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Bianchi type-I cosmological models with perfect fluid in general relativity

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Abstract Einstein's field equations with variable gravitational and cosmological constants are considered in the presence of perfect fluid for the Bianchi type-I universe by assuming that the cosmological term is proportional to R^{-m} (R is a scale factor and m is a constant). A variety of solutions are presented. The physical significance of the respective cosmological models are also discussed.

Key words: cosmological parameters — cosmology: theory — relativity

1 INTRODUCTION

Einstein's field equations are a coupled system of highly nonlinear differential equations and we seek physical solutions to the field equations for applications in cosmology and astrophysics. In order to solve the field equations, we normally assume a form of the matter content or suppose that space-time allows killing vector symmetry (Kramer & Schmutzer 1980).

Solutions to the field equations may also be generated by the law of variation of the scale factor which was proposed by Pavón (1991). The behavior of the cosmological scale factor $R(t)$ in the solution of Einstein's field equations with Robertson-Walker line elements has been the subject of numerous studies. In earlier literature, cosmological models where the cosmological term is proportional to the scale factor have been studied by Hoyle et al. (1997), Olson & Jordan (1987), Pavón (1991), Maia & Silva (1994), Silveira & Waga (1994), Silveira & Waga (1997) and Bloomfield Torres & Waga (1996). Chen & Wu (1990) considered Λ varying with R^{-2} (R is the scale factor), Carvalho et al. (1992) generated it by taking $\Lambda = \alpha R^{-2} + \beta H^2$, where R is the scale factor of the Robertson-Walker metric, H is the Hubble parameter and α and β are adjustable dimensionless parameters on the basis of quantum field estimations in the curved expanding background.

The idea of a variable gravitational constant G in the framework of general relativity was first proposed by Dirac (1937). Lau (1985), working in the framework of general relativity, proposed a modification linking the variation of G with that of Λ. This modification allows us to use Einstein's field equations formally unchanged since variation in Λ is accompanied by a variation of G. A number of authors investigated FRW models and Bianchi models using this approach (Abdel-Rahman 1990; Berman 1990; Sisteró 1991; Kalligas et al. 1992; Abdussattar & Vishwakarma 1997; Vishwakarma 2000, 2005; Pradhan & Otarod 2006; Singh et al. 2007; Singh & Tiwari 2008). Borges & Carneiro (2005) have considered that the cosmological term is proportional to the Hubble parameter in the FRW model and the Bianchi type-I model with variables G and Λ. Recently, I have (present author 2008) considered whether or not the cosmological term is proportional to the Hubble parameter in the Bianchi type-I model with varying G and Λ.

In this paper, we study homogeneous Bianchi type-I space-time with variables G and Λ containing matter in the form of a perfect fluid. We obtain solutions of the field equations assuming that the cosmological term is proportional to R^{-m} (where R is a scale factor and m is constant). The paper is organized as follows. Basic equations of the models are given in Section 2, and their solution in Section 3. We discuss the models and conclude our results in Section 4.

2 MODEL AND FIELD EQUATIONS

We consider the space-time admitting Bianchi type-I group of motions in the form

$$ds^2 = -dt^2 + A^2(t)dx^2 + B^2(t)dy^2 + C^2(t)dz^2. \tag{1}$$

We assume that the cosmic matter is represented by the energy momentum tensor of a perfect fluid

$$T_{ij} = (\rho + p)v_i v_j + p g_{ij}, \tag{2}$$

where ρ is the energy density of the cosmic matter, p is its pressure, and v_i is the four velocity vector such that $v_i v^i = 1$. We take the equation of state

$$p = \omega\rho, \quad 0 \leq \omega \leq 1. \tag{3}$$

The Einstein field equations with time dependent G and Λ given by (Weinberg 1972) are

$$R_{ij} - \frac{1}{2}R g_{ij} = -8\pi G(t)T_{ij} + \Lambda(t)g_{ij}, \tag{4}$$

so that for metric (1) and energy-momentum tensor (2) in a co-moving system of co-ordinates, the field Equation (4) yields

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = -8\pi G p + \Lambda, \tag{5}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} = -8\pi G p + \Lambda, \tag{6}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = -8\pi G p + \Lambda, \tag{7}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} = 8\pi G\rho + \Lambda. \tag{8}$$

In view of the vanishing divergence of the Einstein tensor, we have

$$8\pi G\Big[\dot{\rho} + \Big(\rho + p\Big)\Big(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\Big)\Big] + 8\pi\rho\dot{G} + \dot{\Lambda} = 0. \tag{9}$$

The usual energy conservation equation $T^j_{i;j} = 0$ yields

$$\dot{\rho} + \Big(\rho + p\Big)\Big(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\Big) = 0. \tag{10}$$

Equation (9) together with Equation (10) puts G and Λ in some sort of a coupled field given by

$$8\pi\rho\dot{G} + \dot{\Lambda} = 0, \tag{11}$$

here and elsewhere a dot denotes ordinary differentiation with respect to t. Equation (11) implies that Λ is a constant whenever G is constant. Inserting Equation (3) into Equation (10) and then integrating, we obtain

$$\rho = \frac{k}{R^{3(\omega+1)}}, \tag{12}$$

where $k > 0$ is a constant of integration.

Let R be the average scale factor of the Bianchi type-I universe, i.e.

$$R^3 = ABC. \tag{13}$$

From Equations (5), (6) and (7), we obtain

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = \frac{k_1}{R^3}, \tag{14}$$

and

$$\frac{\dot{B}}{B} - \frac{\dot{C}}{C} = \frac{k_2}{R^3}, \tag{15}$$

where k_1 and k_2 are constants of integration. The Hubble parameter H, volume expansion θ, sheer σ and deceleration parameter q are given by

$$\begin{aligned} H &= \frac{\theta}{3} = \frac{\dot{R}}{R}, \\ \sigma &= \frac{k}{\sqrt{3}R^3}, \\ q &= -1 - \frac{\dot{H}}{H^2} = -\frac{-R\ddot{R}}{\dot{R}^2}. \end{aligned}$$

Equations (5)–(8) and (10) can be written in terms of H, σ and q as

$$H^2\Big(2q - 1\Big) - \sigma^2 = 8\pi Gp - \Lambda, \tag{16}$$

$$3H^2 - \sigma^2 = 8\pi G\rho + \Lambda, \tag{17}$$

$$\dot{\rho} + 3\Big(\rho + p\Big)\frac{\dot{R}}{R} = 0. \tag{18}$$

Overduin & Cooperstock (1998) define

$$\rho_c = \frac{3H^2}{8\pi G}, \tag{19}$$

$$\rho_v = \frac{\Lambda}{8\pi G}, \tag{20}$$

and

$$\Omega = \frac{\rho}{\rho_c} = \frac{8\pi G\rho}{3H^2}, \tag{21}$$

which are, respectively, the critical density, vacuum density and density parameter. From Equation (16), we obtain

$$\frac{\sigma^2}{\theta^2} = \frac{1}{3} - \frac{8\pi G\rho}{\theta^2} - \frac{\Lambda}{\theta^2}.$$

Therefore, $0 \leq \frac{\sigma^2}{\theta^2} \leq \frac{1}{3}$ and $0 \leq \frac{8\pi G\rho}{\theta^2} \leq \frac{1}{3}$ for $\Lambda \geq 0$. Thus, the presence of a positive Λ puts a restriction on the upper limit of anisotropy, whereas a negative Λ contributes to the anisotropy. From Equations (16) and (17), we have $\frac{d\theta}{dt} = -12\pi Gp - \frac{\theta^2}{2} + \frac{3\Lambda}{2} - \frac{3}{2}\sigma^2 = -12\pi G(\rho + p) - 3\sigma^2$. Thus, the universe will be in a decelerating phase for negative Λ, and for positive Λ, the universe will slow its rate of decrease. Also $\dot{\sigma} = -\frac{3\sigma\dot{R}}{R}$ implies that σ decreases in an evolving universe and it is negligible for an infinitely large value of R.

3 SOLUTION OF THE FIELD EQUATIONS

The system of Equations (3), (5)–(8), and (11) supply only six equations in seven unknowns (A, B, C, ρ, p, G and Λ). One extra equation is needed to solve the system completely. The phenomenological Λ decay scenarios have been considered by a number of authors. Chen & Wu (1990) considered $\Lambda \propto a^{-2}$ (a is the scale factor of the Robertson-Walker metric).

Hoyle et al. (1997) considered $\Lambda \propto a^{-3}$ whereas $\Lambda \propto a^{-m}$ (a is the scale factor and m is constant) considered by Olson & Jordan (1987), Pavón (1991), Maia & Silva (1994), Silveira & Waga (1994), Silveira & Waga (1997), and Bloomfield Torres & Waga (1996). Thus, we take the decaying vacuum energy density

$$\Lambda = \frac{a}{R^m}, \tag{22}$$

where a and m are positive constants. Substituting Equations (12) and (22) into Equation (11), we get

$$G = \frac{am}{8\pi k}\frac{R^{3\omega+3-m}}{(3\omega+3-m)}. \tag{23}$$

From Equations (16), (17), (22) and (23), we get

$$\frac{\ddot{R}}{R} + 2\Big(\frac{\dot{R}}{R}\Big)^2 - \frac{am(1-\omega)}{2(3\omega+3-m)R^m} - \frac{a}{R^m} = 0. \tag{24}$$

Now we analyze for different values of ω.

3.1 Matter Dominated Solution (Cosmology for $\omega = 0$)

For $\omega = 0$, Equation (24) becomes

$$\frac{\ddot{R}}{R} + 2\Big(\frac{\dot{R}}{R}\Big)^2 - \frac{a\Big(m-6\Big)}{2\Big(m-3\Big)}\frac{1}{R^m} = 0. \tag{25}$$

To determine the time evolution of the Hubble parameter, integrating Equation (25), we get

$$\frac{\dot{R}}{R} = H = \sqrt{\frac{a}{3-m}}\Big[\frac{m}{2}\sqrt{\Big(\frac{a}{3-m}\Big)}t + t_0\Big]^{-1}, \tag{26}$$

where t_0 is a constant of integration. The integration constant is related to the choice of origin of time. From Equation (26), we obtain the scale factor

$$R = \Big[\frac{m}{2}\sqrt{\Big(\frac{a}{3-m}\Big)}t + t_0\Big]^{2/m}. \tag{27}$$

By inserting Equation (27) into Equations (14) and (15), the metric (1) has the form

$$\begin{aligned} ds^2 \; = \; & -dt^2 + \Big(\frac{m}{2}\sqrt{\frac{a}{3-m}}t + t_0\Big)^{\frac{4}{m}} \times \\ & \Big[m_1^2 \exp 2\Big\{\frac{(2k_1+k_2)}{3}2\sqrt{\frac{3-m}{a}}\frac{1}{m-6}\Big(\frac{m}{2}\sqrt{\frac{a}{3-m}}t+t_0\Big)^{\frac{m-6}{m}}\Big\}dx^2 \\ & + \; m_2^2 \exp 2\Big\{\frac{(k_2-k_1)}{3}2\sqrt{\frac{3-m}{a}}\frac{1}{m-6}\Big(\frac{m}{2}\sqrt{\frac{a}{3-m}}t+t_0\Big)^{\frac{m-6}{m}}\Big\}dy^2 \\ & + \; m_3^2 \exp 2\Big\{\frac{-(k_1+2k_2)}{3}2\sqrt{\frac{3-m}{a}}\frac{1}{m-6}\Big(\frac{m}{2}\sqrt{\frac{a}{3-m}}t+t_0\Big)^{\frac{m-6}{m}}\Big\}dz^2\Big], \end{aligned} \tag{28}$$

where m_1, m_2 and m_3 are constants.

For the model (28), the spatial volume V, matter density ρ, pressure p, gravitational constant G, and cosmological constant Λ are given by

$$V = \left[\frac{m}{2}\sqrt{\left(\frac{a}{3-m}\right)}\mathrm{t} + \mathrm{t}_0\right]^{6/\mathrm{m}}, \tag{29}$$

$$\rho = \frac{k}{\left[\frac{m}{2}\sqrt{\left(\frac{a}{3-m}\right)}\mathrm{t} + \mathrm{t}\right]^{\frac{6}{\mathrm{m}}}}, \tag{29$'$}$$

$$p = 0, \tag{30}$$

$$G = \frac{am}{8\pi k(3-m)}\left[\frac{m}{2}\sqrt{\left(\frac{a}{3-m}\right)}\mathrm{t} + \mathrm{t}_0\right]^{\frac{2}{\mathrm{m}}(3-\mathrm{m})}, \tag{31}$$

$$\Lambda = a\left[\frac{m}{2}\sqrt{\left(\frac{a}{3-m}\right)}t + t_0\right]^{-2}. \tag{32}$$

Expansion scalar θ and shear σ are given by

$$\theta = 3\sqrt{\frac{a}{3-m}}\left[\frac{m}{2}\sqrt{\left(\frac{a}{3-m}\right)}t + t_0\right]^{-1}, \tag{33}$$

$$\sigma = \frac{k}{\sqrt{3}}\left[\frac{m}{2}\sqrt{\left(\frac{a}{3-m}\right)}t + t_0\right]^{-6/m}. \tag{34}$$

The density parameter is given by

$$\Omega = \frac{\rho}{\rho_c} = \frac{m}{3}. \tag{35}$$

The deceleration parameter q for the model is

$$q = \frac{m}{2} - 1. \tag{36}$$

The vacuum energy density ρ_v and critical density ρ_c are given by

$$\rho_v = \frac{k(3-m)}{m}\left[\frac{m}{2}\sqrt{\left(\frac{a}{3-m}\right)}t + t_0\right]^{-6/m}, \tag{37}$$

$$\rho_c = \frac{3k}{m}\left[\frac{m}{2}\sqrt{\left(\frac{a}{3-m}\right)}\mathrm{t} + \mathrm{t}_0\right]^{-6/\mathrm{m}}. \tag{38}$$

Thus for the model (28), we find that for $0 < m < 3$ the spatial volume V is zero at $t = t'$, where $t' = \frac{-t_0}{\frac{m}{2}\sqrt{\frac{a}{3-m}}}$ and expansion scalar θ is infinite, which shows that the universe starts evolving with zero volume at $t = t'$ with an infinite rate of expansion. The scale factors also vanish at $t = t'$ and hence the space-time exhibits a point type singularity during the initial epoch. The energy density shear scalar diverges at the initial singularity. As t increases, the scale factors and spatial volume increase but the expansion scalar decreases. Thus, the rate of expansion slows down with an increase in time. Also ρ, σ, ρ_v, ρ_c, and Λ decrease as t increases. As $t \to\infty$, scale factors and volume become infinite whereas ρ, σ, ρ_v, ρ_c, and Λ tend to zero. Therefore, the model would essentially give an empty universe for large time t. Gravitational constant $G(t)$ is zero at $t = t'$ and as t increases, $G(t)$ also increases.

A partial list of cosmological models in which the gravitational constant G is increasing with time are contained in Abdel-Rahman (1990), Chow (1981), Levitt (1980) and Milne (1935). The ratio $\frac{\sigma}{\theta} \to 0$ as $t \to \infty$ provided $m < 3$. So, the model approaches isotropy for large values of t. Thus, the model represents a shearing, non-rotating and expanding model of the universe with a big bang start approaching isotropy at late times.

Further, it is observed that when $2<m<3$, $q>0$; $q = 0$ for $m = 2$ and for $0<m<2$, $q<0$. Therefore, the universe begins with decelerating expansion changes and the expansion changes from a decelerating phase to an accelerating one. This cosmological scenario is in agreement with SNe Ia astronomical observations (Knop et al. 2003; Riess et al. 1998, 2004; Spergel et al. 2007; Tegmark et al. 2004; Perlmutter et al. 1998) and it presents a unified description of the evolution of the universe.

3.2 Zel'dovich Fluid Distribution (Cosmology for $\omega = 1$)

It corresponds to the equation of state $\rho = p$. This equation of state has been widely used in general relativity to obtain stellar and cosmological models for utterly dense matter (Zel'dovich 1968)

$$\frac{\ddot{R}}{R} + 2\Big(\frac{\dot{R}}{R}\Big)^2 - \frac{a}{R^m} = 0. \tag{39}$$

To determine the time evolution of the Hubble parameter, integrating Equation (39) we get

$$\frac{\dot{R}}{R} = H = \sqrt{\frac{2a}{6-m}}\Big[\frac{m}{2}\sqrt{\Big(\frac{2a}{6-m}\Big)}t + t_0\Big]^{-1}, \tag{40}$$

where the integration constant t_0 is related to the choice of origin of time. From Equation (39) we obtain

$$R = \Big[\frac{m}{2}\sqrt{\Big(\frac{2a}{6-m}\Big)}t + t_0\Big]^{\frac{2}{m}}. \tag{41}$$

By substituting Equation (41) into Equations (14) and (15), the metric (1) has the form

$$\begin{aligned} ds^2 \;=\; & -dt^2 + \Big(\frac{m}{2}\sqrt{\frac{2a}{6-m}}t + t_0\Big)^{\frac{4}{m}} \times \\ & \Big[m_1^2 \exp 2\Big\{\frac{\big(2k_1 + k_2\big)}{3} 2\sqrt{\frac{6-m}{2a}}\frac{1}{m-6}\Big(\frac{m}{2}\sqrt{\frac{2a}{6-m}}t + t_0\Big)^{\frac{m-6}{m}}\Big\}dx^2 \\ & + m_2^2 \exp 2\Big\{\frac{\big(k_2 - k_1\big)}{3} 2\sqrt{\frac{6-m}{2a}}\frac{1}{m-6}\Big(\frac{m}{2}\sqrt{\frac{2a}{6-m}}t + t_0\Big)^{\frac{m-6}{m}}\Big\}dy^2 \\ & + m_3^2 \exp 2\Big\{\frac{-\big(k_1 + 2k_2\big)}{3} 2\sqrt{\frac{6-m}{2a}}\frac{1}{m-6}\Big(\frac{m}{2}\sqrt{\frac{2a}{6-m}}t + t_0\Big)^{\frac{m-6}{m}}\Big\}dz^2\Big]. \end{aligned} \tag{42}$$

For the model (42), spatial volume V, matter density ρ, pressure p, gravitational constant G, and cosmological constant Λ are given by

$$V = \Big[\frac{m}{2}\sqrt{\Big(\frac{2a}{6-m}\Big)}t + t_0\Big]^{\frac{6}{m}}, \tag{43}$$

$$\rho = p = \frac{k}{\Big(\frac{m}{2}\sqrt{\frac{2a}{6-m}}t + t_0\Big)^{\frac{12}{m}}}, \tag{43$'$}$$

$$G = \frac{am}{8\pi k(6-m)}\Big[\frac{m}{2}\sqrt{\frac{2a}{6-m}}t + t_0\Big]^{\frac{2}{m}(6-m)}, \tag{44}$$

$$\Lambda = \frac{a}{\left[\frac{m}{2}\sqrt{\frac{a}{6-m}}t + t_0\right]^2}. \tag{45}$$

Expansion scalar θ and shear σ are obtained by

$$\theta = 3\sqrt{\frac{2a}{6-m}}\left[\frac{m}{2}\sqrt{\frac{2a}{6-m}}t + t_0\right]^{-1}, \tag{46}$$

$$\sigma = \frac{k}{\sqrt{3}}\left[\frac{m}{2}\sqrt{\frac{2a}{6-m}}t + t_0\right]^{-6/m}. \tag{47}$$

The density parameter is given by

$$\Omega = \frac{\rho}{\rho_c} = \frac{m}{6}. \tag{48}$$

The deceleration parameter q for the model is

$$q = \frac{m}{2} - 1. \tag{49}$$

The vacuum energy density ρ_v and critical density ρ_c are given by

$$\rho_v = \frac{k(6-m)}{m}\left[\frac{m}{2}\sqrt{\frac{2a}{6-m}}t + t_0\right]^{-12/\mathrm{m}}, \tag{50}$$

$$\rho_c = \frac{6k}{m}\left[\frac{m}{2}\sqrt{\frac{2a}{6-m}}t + t_0\right]^{-12/\mathrm{m}}. \tag{51}$$

In the model, we observe that for $m < 6$, the spatial volume V is zero at $t = \frac{-t_0}{\frac{m}{2}\sqrt{\frac{2a}{6-m}}} = t''$ and expansion scalar θ is infinite at $t = t''$ which shows that the universe starts evolving with zero volume and infinite rate of expansion at $t = t''$. Initially at $t = t''$, the energy density ρ, pressure p, Λ and shear scalar σ are infinite. As t increases, the spatial volume increases but the expansion scalar decreases. Thus, the expansion rate decreases as time increases. As t tends to ∞, the spatial volume V becomes infinitely large. As t increases all the parameters p, ρ, Λ, θ, ρ_c, and ρ_v, decrease and tend to zero asymptotically. Therefore, the model essentially gives an empty universe for large t. The ratio $\frac{\sigma}{\theta} \to 0$ as $t \to \infty$, which shows that the model approaches isotropy for large values of t. The gravitational constant $G(t)$ is zero at $t = t''$ and as t increases, G increases and it becomes infinitely large at late times.

Further, we observe that $\Lambda \propto \frac{1}{t^2}$ which follows from the model of Kalligas et al. (1992), Berman (1990), Berman & Som (1990), Berman et al. (1989) and Bertolami (1986a,b). This form of Λ is physically reasonable as observations suggest that Λ is very small in the present universe.

3.3 Radiation Dominated Solution ($\rho = 3p$) (Cosmology for $\omega = 1/3$)

In this case, Equation (24) becomes

$$\frac{\ddot{R}}{R} + 2\left(\frac{\dot{R}}{R}\right)^2 + \frac{2(m-6)a}{3(4-m)R^m} = 0. \tag{52}$$

To determine the time evolution of Hubble's parameter, integrating Equation (52), we get

$$\frac{\dot{R}}{R} = H = \sqrt{\frac{4a}{3(4-m)}}\left[\frac{m}{2}\sqrt{\frac{4a}{3(4-m)}}t + t_0\right]^{-1}, \tag{53}$$

where t_0 is the constant of integration and the integration constant t_0 is related to the choice of origin of time. From Equation (53) we obtain

$$R = \left[\frac{m}{2}\sqrt{\frac{4a}{3(4-m)}}t + t_0\right]^{2/m}. \tag{54}$$

By inserting Equation (54) into Equations (14) and (15), the metric (1) has the form

$$\begin{aligned} ds^2 \;=\; & -dt^2 + \left(\frac{m}{2}\sqrt{\frac{4a}{3(4-m)}}t + t_0\right)^{\frac{m}{4}} \times \\ & \Bigg[m_1^2 \exp 2\Bigg\{\frac{\left(2k_1 + k_2\right)}{3}\frac{1}{(m-6)}\sqrt{\frac{3(4-m)}{a}}\left(\frac{m}{2}\sqrt{\frac{4a}{3(4-m)}}t + t_0\right)^{\frac{m-6}{m}}\Bigg\}dx^2 \\ & + m_2^2 \exp 2\Bigg\{\frac{\left(k_2 - k_1\right)}{3}\frac{1}{(m-6)}\sqrt{\frac{3(4-m)}{a}}\left(\frac{m}{2}\sqrt{\frac{4a}{3(4-m)}}t + t_0\right)^{\frac{m-6}{m}}\Bigg\}dy^2 \\ & + m_3^2 \exp 2\Bigg\{\frac{-\left(k_1 + 2k_2\right)}{3}\frac{1}{(m-6)}\sqrt{\frac{3(4-m)}{a}}\left(\frac{m}{2}\sqrt{\frac{4a}{3(4-m)}}t + t_0\right)^{\frac{m-6}{m}}\Bigg\}dz^2\Bigg]. \end{aligned} \tag{55}$$

For the model (55), matter density ρ, pressure p, gravitational constant G, and cosmological constant Λ are given by

$$\rho = \frac{k}{\left(\frac{m}{2}\sqrt{\frac{4a}{3(4-m)}}t + t_0\right)^{\frac{8}{m}}}, \tag{56}$$

$$p = \frac{k}{3\left(\frac{m}{2}\sqrt{\frac{4a}{3(4-m)}}t + t_0\right)^{\frac{8}{m}}}, \tag{57}$$

$$G = \frac{am}{8\pi k(4-m)}\left[\frac{m}{2}\sqrt{\frac{4a}{3(4-m)}}t + t_0\right]^{\frac{2}{m}(4-m)}, \tag{58}$$

$$\Lambda = \frac{a}{\left[\frac{m}{2}\sqrt{\frac{4a}{3(4-m)}}t + t_0\right]^2}. \tag{59}$$

Expansion scalar θ and shear σ are obtained by

$$\theta = 3\sqrt{\frac{4a}{3(4-m)}}\left[\frac{m}{2}\sqrt{\frac{4a}{3(4-m)}}t + t_0\right]^{-1}, \tag{60}$$

$$\sigma = \frac{k}{\sqrt{3}}\left[\frac{m}{2}\sqrt{\frac{4a}{3(4-m)}}t + t_0\right]^{-6/m}. \tag{61}$$

The density parameter is given by

$$\Omega = \frac{\rho}{\rho_c} = \frac{m}{4}. \tag{62}$$

The deceleration parameter q for the model is

$$q = \frac{m}{2} - 1. \tag{63}$$

The vacuum energy density ρ_v and critical density ρ_c are given by

$$\rho_v = \frac{k(4-m)}{m\left(\frac{m}{2}\sqrt{\frac{4a}{3(4-m)}}t + t_0\right)^{\frac{8}{m}}}, \tag{64}$$

$$\rho_c = \frac{4k}{m\left(\frac{m}{2}\sqrt{\frac{4a}{3(4-m)}}t + t_0\right)^{\frac{8}{m}}}. \tag{65}$$

Thus, for the model (55), we observe that for $0<m<4$ the spatial volume V is zero at $t = t'''$, where $t''' = \frac{-t_0}{\frac{m}{2}\sqrt{\frac{4a}{3(4-m)}}}$ and the expansion scalar θ is infinite, which shows that the universe starts evolving with zero volume at $t = t'''$ with an infinite rate of expansion. The scale factors also vanish at $t = t'''$ and hence the space-time exhibits a point type singularity at the initial epoch. The energy density shear scalar diverges at the the initial singularity. As t increases, the scale factors and spatial volume increase but the expansion scalar decreases. Thus, the rate of expansion slows down with an increase in time. Also ρ, σ, ρ_v, ρ_c, and Λ decrease as t increases. As $t \rightarrow \infty$, scale factors and volume become infinite whereas ρ, σ, ρ_v, ρ_c, and Λ tend to zero. Therefore, the model would essentially give an empty universe for large time t. Gravitational constant $G(t)$ is zero at $t = t'''$ and as t increases, $G(t)$ also increases, which is a similar result as that obtained by Abdel-Rahman (1990), Chow (1981), Levitt (1980) and Milne (1935). The ratio $\frac{\sigma}{\theta} \rightarrow 0$ as $t \rightarrow \infty$ provided $m<4$. So, the model approaches isotropy for a large value of t. Thus, the model represents a shearing, non-rotating and expanding model of the universe with a big bang start approaching isotropy at late times.

4 CONCLUSIONS

In this paper, we have studied a spatially homogeneous and isotropic Bianchi type-I space-time with variable gravitational constant $G(t)$ and cosmological constant $\Lambda(t)$. The field equations have been solved exactly by using a law of variation of scale factor with a variable cosmological term, i.e. a cosmological term that scales as $\Lambda \propto R^{-m}$ (where R is a scale factor). Three exact Bianchi type-I models have been obtained in Sections 3.1, 3.2 and 3.3. Expressions for some important cosmological parameters have been obtained for all the models and the physical behavior of the models is discussed in detail. In all the cases, the models represent a shearing, non-rotating and expanding model of the universe with a big-bang start approaching isotropy at late times. It is interesting that the proposed variation law provides an alternative approach to obtaining exact solutions of Einstein's field equations. It presents a unified description of the evolution of the universe which starts with a decelerating expansion and expands with acceleration at late times. Recent observational data (Knop et al. 2003; Riess et al. 1998, 2004; Spergel et al. 2007; Tegmark et al. 2004; Perlmutter et al. 1998) strongly suggest this acceleration. Also, gravitational constant $G(t)$ is zero at the initial singularity and it is increasing with the increase of time. The cosmological constant $\Lambda(t) \propto 1/t^2$ which follows from the model of Kalligas et al. (1992), Berman (1990), Berman & Som (1990), Berman et al. (1989) and Bertolami (1986a,b). This form of Λ is physically reasonable as observations suggest that Λ is very small in the present universe. Finally, the solutions presented in the paper are new and useful for a better understanding of the evolution of the universe in Bianchi type-I space-time with variables G and Λ.

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