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Propagation of a strong cylindrical shock wave in a rotational axisymmetric dusty gas with exponentially varying density

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Abstract Non-similarity solutions are obtained for one-dimensional isothermal and adiabatic flow behind strong cylindrical shock wave propagation in a rotational axisymmetric dusty gas, which has a variable azimuthal and axial fluid velocity. The dusty gas is assumed to be a mixture of small solid particles and perfect gas. The equilibrium flow conditions are assumed to be maintained, and the density of the mixture is assumed to be varying and obeying an exponential law. The fluid velocities in the ambient medium are assumed to obey exponential laws. The shock wave moves with variable velocity. The effects of variation of the mass concentration of solid particles in the mixture, and the ratio of the density of solid particles to the initial density of the gas on the flow variables in the region behind the shock are investigated at given times. Also, a comparison between the solutions in the cases of isothermal and adiabatic flows is made.

Key words: shock wave — equation of state — stars: rotation — radiative transfer — interplanetary medium

1 INTRODUCTION

Hayes (1968), Laumbach & Probstein (1969), Deb Ray (1974), Verma & Vishwakarma (1976, 1980), and Vishwakarma (2000) have discussed the propagation of shock waves in a medium where density varies exponentially and obtained similarity and non-similarity solutions. These authors have not taken into account the effects of rotation of the medium. The formation of self-similar problems and examples describing the adiabatic motion of non-rotating gas models of stars are considered by Sedov (1982), Zel'dovich & Raizer (1967), Lee & Chen (1968) and Summers (1975). The experimental studies and astrophysical observations show that the outer atmosphere of the planets rotates due to rotation of the planets. Macroscopic motion with supersonic speed occurs in an interplanetary atmosphere and shock waves are generated. Thus rotation of planets or stars significantly affects the process taking place in their outer layers; therefore questions connected with the explosions in rotating gas atmospheres are of definite astrophysical interest. Chaturani (1970) studied the propagation of cylindrical shock waves through a gas having solid body rotation, and obtained the solutions by a similarity method adopted by Sakurai (1956). Nath et al. (1999) obtained the similarity solutions for the flow behind spherical shock waves propagating in a non-uniform rotating interplanetary atmosphere with increasing energy. Vishwakarma & Vishwakarma (2007) obtained the similarity solution for magnetogasdynamic cylindrical shock waves propagating in a rotating medium which is a perfect gas with variable density.

The study of shock waves in a mixture of a gas and small solid particles is of great importance due to its applications to nozzle flow, lunar ash flow, bomb blasts, coal-mine blasts, underground, volcanic and cosmic explosions, metallized rocket propellant, supersonic flights in polluted air, collisions of coma with a planet and many other engineering problems (see Pai et al. 1980; Higashino & Suzuki 1980; Miura & Glass 1983; Gretler & Regenfelder 2005; Popel & Gisko 2006; Vishwakarma & Nath 2006, 2009; Vishwakarma et al. 2008). Miura & Glass (1985) obtained an analytical solution for a planar dusty gas flow with constant velocities of the shock and the piston moving behind it. Because they neglected the volume occupied by the solid particles mixed into the perfect gas, the dust has a non-negligible mass fraction but virtually no volume fraction. Their results reflect the influence of the additional inertia of the dust upon the shock propagation. Pai et al. (1980) generalized the well known solution of a strong explosion due to an instantaneous release of energy in gas (Sedov 1982; Korobeinikov 1976) to the case of a two-phase flow of a mixture of perfect gas and small solid particles, and brought out the essential effects due to the presence of dusty particles on such a strong shock wave. As they considered a non-zero volume fraction of solid particles in the mixture, their results reflect the influence of both the decrease of the mixture's compressibility and the increase of the mixture's inertia on the shock propagation (Steiner & Hirchler 2002; Vishwakarma & Nath 2006, 2009). Vishwakarma (2000) studied the propagation of shock waves in a dusty gas with exponentially varying density, using a non-similarity method.

In all of the work mentioned above, the ambient medium is supposed to have only one component of velocity which is the azimuthal component. The effects of rotation of the ambient medium are not taken into account by any of the authors in the case of a dusty gas with exponentially varying density.

In the present work, the non-similarity solutions for the flow behind the cylindrical shock wave propagating in a rotational axisymmetric dusty gas (a mixture of small solid particles and perfect gas), which has a variable azimuthal fluid velocity together with a variable axial fluid velocity (Levin & Skopina 2004) are obtained. The fluid velocities and the density in the ambient medium are assumed to obey the exponential laws. In order to get some essential features of the shock propagation, small solid particles are considered to be a pseudo-fluid, and the mixture is at velocity and temperature equilibrium with a constant ratio of specific heats (Pai 1977). For this gas- particle mixture to be treated as a so-called idealized equilibrium gas (Geng & Groening 2000), it is necessary to consider the particle diameter to be much smaller than the characteristic length of the flow-field and their number density is small in relation to that of the gas particles. The Brownian motion of the solid particles is negligibly small. No deformations and no phase changes of the solid particles occur. Gas and solid particles are chemically inert. In this case, we may assume that the viscous stress and heat conduction of the medium are negligible (Pai et al. 1980; Higashino & Suzuki 1980; Vishwakarma & Nath 2006, 2009; Steiner & Hirschler 2002; Hirschler & Steiner 2003). Although the density of the mixture is assumed to be increasing exponentially, the volume occupied by the solid particles may be very small under ordinary conditions owing to the large density of the particle material. Hence for simplicity, the initial volume fraction of solid particles Z_a is assumed to be a small constant (Vishwakarma 2000; Vishwakarma et al. 2008).

Due to high temperature in the flow, intense radiation heat transfer takes place behind a strong shock. For such flows, the assumption of adiabaticity may not be valid. Therefore, an alternative assumption of zero temperature gradient throughout the flow (flows which satisfy this condition are also known as isothermal flows) may approximately be taken (as in Vishwakarma & Nath 2006, 2007, 2009; Gretler & Regenfelder 2005; Korobeinikov 1976; Laumbach & Probstin 1970; Sachdev & Ashraf 1971). With this assumption, we therefore obtain, in Sections 2 and 3, the solution of the problem treated by Vishwakarma (2000). In Section 4, we present the solutions for the flow taken to be adiabatic. The effects of an increase in the mass concentration of solid particles in the mixture K_p, and the ratio of the density of solid particles to the initial density of the gas G_a on the flow variables behind the shock are investigated at different times. A comparative study between the solutions of isothermal and adiabatic flows is also made.

2 FUNDAMENTAL EQUATIONS AND BOUNDARY CONDITIONS-ISOTHERMAL FLOW

The fundamental equations for one-dimensional, unsteady and cylindrically symmetric isothermal flow of a mixture of perfect gas and small solid particles, which is rotating about the axis of symmetry, can be written as (c.f. Pai et al. 1980; Zhuravskaya & Levin 1996; Chaturani 1970; Vishwakarma & Nath 2006, 2009; Gretler & Regenfelder 2005; Naidu et al. 1985)

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + \rho\frac{\partial u}{\partial r} + \frac{u\rho}{r} = 0, \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{(1-K_p)R^*T}{\rho(1-Z)^2}\frac{\partial \rho}{\partial r} - \frac{\nu^2}{r} = 0, \tag{2}$$

$$\frac{\partial \nu}{\partial t} + u\frac{\partial \nu}{\partial r} + \frac{u\nu}{r} = 0, \tag{3}$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial r} = 0, \tag{4}$$

$$\frac{\partial T}{\partial r} = 0, \tag{5}$$

where $\left(\frac{\partial p}{\partial \rho}\right)_T = \frac{p}{\rho(1-Z)}$, which expresses the isothermal sound speed

$$a_{\text{iso}}^2 = \frac{(1-K_p)R^*T}{(1-Z)^2}, \tag{6}$$

is replaced with the equation of state (7) of the mixture under the equilibrium condition; ρ, p, and T are the density, the pressure and the temperature of the mixture, u, ν, and w are the radial, azimuthal and axial components of the fluid velocity $\boldsymbol{q}$ in the cylindrical coordinates (r, θ, z^*), r and t are the distance and time, R^* the gas constant, K_p the mass concentration of solid particles and 'Z' the volume fraction of solid particles in the mixture.

The equation of state of the mixture of perfect gas and small solid particles can be written as (Pai 1977)

$$p = \frac{(1-K_p)}{(1-Z)}\rho R^* T. \tag{7}$$

The relation between K_p and Z is given by

$$K_p = \frac{Z\rho_{\text{sp}}}{\rho}, \tag{8}$$

where ρ_{sp} is the species density of solid particles. In the equilibrium flow, K_p is a constant in the whole flow-field. Therefore, from Equation (8)

$$\frac{Z}{\rho} = \text{constant}, \tag{9}$$

in the whole flow-field. Also, we have the relation

$$Z = \frac{K_p}{(1-K_p)G + K_p}, \tag{10}$$

where $G = \frac{\rho_{\rm sp}}{\rho_{\rm g}}$ is the ratio of the density of the solid particles to the species density of the gas.

The internal energy of the mixture may be written as

$$U_{\rm m} = [K_p C_{\rm sp} + (1 - K_p)C_{\rm v}]T = C_{\rm vm}T, \tag{11}$$

where $C_{\rm sp}$ is the specific heat of the solid particles, $C_{\rm v}$ the specific heat of the gas at constant volume and $C_{\rm vm}$ the specific heat of the mixture at constant volume.

The specific heat of the mixture at constant pressure is

$$C_{\rm pm} = K_p C_{\rm sp} + (1 - K_p)C_{\rm p}, \tag{12}$$

where $C_{\rm p}$ is the specific heat of the gas at constant pressure.

The ratio of the specific heats of the mixture is given by (Pai et al. 1980; Pai 1977; Marble 1970)

$$\Gamma = \frac{C_{\rm pm}}{C_{\rm vm}} = \gamma \frac{1 + \frac{\delta\beta'}{\gamma}}{1 + \delta\beta'}, \tag{13}$$

where $\gamma = \frac{C_{\rm p}}{C_{\rm v}}, \delta = \frac{K_p}{(1-K_p)}$ and $\beta' = \frac{C_{\rm sp}}{C_{\rm v}}$. Now,

$$C_{\rm pm} - C_{\rm vm} = (1 - K_p)(C_{\rm p} - C_{\rm v}) = (1 - K_p)R^*. \tag{14}$$

The internal energy per unit mass of the mixture is therefore given by

$$U_{\rm m} = \frac{p(1 - Z)}{(\Gamma - 1)\rho}. \tag{15}$$

Also,

$$\nu = Ar, \tag{16}$$

where 'A' is the angular velocity of the medium at radial distance r from the axis of symmetry. In this case the vorticity vector $\boldsymbol{\varsigma} = \frac{1}{2}$ Curl $\boldsymbol{q}$ has the components

$$\varsigma_r = 0, \quad \varsigma_\theta = -\frac{1}{2}\frac{\partial w}{\partial r}, \quad \varsigma_{z^*} = \frac{1}{2r}\frac{\partial}{\partial r}(r\nu). \tag{17}$$

The initial density of the medium is assumed to obey the exponential law, namely,

$$\rho_a = \rho_0 e^{\delta R}, \tag{18}$$

where ρ_0 and δ are suitable constants, and R is the shock radius.

In order to obtain a solution, it is assumed that a strong cylindrical shock wave is propagating outwards from the axis of symmetry in the undisturbed medium (mixture of a perfect gas and small solid particles) with density varying exponentially, which has zero radial velocity and variable azimuthal and axial velocities. Thus

$$u = 0, \tag{19}$$

$$\nu_a = Be^{\mu R}, \tag{20}$$

$$w_a = Ce^{\alpha R}, \tag{21}$$

where B, C, λ and α are constants and the subscript 'a' refers to the values in the initial state.

Ahead of the shock, the components of the vorticity vector, therefore, vary as

$$\varsigma_{r_a} = 0, \tag{22}$$

$$\varsigma_{\theta_a} = -\frac{C\alpha}{2}e^{\alpha R}, \tag{23}$$

$$\varsigma_{z_a^*} = -\frac{B(1+\mu R)}{2R} e^{\mu R}\,. \tag{24}$$

From Equations (20) and (16), we find that the initial angular velocity varies as

$$A_a = \frac{B e^{\mu R}}{R}. \tag{25}$$

The initial volume fraction of solid particles Z_a is, in general, not a constant. However, the volume occupied by the solid particles is very small because the density of solid particles is much larger than that of the gas (Miura & Glass 1985), hence Z_a may be assumed to be a small constant (Vishwakarma 2000; Vishwakarma et al. 2008). The expression for Z_a is given by (Pai 1977; Naidu et al. 1985)

$$Z_a = \frac{V_{\rm sp}}{V_a} = \frac{K_p}{(1-K_p)G_a + K_p}\,, \tag{26}$$

where $G_a = \frac{\rho_{\rm sp}}{\rho_{{\rm g}_a}}$ is the ratio of the density of the solid particles to the initial density of the gas $\rho_{{\rm g}_a}$. Values of Z_a for some typical values of K_p and G_a are given in Table 1.

Table 1 Values of Z_a for Some Typical Values of K_p and G_a

K_p	G_a	Z_a
0.1	5	0.0217391
	10	0.0109890
	50	0.00221729
	100	0.00110988
	1000	0.0001111
0.3	5	0.0789474
	10	0.0410959
	50	0.00849858
	100	0.00426743
	1000	0.00042839

The deviation of the behavior of a mixture of perfect gas and small solid particles from that of a perfect gas is indicated in Equation (2) by the isothermal compressibility

$$\tau_{\rm iso} = \frac{1}{\rho a_{\rm iso}^2} = \frac{1-Z}{p}. \tag{27}$$

The volume fraction of solid particles lowers the compressibility of the mixture, while the mass of the solid particles increases the total mass, and therefore may add to the inertia of the mixture.

The jump conditions at the shock wave are given by the principle of conservation of mass, momentum and energy across the shock (Chaturani 1970; Vishwakarma & Nath 2006, 2009), namely

$$\begin{aligned} \rho_a V &= \rho_n (V - u_n), \\ p_a + \rho_a V^2 &= p_n + \rho_n (V - u_n)^2, \\ U_{\mathrm{m}_a} + \frac{p_a}{\rho_a} + \frac{1}{2}V^2 - \frac{F_a}{\rho_a V} &= U_{\mathrm{m}_n} + \frac{p_n}{\rho_n} + \frac{1}{2}(V - u_n)^2 - \frac{F_n}{\rho_a V}, \\ \nu_a &= \nu_n, \\ w_a &= w_n, \\ \frac{Z_a}{\rho_a} &= \frac{Z_n}{\rho_n}, \end{aligned} \tag{28}$$

where the subscript 'n' denotes the conditions immediately behind the shock front, and $V(= \frac{dR}{dt})$ denotes the velocity of the shock front; Z_a is given by Equation (24) and 'F' is the radiation heat flux.

If the shock is a strong one, then the jump conditions (28) become

$$\begin{aligned} u_n &= (1 - \beta)V, \\ \rho_n &= \frac{\rho_a}{\beta}, \\ p_n &= (1 - \beta)\rho_a V^2, \\ \nu_n &= Be^{\mu R}, \\ w_n &= Ce^{\alpha R}, \\ Z_n &= \frac{Z_a}{\beta}, \end{aligned} \tag{29}$$

where the quantity β (0< β <1) is obtained by the relation

$$\frac{2(\Gamma\beta - Z_a)}{(\Gamma - 1)} - (1 + \beta) = \frac{2(F_n - F_a)}{p_n V}. \tag{30}$$

Because the shock is strong, we assume that $(F_n - F_a)$ is negligible in comparison with the product of p_n and V (Vishwakarma & Nath 2006, 2009; Laumbach & Probstein 1970). Therefore Equation (30) reduces to

$$\beta = \frac{(\Gamma - 1 + 2Z_a)}{(\Gamma + 1)}. \tag{31}$$

Following Levin & Skopina (2004), we obtain the jump conditions for the components of the vorticity vector across the shock front as

$$\varsigma_{\theta_n} = \frac{\varsigma_{\theta_a}}{\beta}, \tag{32}$$

$$\varsigma_{z_n^*} = \frac{\varsigma_{z_a^*}}{\beta}. \tag{33}$$

Equation (5) together with Equation (7) gives

$$\frac{p}{p_n} = \frac{\rho(1 - Z_n)}{\rho_n(1 - Z)}. \tag{34}$$

Let the solution of Equations (1) to (5) be of the form (Vishwakarma 2000; Vishwakarma et al. 2008; Verma & Vishwakarma 1976, 1980)

$$u = \frac{1}{t}U(\eta), \tag{35}$$

$$\rho = t^{\Omega}D(\eta), \tag{36}$$

$$\nu = \frac{1}{t}\phi(\eta), \tag{37}$$

$$w = \frac{1}{t}W(\eta), \tag{38}$$

$$p = t^{\Omega-2}P(\eta), \tag{39}$$

where

$$\eta = t\exp(\lambda r),\ \lambda \neq 0 \tag{40}$$

and the constants Ω and λ are to be determined subsequently. We choose the shock surface to be given by

$$\eta_0 = \text{constant}, \tag{41}$$

so that its velocity is given by

$$V = -\frac{1}{\lambda t}, \tag{42}$$

which represents the outgoing shock surface, if $\lambda < 0$.

The solutions of Equations (1) to (5) in the forms (35) to (42) are compatible with the shock conditions and Equations (20) and (21), if

$$\Omega = 2, \lambda = \mu = \alpha = -\frac{\delta}{2}. \tag{43}$$

Since necessarily $\lambda < 0$, relation (43) shows that $\delta > 0$, meaning thereby that the shock surface expands outward in an exponentially increasing medium (Hayes 1968; Deb Ray 1974; Vishwakarma 2000).

From Equations (42) and (43), we obtain

$$R = \frac{2}{\delta}\log\left(\frac{t}{t_0}\right), \tag{44}$$

where t_0 is the duration of the almost instantaneous explosion.

3 SOLUTION OF THE EQUATIONS

The flow variables in the flow-field behind the shock front may be obtained by solving Equations (1) to (5) and (17). From Equations (35) to (40), and (42) and (43), we obtain

$$\frac{\partial u}{\partial t} = \lambda u V - V\frac{\partial u}{\partial r}, \tag{45}$$

$$\frac{\partial \rho}{\partial t} = -2\rho\lambda V - V\frac{\partial \rho}{\partial r}, \tag{46}$$

$$\frac{\partial p}{\partial t} = -V\frac{\partial p}{\partial r}, \tag{47}$$

$$\frac{\partial \nu}{\partial t} = \lambda \nu V - V\frac{\partial \nu}{\partial r}, \tag{48}$$

$$\frac{\partial w}{\partial t} = \lambda w V - V\frac{\partial w}{\partial r}. \tag{49}$$

Equation (34) with the aid of Equation (29) and the transformations

$$r' = \frac{r}{R}, \quad u' = \frac{u}{V}, \quad \nu' = \frac{\nu}{V}, \quad w' = \frac{w}{V}, \quad \rho' = \frac{\rho}{\rho_n}, \quad p' = \frac{p}{p_n}, \tag{50}$$

yield a relation between p' and ρ' in the form

$$p' = \frac{\rho'(\beta - Z_a)}{(\beta - Z_a\rho')}. \tag{51}$$

Using Equations (45) to (51) in the fundamental Equations (1) to (4), we obtain

$$\frac{d\rho'}{dr'} = \frac{\rho'}{(1-u')}\left[\frac{du'}{dr'} + 2\log\left(\frac{t}{t_0}\right) + \frac{u'}{r'}\right], \tag{52}$$

$$\frac{du'}{dr'} = \frac{\left\{\begin{array}{c}\left[2\beta^2(1-\beta)(\beta-Z_a) - u'(1-u')(\beta - Z_a\rho')^2\right]\log\left(\frac{t}{t_0}\right) \\ +\beta^2(1-\beta)(\beta-Z_a)\frac{u'}{r'} - (1-u')(\beta-Z_a\rho')^2\frac{\nu'^2}{r'}\end{array}\right\}}{[(\beta - Z_a\rho')^2(1-u')^2 - \beta^2(1-\beta)(\beta-Z_a)]}, \tag{53}$$

$$\frac{d\nu'}{dr'} = \frac{\nu'}{(u'-1)}\left[\log\left(\frac{t}{t_0}\right) - \frac{u'}{r'}\right], \tag{54}$$

$$\frac{dw'}{dr'} = \frac{w'}{(u'-1)}\log\left(\frac{t}{t_0}\right). \tag{55}$$

Applying the transformations (50) on Equation (17), we obtained the non-dimensional components of the vorticity vector $l_r = \frac{\varsigma_r}{V/R}$, $l_\theta = \frac{\varsigma_\theta}{V/R}$, $l_{z^*} = \frac{\varsigma_{z^*}}{V/R}$ in the flow-field behind the shock as

$$l_r = 0, \tag{56}$$

$$l_\theta = \frac{w'}{2(1-u')}\log\left(\frac{t}{t_0}\right), \tag{57}$$

$$l_{z^*} = \frac{\nu'[r'\log\left(\frac{t}{t_0}\right) - 1]}{2r'(u'-1)}. \tag{58}$$

By using Equations (29) and (50) in (28), we get the expression for the isothermal compressibility as

$$(\tau_{\text{iso}})\rho_a V^2 = \frac{(\beta - Z_a\rho')}{\beta(1-\beta)p'}. \tag{59}$$

In terms of dimensionless variables r', u', ν', w', ρ' and p', the shock conditions (29) take the form

$$r' = 1, \quad u' = (1-\beta), \quad \nu' = \frac{B\delta\eta_0}{2}, \quad w' = \frac{C\delta\eta_0}{2}, \quad \rho' = 1, \quad p' = 1. \tag{60}$$

Equations (52) to (55) can be numerically integrated with the boundary conditions (60) to obtain the solution of the problem.

4 ADIABATIC FLOW

In this section, we present the non-similarity solution for the adiabatic flow behind a strong cylindrical shock in the mixture of small solid particles and perfect gas, which is rotating about the axis of symmetry.

The strong shock conditions, which serve as the boundary conditions for the problem, will be the same as the shock conditions (60) in the case of an isothermal flow.

For adiabatic flows, Equations (2) and (5) are replaced by (Vishwakarma & Nath 2006, 2009; Steiner & Hirschler 2002)

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{1}{\rho}\frac{\partial p}{\partial r} - \frac{\nu^2}{r} = 0, \tag{61}$$

$$\frac{\partial U_{\mathrm{m}}}{\partial t} + u\frac{\partial U_{\mathrm{m}}}{\partial r} - \frac{p}{\rho^2}\left(\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r}\right) = 0. \tag{62}$$

For isentropic change of state of the mixture of small solid particles and perfect gas, under the thermodynamic equilibrium condition, we may calculate the equilibrium sound speed of the mixture, as follows

$$a_{\mathrm{m}} = \left(\frac{\partial p}{\partial \rho}\right)_S = \left[\frac{\Gamma p}{\rho(1-Z)}\right]^{\frac{1}{2}}, \tag{63}$$

where the subscript 'S' refers to the process of constant entropy.

The adiabatic compressibility of the mixture of small solid particles and perfect gas may be calculated as (c.f. Moelwyn-Hughes 1961)

$$C_{\mathrm{adi}} = -\rho\left[\frac{\partial}{\partial p}\left(\frac{1}{\rho}\right)\right]_S = \frac{1}{\rho a_{\mathrm{m}}^2} = \frac{1-Z}{\Gamma p}. \tag{64}$$

Using Equations (45) to (50) in the Equations (1), (3), (4), (61) and (62), we obtain

$$\frac{d\rho'}{dr'} = \frac{\rho'}{(1-u')}\left[\frac{du'}{dr'} + 2\log\left(\frac{t}{t_0}\right) + \frac{u'}{r'}\right], \tag{65}$$

$$\frac{dp'}{dr'} = \frac{\rho'}{\beta(1-\beta)}\left[(1-u')\frac{du'}{dr'} + u'\log\left(\frac{t}{t_0}\right) + \frac{\nu'^2}{r'}\right], \tag{66}$$

$$\frac{d\nu'}{dr'} = \frac{\nu'}{(u'-1)}\left[\log\left(\frac{t}{t_0}\right) - \frac{u'}{r'}\right], \tag{67}$$

$$\frac{dw'}{dr'} = \frac{w'}{(u'-1)}\log\left(\frac{t}{t_0}\right), \tag{68}$$

$$\frac{du'}{dr'} = \frac{\left\{\begin{array}{c}\left[\Gamma\beta^2(1-\beta)p' - u'\rho'(1-u')(\beta - Z_a\rho')\right] r'\log\left(\frac{t}{t_0}\right) \\ +\Gamma\beta^2(1-\beta)p'u' - \rho'(1-u')(\beta - Z_a\rho')\nu'^2\end{array}\right\}}{\left[(\beta - Z_a\rho')\rho'(1-u')^2 - \Gamma\beta^2(1-\beta)p'\right]}. \tag{69}$$

The transformed shock conditions and non-dimensional components of the vorticity vector will be the same as in the case of an isothermal flow. By using Equations (29) and (50) in (64), we get the expression for the adiabatic compressibility as

$$(C_{\mathrm{adi}})\rho_a V^2 = \frac{(\beta - Z_a\rho')}{\beta(1-\beta)\Gamma p'}. \tag{70}$$

The ordinary differential Equations (65) to (69) with the boundary conditions (60) can now be numerically integrated to obtain the solution for the adiabatic flow behind the shock front.

Also, the total energy of the disturbance is given by

$$E = 2\pi\int_{\bar{r}}^{R} \rho[U_{\mathrm{m}} + \frac{1}{2}(u^2 + \nu^2 + w^2)]r dr, \tag{71}$$

where $\bar{r}$ is the position of the inner boundary of the disturbance. Using Equations (15) and (50), (71) becomes

$$E = \frac{8\pi\rho_0}{\delta^2\eta_0^2\beta}R^2\int_{\bar{r}'}^{1}\left[\frac{(1-\beta)(\beta - Z_a\rho')p'}{(\Gamma - 1)} + \frac{1}{2}\rho'(u'^2 + \nu'^2 + w'^2)\right] r'\, dr'. \tag{72}$$

Hence, the total energy of the shock wave is non-constant and varies as R^2. The increase of total energy may be achieved by the pressure exerted on the fluid by the inner expanding surface (a contact surface or a piston). This surface may be, physically, the surface of the stellar corona or the condensed explosives or the diaphragm containing a very high-pressure driver gas. By sudden expansion of the stellar corona or the detonation products or the driver gas into the ambient gas, a shock wave is produced in the ambient gas. The shocked gas is separated from this expanding surface which is a contact discontinuity. This contact surface acts as a 'piston' for the shock wave. Thus, the flow is headed by a shock front and has an expanding surface as an inner boundary. A situation very much of the same kind may prevail during the formation of a cylindrical spark channel from exploding wires. In addition, in the usual cases of spark break down, time-dependent energy input is a more realistic assumption than instantaneous energy input (Freeman & Cragges 1969; Director & Dabora 1977).

5 RESULTS AND DISCUSSION

The distribution of the flow variables behind the shock front is obtained by the numerical integration of Equations (52)–(55), (57) and (58) for an isothermal flow, and by Equations (65)–(69) for an adiabatic flow with the boundary conditions (60) by the Runge-Kutta method of the fourth order. For the purpose of numerical integration, the values of the constant parameters are taken to be (Pai et al. 1980; Miura & Glass 1983; Vishwakarma & Nath 2006; Vishwakarma et al. 2008) $\gamma = 1.4$; $K_p = 0, 0.1, 0.3$; G_a=5, 10, 50, 100; $\beta' = 1$ and $\frac{t}{t_0} = 1.7$. The values $\gamma = 1.4$, $\beta' = 1$ may correspond to the mixture of air and glass particles (Miura & Glass 1985). In our analysis, we have assumed the initial volume fraction of solid particles Z_a to be a small constant. The values $K_p = 0.1, 0.3$ with G_a=5, 10, 50, 100, 1000 give small values of Z_a (see Table 1). The value $K_p = 0$ corresponds to the dust-free case.

Figures 1 and 2 show the variation of the flow variables ρ', p', u', ν', w', l_θ, l_{z^*} and the compressibility for $\frac{t}{t_0} = 1.7$ and for various values of the parameters K_p, G_a in isothermal and adiabatic cases, respectively. Figure 2(a) shows that there is an abrupt fall of density distribution near the inner boundary surface, and the derivative of the density tends to negative infinity. This is quite expected and may be explained as follows: the path of the decelerated inner boundary surface (piston) diverges from the path of the particle immediately ahead which rarefies the gas.

It is evident from Figure 1(a) that in the case of an isothermal flow the density distribution is finite at the inner boundary surface for all values of K_p and G_a. Thus one may note that the feature of abrupt fall of density near the inner boundary surface in the adiabatic flow is absent when the flow is isothermal for all values of K_p and G_a. This seems to be necessary because with an unbounded density near the inner boundary surface, the temperature there approaches infinity, which violates the basic assumption of zero temperature gradient throughout the flow. Thus it may be observed that the assumption of zero temperature gradient brings a profound change in the density distribution as compared to the adiabatic flow; whereas the pressures, compressibility, components of velocity and vorticity vector are little affected.

Table 2 shows the variation of the density ratio β across the shock front and the position of the inner boundary surface $\overline{r'}$ for different values of K_p, G_a with $\frac{t}{t_0} = 1.7$, $\beta' = 1$ and $\gamma = 1.4$.

It is found that the effects of an increase in the ratio of the density of the solid particles to the initial density of the gas G_a at a given instant are

(i) to decrease β (i.e. to increase the shock strength, see Table 2);

(ii) to decrease the flow variables ρ', p' and ν' in general, but to increase the pressure p' in the case of isothermal flow, when $G_a \leq 10$ (see Figs. 1(a, b, d) and 2(a, b, d));

(iii) to increase the flow variables u', w', l_θ, l_{z^*} and the compressibility. The increase in the compressibility causes stronger compression of the gas behind the shock wave and, hence, an increase in the shock strength (see Figs. 1(c, e, f, g, h) and 2(c, e, f, g, h));

(iv) to decrease the distance of the inner boundary surface and the shock front (see Table 2). This means that an increase in the ratio of the density of the solid particles to the initial density of the gas has an effect of increasing the shock strength, which is the same as indicated in (i) and (iii) above.

The above effects are more impressive at higher values of K_p. These effects may be physically interpreted as follows.

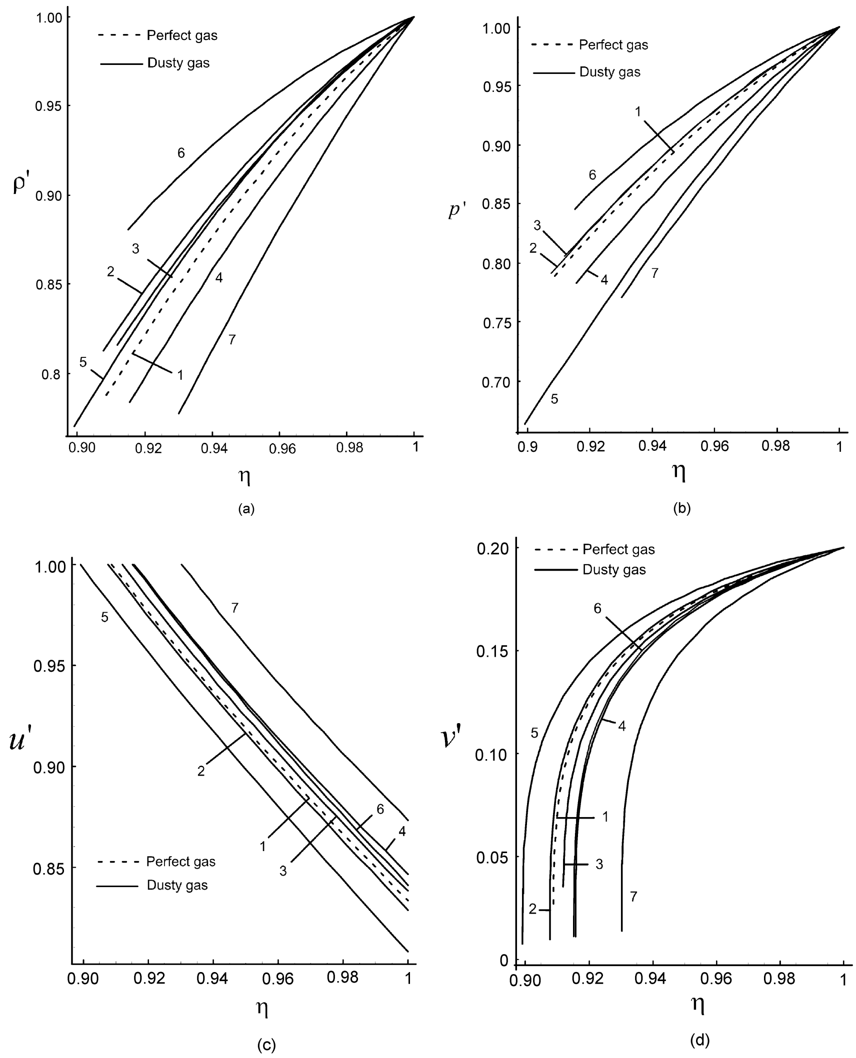

Fig. 1 Variation of the flow variables in the region behind the shock front for $\frac{t}{t_0} = 1.7$ in the case of isothermal flows: (1). $K_p = 0$, $G_a = 0$; (2). $K_p = 0.1$, $G_a = 5$; (3). $K_p = 0.1$, $G_a = 10$; (4). $K_p = 0.1$, $G_a = 100$; (5). $K_p = 0.3$, $G_a = 5$; (6). $K_p = 0.3$, $G_a = 10$; (7). $K_p = 0.3$, $G_a = 100$. (a) density ρ', (b) pressure p', (c) radial component of fluid velocity u', (d) azimuthal component of fluid velocity ν', (e) axial component of fluid velocity w' (f) azimuthal component of vorticity vector l_θ, (g) axial component of vorticity vector l_{z^*}, and (h) isothermal compressibility $(\tau_{\text{iso}})\ \rho_a\ V^2$.

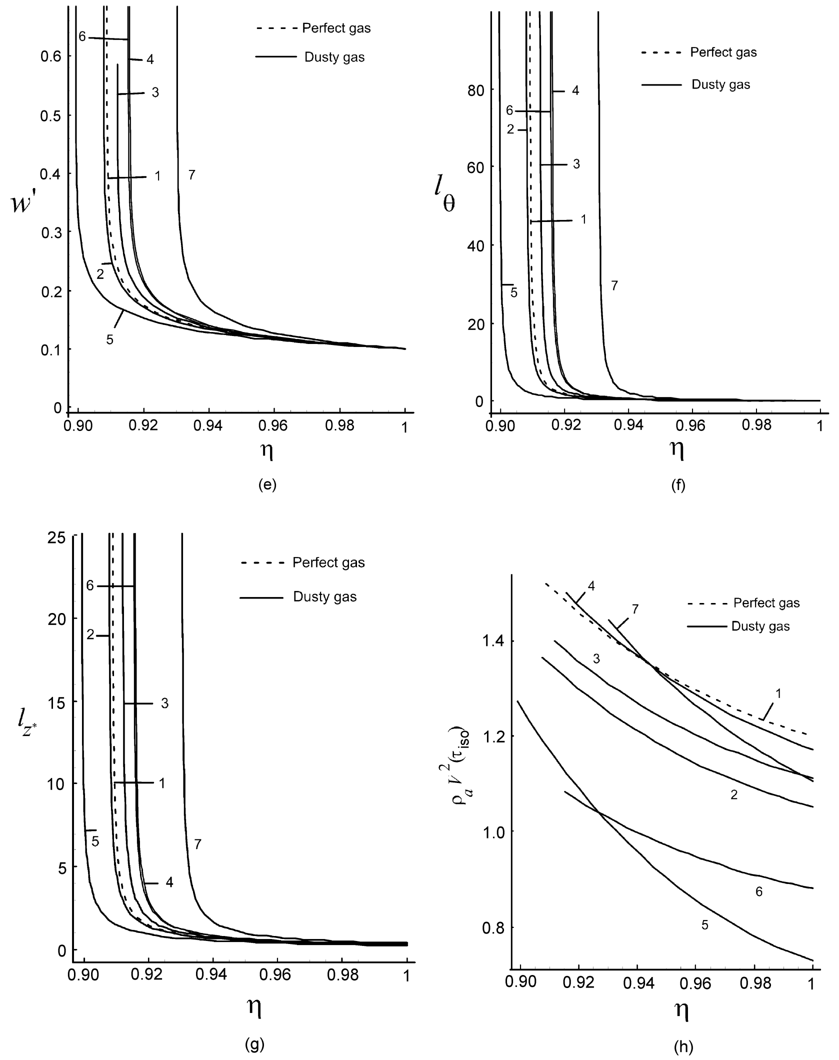

Fig. 1 *Continued.*

With an increase in G_a (at constant K_p), there is a strong decrease in Z_a, i.e. the volume fraction of solid particles in the undisturbed medium becomes comparatively very small. This causes comparatively more compression of the mixture in the region between shock and the inner boundary surface, which displays the above effects.

Effects of an increase in the mass concentration of the solid particles K_p at a given instant are

(i) to decrease the shock strength (to increase the value of β) when $G_a = 5$, and to increase it, when $G_a \geq 10$ (see Table 2);
(ii) to increase the distance of the inner boundary surface and the shock front (see Table 2) when $G_a = 5$. At higher values of G_a the effect is of an opposite nature;
(iii) to decrease the flow variables u', w', l_θ, l_{z^*} when $G_a = 5$, and to increase them, when $G_a \geq 10$ (see Figs. 1(c, e, f, g) and 2(c, e, f, g));
(iv) to decrease the flow variables ρ', p' and ν' in general when $G_a \geq 10$, but to increase ν', ρ' and p' (in the case of isothermal flow) when $G_a = 5$ (see Figs. 1(a, b, d) and 2(a, b, d));

(v) to decrease the compressibility when $G_a = 5,\ 10$, but to increase it when $G_a = 100$ (see Figs. 1(h) and 2(h)).

Physical interpretations of these effects are as follows.

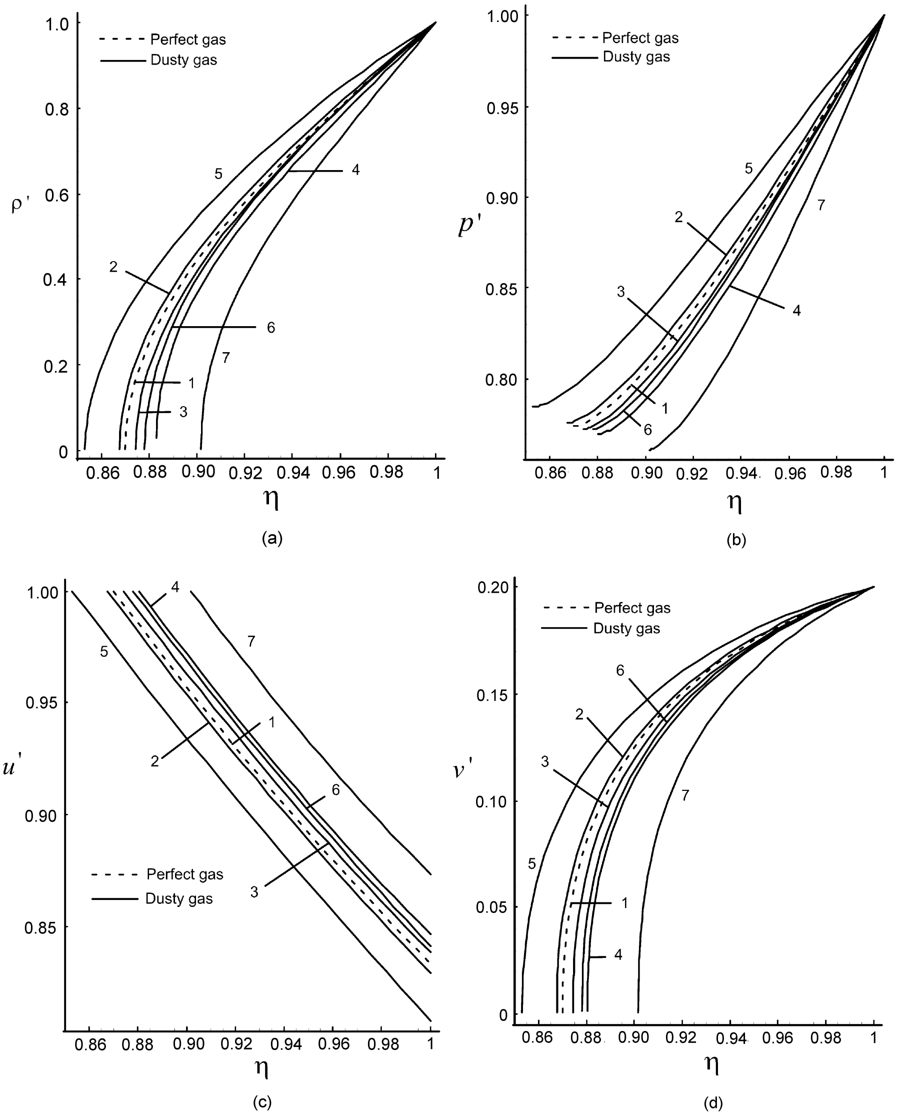

Fig. 2 Variation of the flow variables in the region behind the shock front for $\frac{t}{t_0} = 1.7$ in the case of adiabatic flows: (1). $K_p = 0$, $G_a = 0$; (2). $K_p = 0.1$, $G_a = 5$; (3). $K_p = 0.1$, $G_a = 10$; (4). $K_p = 0.1$, $G_a = 100$; (5). $K_p = 0.3$, $G_a = 5$; (6). $K_p = 0.3$, $G_a = 10$; (7). $K_p = 0.3$, $G_a = 100$. (a) density ρ', (b) pressure p', (c) radial component of fluid velocity u', (d) azimuthal component of fluid velocity ν', (e) axial component of fluid velocity w', (f) azimuthal component of vorticity vector l_θ, (g) axial component of vorticity vector l_{z^*}, and (h) adiabatic compressibility $(C_{\text{adi}})\ \rho_a\ V^2$.

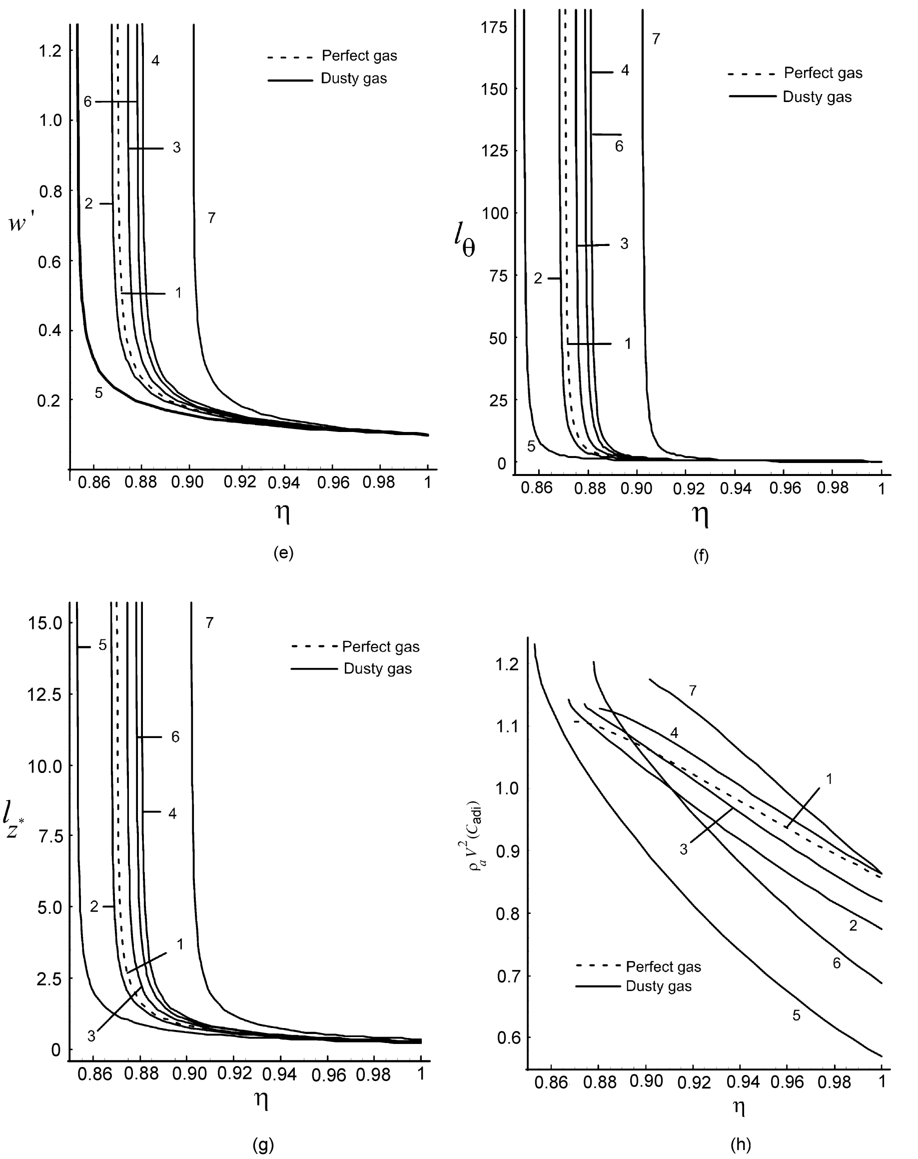

Fig. 2 *Continued.*

In the case of $G_a = 5$, small solid particles of density equal to five times that of the perfect gas in the mixture occupy a significant portion of the volume which remarkably lowers the compressibility of the medium. Then, an increase in K_p further reduces the compressibility which causes an increase in the distance between the shock front and the inner boundary surface, a decrease in the shock strength, and the above behavior of the flow variables. In the case of $G_a = 100$, small solid particles of density equal to one hundred times that of the perfect gas in the mixture occupy a very small portion of the volume, and therefore compressibility is not lowered much; but the inertia of the mixture is increased significantly due to the particle load. An increase in K_p, from 0.1 to 0.3 for $G_a = 100$, means that the perfect gas in the mixture constituting 90% of the total mass and occupying 99.889% of the total volume now constitutes 70% of the total mass and occupies 99.573% of

Table 2 Variation of the density ratio β across the shock front and the position of the inner boundary surface for different values of K_p and G_a with $\frac{t}{t_0} = 1.7$, $\beta^{'} = 1$ and $\gamma = 1.4$.

K_p	Γ	G_a	Z_a	β	Position of the inner boundary surface $\overline{r'}$	
					Isothermal flow	Adiabatic flow
0	1.4	–	0	0.16667	0.908668	0.869988
0.1	1.36	5	0.0217391	0.170965	0.907617	0.867523
		10	0.0109890	0.161855	0.911818	0.8742688
		50	0.00221729	0.154421	0.915221	0.879727
		100	0.00110988	0.153483	0.915649	0.880413
0.3	1.28	5	0.0789474	0.192059	0.899084	0.853056
		10	0.0410959	0.158856	0.915089	0.8781055
		50	0.00849858	0.130262	0.928493	0.899024
		100	0.00426743	0.126550	0.930194	0.9016962

the total volume. Due to this fact, the density of the perfect gas in the mixture is highly decreased which overcomes the effect of incompressibility of the mixture and ultimately causes a small decrease in the distance between the inner boundary surface and the shock front, an increase in the shock strength, and the above nature of the flow variables.

The present non-similar model may be used to describe some of the overall features of a "driven" shock wave produced by a flare energy release 'E' (c.f. Eq. (72)) that is time dependent. The energy 'E' increases with time and the solutions then correspond to a blast wave produced by intense, prolonged flare activity in a rotating star when the wave is driven by fresh erupting plasma for some time and its energy tends to increase as it propagates from the star into a cold atmosphere whose density varies exponentially with altitude. The atmospheric scale height of a star is generally small compared to its radius so that the solutions still describe a stellar explosion.

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