9.79
2008EX5	23.8	50–110	0.00428753 (2042/10)	2.87449	−9.46
2009EU	26.6	10–30	0.00443390 (2043/03)	2.86389	−9.27
153814	18.2	700	0.0016667 (2028/06)	2.93194	−8.45
2007UT3	25.7	10–40	0.00753056 (2028/08)	2.94368	−8.06
2008LH2	24.4	30–80	0.00640139 (2039/06)	2.95929	−7.87
2009TM8	28.6	0–10	0.00283001 (2011/10)	2.91328	−7.23
2007JB21	25.4	20–50	0.00649834 (2054/05)	2.96018	−6.86
2004UT1	26.4	10–30	0.00770637 (2022/10)	2.92514	−6.38
162162	19.7	160–370	0.0065285 (2048/02)	2.9226	−6.15
2008TC3*	–	0–0	0.00003910 (2008/10)	2.98029	−2.04
2005VL1	26.7	10–20	0.00558563 (2049/02)	2.91503	−5.4
2008GM2	28.4	0–10	0.00538838 (2035/04)	2.96996	−4.3
2008LG2	25.2	20–60	0.00673584 (2056/06)	3.02363	−4.17
2005TA	27.2	10–20	0.00297955 (2034/10)	2.97108	−4.15
2005TA	27.2	10–20	0.00265873 (2041/10)	2.97172	−4.1
2009QR	27.3	0–20	0.00235132 (2023/08)	3.00272	−4.03
2009WR52	28.3	0–10	0.00303283 (2028/05)	3.00523	−4.03
2006HE2	26.5	10–30	0.00518549 (2029/09)	2.98176	−3.76
2003LN6	24.5	30–70	0.00672417 (2053/04)	2.99763	−3.66
2006WB	22.8	80–180	0.00595314 (2024/11)	2.93867	−3.64
2007XB23	27.1	10–20	0.00292837 (2024/12)	2.93334	−3.53
2008HQ3	18.1	0–20	0.00717914 (2038/04)	2.98815	−3.3
2001AV43	24.4	30–70	0.00742242 (2013/11)	2.95181	−3.24
2007UD6	28.3	–	0.00026283 (2048/10)	2.95841	−2.59
99942	19.2	270	0.00025418 (2029/4)	2.97612	−2.39
2008EA9	27.7	0–10	0.00694714 (2049/02)	2.95187	−1.0
2009BD	28.3	0–10	0.00231372 (2011/06)	3.03807	−0.67
2009BD	28.3	0–10	0.00458782 (2009/01)	2.95112	−0.41

According to our calculations, on 2008 Oct the minimum distance between 2008TC3 and the Earth is 0.00003910 AU, which is less than the radius of the Earth. In fact, this object has impacted the Earth and exploded over the Sudan on 2008 October 7. Parameters are referenced from NEO Information Services of Pisa University and the JPL Near Earth Object Program.
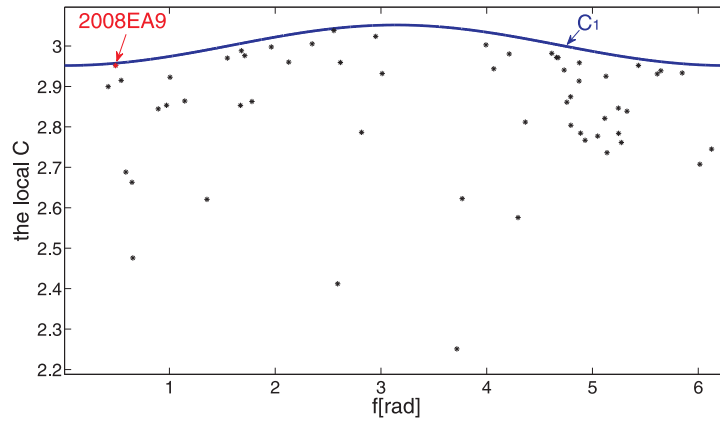
Fig. 3 Local C of all the NEOs (Earth approaching distance ≤ 0.008 AU).

The NEOs' diameter data listed in Table 1 can be computed using the following equation (Steven et al. 2002)

$$D = \frac{1329}{\sqrt{P}} \times 10^{-0.2H},$$

where H is the absolute magnitude of the NEO, P is the geometric albedo of the NEO, and D is the diameter of the NEO in kilometers.

It should be addressed that the orbital data we adopted are not so exact, because, due to the limitation of the observations, some orbits of NEOs are poorly determined.

It is obvious from Equation (11) that the Jacobian constant is actually not a constant in the elliptical system, but it is varying with the true anomalous behavior of the Earth's orbit. However, when the true anomaly is fixed at a close approaching moment, one can get a local Jacobian constant. Figure 3 shows the time varying Jacobian constant of the close approaching NEOs vs. the Sun-Earth C_1 at the corresponding true anomaly.

4 CAPTURE EXAMPLE

4.1 2008EA9

2008EA9 is a 10-m near Earth object, and its orbital data are shown in Table 2. Figure 4 shows the orbits of the Earth and the NEO 2008EA9 in the Sun centered coordinates. From Figure 3 and Table 1, it can be seen that the velocity increment of the 2008EA9 is relatively small ($-1.00\,\mathrm{km\,s^{-1}}$) and it will very closely approach (0.00694761 AU) the Earth in 2049/02. Moreover, the size of the NEO 2008EA9 is very small so that capturing it is relatively easy.

Table 2 Orbital Data of the NEO 2008EA9 (Epoch: MJD 55200)

a (AU)	e	$i(°)$	$\Omega(°)$	$\omega(°)$	$M(°)$
1.05916	0.07978	0.424	129.426	335.944	298.104

We simulated the trajectory before and after the orbit maneuver using the accurate dynamic model, and the result is shown in Figure 5. Figure 5 shows that the orbit of 2008EA9 after the maneuver is very close to the Earth's orbit in the xy plane, only showing pulsations on the order of 10^{-3} AU on the z axis. Figure 6 shows the trajectory of the NEO 2008EA9 after maneuvering it
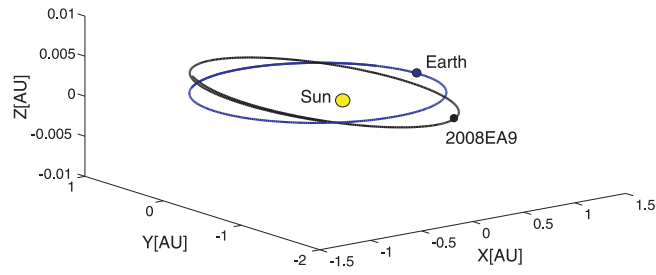
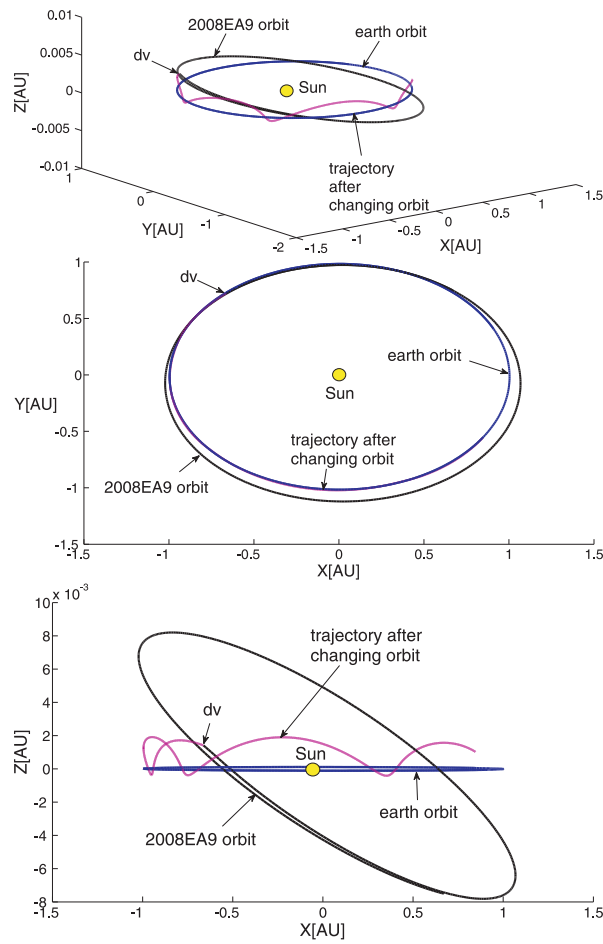**Fig. 4** Orbits of the Earth and the NEO 2008EA9.

Fig. 5 *Upper:* trajectory of 2008EA9 before and after the orbit maneuver in the Sun-centered co-ordinates; *Middle:* trajectory of 2008EA9 before and after the orbit maneuver in the Sun-centered coordinates (xy); *Bottom:* trajectory of 2008EA9 before and after the orbit maneuver in the Sun-centered coordinates (xz).
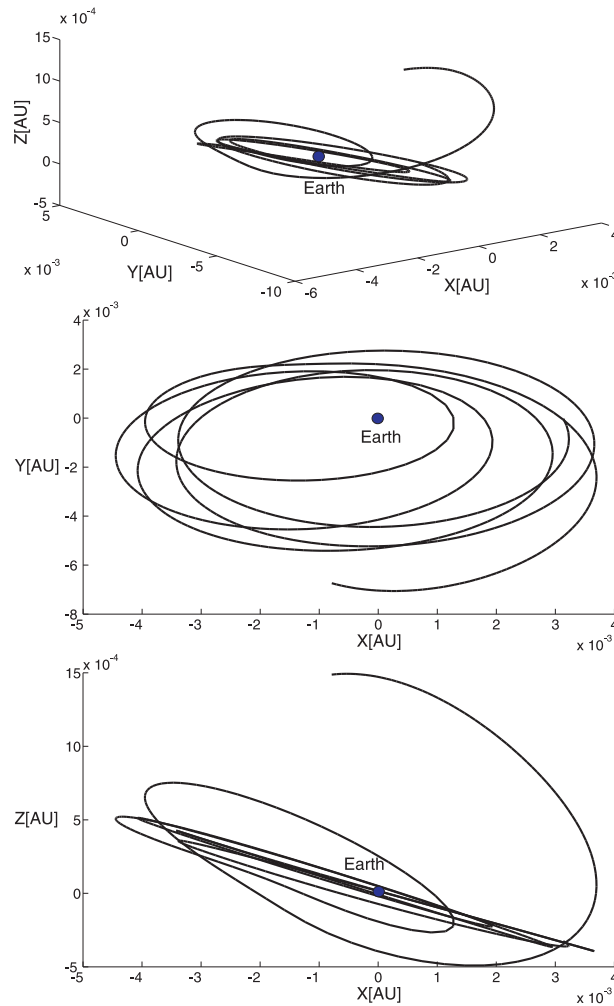
Fig. 6 *Upper:* trajectory of 2008EA9 after orbit change in geocentric inertial coordinates; *Middle:* trajectory of 2008EA9 after orbit change in geocentric inertial coordinate (xy); *Bottom:* trajectory of 2008EA9 after orbit change in geocentric inertial coordinate (xz).

into a geocentric inertial coordinate system. After the orbital maneuver, the NEO becomes an Earth temporary orbiting satellite with an orbital altitude of about 0.005 AU, a distance twice that of the Earth-Moon distance.

5 CAPTURE METHODS

In order to capture a near Earth object, there are several alternatives. These alternatives are broadly classified as "impulsive" if they act nearly instantaneously, or "slow push" if they act over an extended period of time (NASA 2006). The impulsive techniques generally include conventional explosives, kinetic impactors and nuclear explosives. The slow push techniques include the enhanced Yarkovsky effect, a focused solar method, gravity tractor, mass driver, pulsed laser and space tug. Considering that the required impulsive velocity increment is not so small and the diameters of

NEOs can be relatively large, there are two impulsive capture methods available, kinetic impactor and nuclear explosion, but these techniques have never tested or applied. Among them, the nuclear explosion method may not be ideal for the mentioned small NEO, because the nuclear explosion can release a very large amount of energy, and the result may be a fragmentation of the target NEO. So, the kinetic impactor is often considered as a better method of maneuvering the NEO, especially for NEOs smaller than 50 m in diameter.

5.1 Kinetic Impact

A space probe or a specially designed projectile which would hit an NEO at a very high velocity could, therefore, deliver an impulse that would change the orbit of the NEO. The relative velocity between the impactor and the NEO depends on the orbit of the impactor, the limited payload capacity of the available launch systems, and the application of new technology. For example, the relative velocity can reach $60 \, \text{km s}^{-1}$ by utilizing solar sailing (McInnes 2004).

For the kinetic impactor method, the transfer of momentum can be calculated by

$$\Delta V = \beta \frac{m_i v_i}{M_a}, \tag{14}$$

where β is the impact efficiency constant, and the value of the constant greatly depends on the structure and the material properties of the NEO. If $\beta=1$, the collision is perfectly plastic, no ejecta are produced, and momentum is imparted directly. If an NEO is sufficiently soft and the impactor penetrates into it, then $\beta<1$ and the impact is less effective. If $\beta>1$, ejecta are released by the impact, and the impact is more effective. Some estimate that β could have a magnitude of 10 or more.

The effective momentum changes for kinetic impacts are calculated to range from about 5×10^7 to $2 \times 10^{10} \, \text{kg m s}^{-1}$(NASA 2006). Taking the NEO 2008EA9 as an example, capturing it needs a velocity increment of $-1.00 \, \text{km s}^{-1}$, if its density is $2 \times 10^3 \, \text{kg m}^{-3}$ and diameter is 10 m. The momentum changes can be given as

$$\Delta p = M \Delta v = \rho V \Delta v = 8 \times 10^9 \, \text{kg} \cdot \text{m s}^{-1}. \tag{15}$$

So the kinetic impact is a possible way for capturing 2008EA9, and, assuming the relative velocity $v_i = 60 \, \text{km s}^{-1}$ and the impact constant $\beta=5$, we can calculate the mass of an impactor needed by Equation (14)

$$m_i = \frac{\Delta p}{v_i \beta} = 26.4 \times 10^3 \, \text{kg}. \tag{16}$$

This is still likely a practical value.

5.2 Sampling from a Close Approaching NEO

Sampling a large amount of NEOs requires a low fuel cost trajectory. However, the near Earth objects are not all in desirable energy orbits. Mining some NEOs needs more velocity increment while others need less. However, if it is possible, mining during the time of closest approach can greatly shorten mission time and may significantly reduce risks. Here, we assume that we are mining an NEO and we depart from the NEO at its closest point from Earth. At this point, the spacecraft changes its velocity into a big elliptical Earth centered orbit whose perigee intersects the parking orbit (for example 200 km), and after it reaches the perigee, we apply another velocity change to get to the parking orbit. The L_1 closing velocity increment can be obtained by Equation (13). According to the orbital energy equation, at the apogee and perigee, the velocity increments can be conservatively

estimated by

$$\Delta v_{\mathrm{a}} = \sqrt{\frac{\mu_{\mathrm{e}}}{r_{\mathrm{a}}}} \left[1 - \sqrt{\frac{2r_p}{(r_{\mathrm{a}} + r_p)}} \right],$$ (17a)

$$\Delta v_{\mathrm{p}} = \sqrt{\frac{\mu_{\mathrm{e}}}{r_{\mathrm{p}}}} \left[1 - \sqrt{\frac{2r_{\mathrm{a}}}{(r_{\mathrm{a}} + r_{\mathrm{p}})}} \right],$$ (17b)

where μ_{e} is the gravitational constant of the Earth, r_{a} is the apogee radius and r_{p} is the perigee radius of the Earth centered orbit. The transfer mentioned above is similar to a Hohmann transfer between a big and a small circular orbit. One can obtain $\Delta v_{\mathrm{a}} = 0.73$ km s^{-1} and $\Delta v_{\mathrm{p}} = 2.19$ km s^{-1} for NEO 2008EA9. Counting L_1 as the closing velocity increment, it totally needs a velocity increment of 3.92 km s^{-1} for returning from the NEO, and it would be much lower if aerobraking is considered.

6 CONCLUSIONS

In this paper, we examined Near Earth Objects' orbital dynamics using the framework of an accurate solar system model and the Sun-Earth-NEO three-body system to search for low-energy NEOs that may be temporarily captured by the Earth or might be able to orbit the Earth after the exertion of a small velocity increment. The results showed that none of the more than six thousand NEOs could naturally be captured, but we did find that some NEOs could be captured by the Earth after exerting a small velocity change (less than 1 km s^{-1}) while close to the Earth. These NEOs are prime candidates for short sampling missions conducted by spacecraft.

References

Ahrens, T. J., & Harris, A. W. 1992, Nature, 360, 429
Brunini, A. 1996, Celestial Mech. Dyn. Astr., 64, 79
Chapman, C. R. 2004, Earth and Planetary Science Letters, 222, 1
Gaffey, M. J., & McCord, T. B. 1977, Mercury, 6, 1
Hartmann, W., & Sokolov, A. 1994, Hazards Due to Comets and Asteroids (Tucson: The Univ. of Arizona Press)
Ivashkin, V. V., & Smirnov, V. V. 1995, Planet. Space Sci., 43, 821
Koon, W. S., Lo, M. W., Marsden, J. E., & Ross, S. D. 2000, Chaos, 10, 427
Makó, Z., & Szenkovits, F. 2004, Celestial Mech. Dyn. Astr., 90, 51
McInnes, C. R. 2004, Planet. Space Sci., 52, 587
Murison, M. A. 1989, AJ, 98, 2346
Steven, R., Chesly, P. W., Chodas, A. M., & Giovanni, B. 2002, ES, I159, 423
Szebehely, V. 1967, Theory of orbits (New-York: Academic Press)
http://www.nasa.gov/pdf/171331main_NEO_report_march07.pdf, 2006, NASA