

# On the DB gap of white dwarf evolution: effects of hydrogen mass fraction and convective overshooting

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**Abstract** We investigate the spectral evolution of white dwarfs by considering the effects of hydrogen mass in the atmosphere and convective overshooting above the convection zone. Our numerical results show that white dwarfs with  $M_{\text{H}} \sim 10^{-16} M_{\odot}$  show the DA spectral type between  $46\,000 \lesssim T_{\text{eff}} \lesssim 26\,000$  K and the DO or DB spectral type may appear on either side of this temperature range. White dwarfs with  $M_{\text{H}} \sim 10^{-15} M_{\odot}$  appear as DA stars until they cool to  $T_{\text{eff}} \sim 31\,000$  K; from then on they will evolve into DB white dwarfs as a result of convective mixing. If  $M_{\text{H}}$  in the white dwarfs is more than  $10^{-14} M_{\odot}$ , the convective mixing will not occur when  $T_{\text{eff}} > 20\,000$  K, thus these white dwarfs always appear as DA stars. White dwarfs within the temperature range  $46\,000 \lesssim T_{\text{eff}} \lesssim 31\,000$  K always show the DA spectral type, which coincides with the DB gap. We notice the importance of the convective overshooting and suggest that the overshooting length should be proportional to the thickness of the convection zone to better fit the observations.

**Key words:** convection — stars: evolution — stars: white dwarfs

## 1 INTRODUCTION

White dwarfs can be classified into several spectral types in terms of their spectral characteristics. The current classification system was introduced by Sion et al. (1983) and has been modified several times. In this system, the spectral type of a white dwarf is denoted by a letter D plus another letter indicating its spectral characteristics. Sometimes, a suffix is added to indicate some other features (polarization, magnetic field, pulse, etc.). Table 1 lists a spectral classification scheme from McCook & Sion (1999).

Most white dwarfs are of the DA type which have hydrogen-dominated atmospheres. They are found at all effective temperatures from 170 000 K down to about 4 500 K (Kurtz et al. 2008). DA white dwarfs occupy the vast majority (about 75%) of all known white dwarfs.

About 25% of the observed white dwarfs have helium-dominated atmospheres which can be divided further into two spectral types. The DO white dwarfs are found between approximately 100 000 to 45 000 K, with their outer atmospheres being dominated by singly ionized helium (He II). The DB white dwarfs are found between approximately 30 000 to 12 000 K, with their outer atmospheres being dominated by neutral helium (He I).

**Table 1** White Dwarf Spectral Types

Spectral Type	Characteristics
DA	Only Balmer lines; no He I or metals present
DB	He I lines; no H or metals present
DC	Continuous spectrum, no lines deeper than 5% in any part of the electromagnetic spectrum
DO	He II strong; He I or H present
DZ	Metal lines only; no H or He lines
DQ	Carbon features, either atomic or molecular in any part of the electromagnetic spectrum
P (suffix)	Magnetic white dwarfs with detectable polarization
H (suffix)	Magnetic white dwarfs without detectable polarization
X (suffix)	Peculiar or unclassifiable spectrum
E (suffix)	Emission lines are present
? (suffix)	Uncertain assigned classification; a colon (:) may also be used
V (suffix)	Optional symbol to denote variability

There is an interesting fact that few white dwarfs with helium-dominated atmospheres (DO or DB type) are found in the effective temperature range of  $45\,000 \lesssim T_{\text{eff}} \lesssim 30\,000$  K. This is the so-called DB gap. The reason for this phenomenon is not clear. On one side, there is a strong gravitational field in the white dwarf, which may cause a stratified atmosphere. The so-called gravitational settling effect lets the light element (hydrogen) float to the stellar surface and the heavy element (helium) sink to the bottom of the stellar envelope. The typical mass fractions of hydrogen and helium in the white dwarfs are  $M_{\text{H}}/M_{\text{tot}} \lesssim 10^{-4}$  and  $M_{\text{He}}/M_{\text{tot}} \lesssim 10^{-2}$ , which are the mass thresholds for residual nuclear burning (Tremblay & Bergeron 2008). If there is no mixing between hydrogen and helium, we will expect all white dwarfs to be DA stars. On the other side, however, many observational data show that  $M_{\text{H}}$  may be significantly lower than the typical value. The existence of a large number of non-DA white dwarfs indicates that some physical mechanisms are competing with the gravitational settling to change the spectral type of some DA white dwarfs. The DB gap is suspected to be due to the competition between the gravitational settling and convective mixing.

Fontaine & Wesemael (1987) proposed that when a white dwarf starts cooling from the hot PG 1159 type star, hydrogen is mixed within the outer helium envelope and the white dwarf shows the DO spectral type. After that, hydrogen gradually floats to the stellar surface in the strong gravitational field. When the DO star cools to  $T_{\text{eff}} \sim 45\,000$  K, hydrogen has accumulated enough at the surface, and the white dwarf is turned into a DA star. They supposed further that as soon as the DA star cools down to  $T_{\text{eff}} \sim 30\,000$  K, the He I/II ionization zone in the stellar envelope becomes convective. The convective motion may penetrate into the top hydrogen layer and mix the hydrogen atmosphere into the helium envelope, with the less abundant hydrogen being overwhelmed by the more abundant helium, which makes the hydrogen undetectable. So, the DA star then appears as a DB white dwarf.

Shibahashi (2005, 2007) proposed a different scenario for the DB gap. During the early stage of a white dwarf's evolution, the convection in the He II/III ionization zone mixes the hydrogen layer and results in a helium-dominated atmosphere, and the white dwarf appears as a DO star. When the star cools down to around  $T_{\text{eff}} \sim 45\,000$  K, the He II/III ionization zone becomes deep enough that convection disappears in the DO star's atmosphere, so hydrogen floats to the surface and then the white dwarf transforms into a DA star. When the white dwarf further cools down to  $T_{\text{eff}} \sim 30\,000$  K, the He I/II ionization zone again generates a convection zone, as in Fontaine and Wesemael's proposal, a similar mixing between H and He occurs and the white dwarf appears as a DB star.

The difference between the two scenarios of the DB gap is the time scale of the gravitational settling. In Fontaine & Wesemael's assumption, the settling process happens slowly as the white dwarf cools, but in Shibahashi's assumption, it happens quickly as soon as the convection is turned

off. However, recent data from the Sloan Digital Sky Survey (SDSS) suggest that several DB white dwarfs do appear in the DB gap (Eisenstein et al. 2006). These facts imply that the formation mechanism of the DB gap is not clear and more work should be done.

In the present paper, we calculate a series of white dwarf evolutionary models to investigate the spectral evolution of white dwarfs caused by the convective mixing. The details of model calculations and input physics are presented in Section 2. In Section 3, we discuss the results of our numerical models. Conclusions are summarized in Section 4.

## 2 MODEL DETAILS AND INPUT PHYSICS

We have used a modified version of the White Dwarf Evolution Code (WDEC), which was originally described by Martin Schwarzschild, to simulate the evolution of the white dwarf. Some details of the WDEC have been described in Lamb & Van Horn (1975) and Wood (1990). Here, we only present some summaries of the input physics in our models.

The equation of state (EOS) used in the present calculations is composed of two parts which apply to different regions. The first part of the EOS is used for the degenerate, completely ionized interior of the white dwarf. In this region, we use the EOS tables provided by Lamb (1974). For a given chemical composition, the needed values are obtained by two-dimensional, four-point Aitken-Lagrange interpolation in terms of variables  $\lg P$  and  $\lg T$ . For a specific C/O mixture, they are obtained by interpolation between the carbon and oxygen tables using the additive volume technique of Fontaine et al. (1977). The second part of the EOS is used for the partially ionized envelope where the non-ideal effect is important. We use the Saumon et al. (1995) EOS for hydrogen and helium mixtures. The new EOS includes some new physical treatments of partial ionization caused by pressure and temperature. Mixtures of hydrogen and helium are also obtained by the additive volume technique.

The total opacity ( $\kappa$ ) is given by

$$\frac{1}{\kappa} = \frac{1}{\kappa_r} + \frac{1}{\kappa_c}, \quad (1)$$

where  $\kappa_r$  is the radiative opacity and  $\kappa_c$  is the conductive opacity. We use the OPAL radiative opacities in our calculations. The new tables include some new physical factors, e.g. the L-S coupling effect of iron atoms. The conductive opacities consist of two parts which are from Itoh et al. (1983) and Mitake et al. (1984) and Hubbard & Lampe (1969). In the actual calculations, we use Itoh et al. opacities only in the  $\lg \rho \geq 1.8$  region and Hubbard & Lampe opacities in the  $\lg \rho \leq 1.5$  region; in the range  $1.5 < \lg \rho < 1.8$  linear interpolation is performed.

The rates of neutrino energy loss used in our calculations are provided by Itoh and his collaborators. The rates of neutrino energy loss due to pair, photo, plasma and bremsstrahlung processes are from Itoh et al. (1989), and the neutrino energy loss rate of the recombination process is from Kohyama et al. (1993).

The high surface gravity of the white dwarf leads to gravitational segregation of the elements in the stellar envelope, and thus models of the white dwarf must include compositionally stratified envelopes. In our calculations, we adopt approximations of the equilibrium diffusion profiles introduced by Wood (1990) (see Fig. 1). Our calculations do not include the impact of convective mixing on the H/He profile. Although this may not be correct in the details, we still expect it to be a reasonable approximation, because the abundance of hydrogen in the mixing region is several orders of magnitude less than the helium abundance.

We use the standard mixing-length theory of Böhm-Vitense (1958) to deal with the convection. We set the mixing-length  $l$  to be equal to one local pressure scale height, i.e.,

$$l = H_P = -\frac{dr}{d \ln P} = \frac{P}{\rho g}. \quad (2)$$

Boundaries of the convection zones are determined by the Schwarzschild criterion. Also, we set the integration step to be equal to  $H_P/8$  in our calculations.

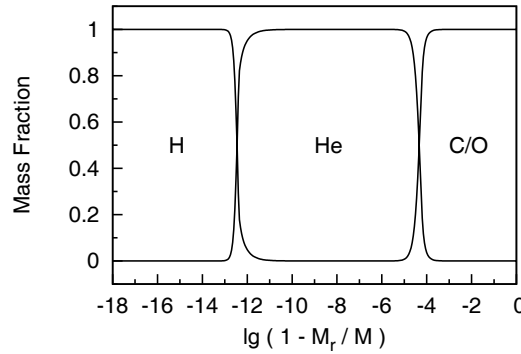
### 3 EVOLUTIONARY RESULTS

We have computed a series of white dwarf evolutionary models with mass  $M = 0.6 M_\odot$ , which is the typical mass of a white dwarf. In order for a DA white dwarf to change its surface chemical composition as a result of the convective mixing, it should have a very thin hydrogen layer, so the hydrogen mass of the model is supposed to vary between  $10^{-16}$  and  $10^{-14} M_\odot$ . The helium mass in the computed model envelope is fixed to be  $5.0 \times 10^{-5} M_\odot$ . We assume that the heavy elements have sunk to the white dwarf's interior during the early phase of the cooling process due to the so-called gravitational settling effect. As a result, the envelope of the white dwarf is only composed of hydrogen and helium, and the metallicity in the envelope is assumed to be  $Z = 0$ . All models are evolved from  $T_{\text{eff}} \sim 90\,000$  K down to  $T_{\text{eff}} = 10\,000$  K. In this section, we present the results of our calculations.

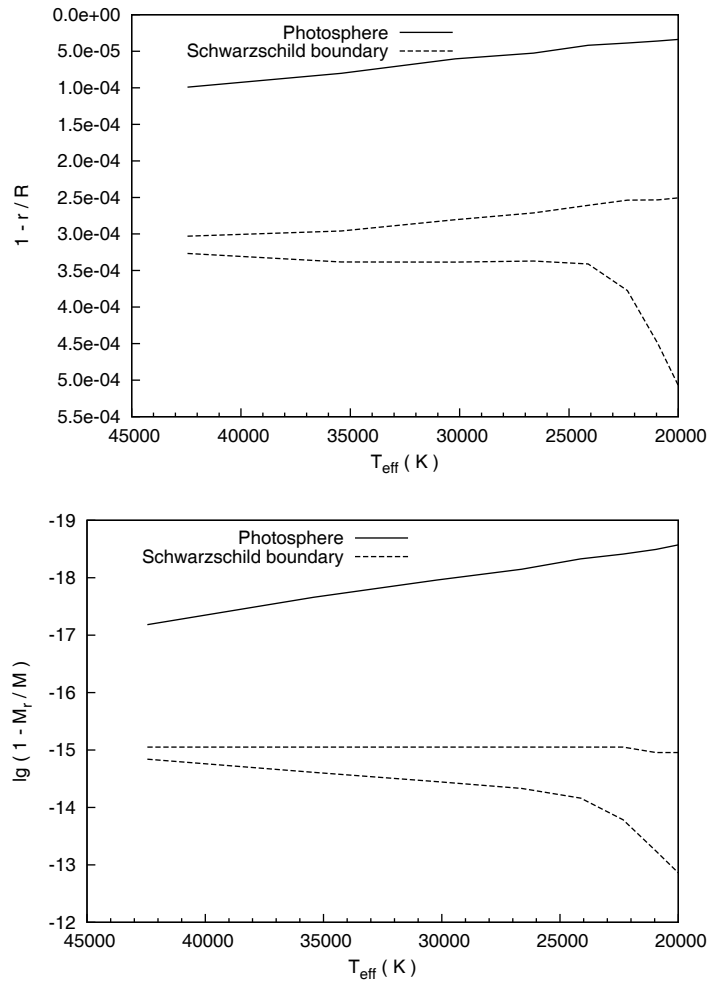
#### 3.1 Importance of the Convective Overshooting

We have examined the development of the convection zone in our white dwarf models. Figure 2 shows the convection zone of a DA model with  $M_H = 10^{-15} M_\odot$  evolving with decreasing effective temperature. The horizontal axis is the effective temperature which denotes the evolutionary sequence, and the vertical axis denotes the location within the envelope of the model. We use  $1 - r/R$  as the vertical axis scale in Figure 2 (a) and use  $\lg(1 - M_r/M)$  in Figure 2 (b). The solid line corresponds to the location of the model's photosphere ( $\tau = 1$ ) and the dashed lines correspond to the Schwarzschild boundaries of the convection zone.

It can be found that convection occurs completely within the helium layer which is located under the photosphere which can be regarded as the innermost point visible to us in the white dwarf. Just above the He convection zone there is a thin layer with a mean molecular weight gradient (the so-called  $\mu$ -barrier). In the case of a very thin hydrogen envelope ( $M_H < 10^{-15} M_\odot$ ), Vauclair & Reisse (1977) have argued that the  $\mu$ -barrier provides extra buoyancy to restrict the upper boundary of the He convection zone just below it. However, at the upper boundary which is determined by the convective stability criterion (at the  $\mu$ -barrier, the Ledoux criterion  $\nabla_{\text{rad}} \geq \nabla_{\text{ad}} + d \ln \mu / d \ln P$  is adopted), the convective motion does not stop but moves further upward due to inertia. In this sense, the fluid parcels with kinetic energy may penetrate into the  $\mu$ -barrier until their velocity drops to zero.



**Fig. 1** Approximations to the diffusive equilibrium profiles.



**Fig. 2** Location of the convection zone.

We perform a rough calculation, supposing that the velocities of the fluid parcels are equal to the mean flow velocity ( $\bar{v}$ ) in the convection zone. When the fluid parcels enter the  $\mu$ -barrier, a force  $F$  (the resultant force of gravity and buoyancy, neglecting the viscous force) acts on them and reduces the velocity. The kinetic energy of the fluid parcels  $E_K \sim \bar{v}^2/2$ . When the fluid parcels deplete all their kinetic energy, they can move beyond the convective boundary a length  $L \sim E_K/F \sim \bar{v}^2/2F$ . As shown in Figure 3, the thickness of the  $\mu$ -barrier  $d_\mu$  is around  $3 \times 10^3$  cm and  $L > d_\mu$  when  $T_{\text{eff}} < 40\,000$  K, and the maximum value of  $L$  is about  $3 \times 10^4$  cm which is one order of magnitude greater than  $d_\mu$ . Therefore, it seems reasonable to believe that the convective overshooting can cross the  $\mu$ -barrier. Once He penetrates into the  $\mu$ -barrier, an effective mixing will flatten the composition gradient and weaken the  $\mu$ -barrier. As a result, the upper boundary of the He convection zone will thus be extended outward. We expect that the convective motion can be extended upward to the stellar surface, or at least, to the photosphere in order to mix the upper hydrogen layer and to ensure that helium can be observed. Thus, the role of convective overshooting appears to be decisive. We suppose that the convective overshooting is able to reach the photosphere. This suggestion lets us set

the distance from the photosphere down to the top of the Schwarzschild boundary as the minimum length of the overshooting region ( $l_{\text{ovs}}$ ). The geometrical length between the two Schwarzschild boundaries is regarded as the length of the convection zone ( $l_{\text{con}}$ ). These two lengths vary with the evolution of the effective temperature as shown in Figure 4. We also compare  $l_{\text{ovs}}$  with  $l_{\text{con}}$ ; the ratio is given in Figure 5.

As shown in Figure 4,  $l_{\text{ovs}}$  keeps almost a constant value (about 2 km), while  $l_{\text{con}}$  increases slowly to a few hundred meters during a long evolutionary time scale. That is to say, the convective motion must overshoot to a distance which is several times thicker than the convection zone itself. Figure 6 is similar to Figure 4 but  $l_{\text{ovs}}$  and  $l_{\text{con}}$  are expressed in units of the local pressure scale height ( $H_P$ ). It is shown that  $l_{\text{ovs}}$  is no more than  $4H_P$ .

We discuss the extension of convective overshooting from another point of view that concerns the masses in the convection and overshooting regions. We denote  $M_{\text{ovs}}$  as the mass within the overshooting region and  $M_{\text{con}}$  as the mass of the convection zone. Figure 7 shows that  $\lg M_{\text{ovs}}$  and  $\lg M_{\text{con}}$  vary with the effective temperature, respectively. Figure 8 shows the variation of the ratio of  $M_{\text{ovs}}$  to  $M_{\text{con}}$ . It can be seen that  $M_{\text{ovs}}/M_{\text{con}}$  decreases rapidly with  $T_{\text{eff}}$  and  $M_{\text{ovs}}$  accounts only for a small fraction of  $M_{\text{con}}$  for the models with relatively low  $T_{\text{eff}}$ . The convection zone thickens during the evolutionary process and causes a rapid increase in  $M_{\text{con}}$ . At  $T_{\text{eff}} \sim 38\,000$  K,  $M_{\text{con}}$  is greater than  $M_{\text{ovs}}$ . It is interesting to note that, although  $l_{\text{ovs}}$  is considerably larger than  $l_{\text{con}}$ , the matter in the overshooting region has a low density compared with the dense, turbulent convection zone. For example, when the white dwarf model cools down to  $T_{\text{eff}} \sim 30\,000$  K, the ratio of  $l_{\text{ovs}}$  to  $l_{\text{con}}$  is less than 3.8 and  $M_{\text{ovs}}$  is only about 1/3 of  $M_{\text{con}}$ . Therefore, we may reasonably believe that the convective motion can extend to the photosphere by the force of the convective overshooting.

### 3.2 Determination of the Transition Temperature

If the convection zone in the helium envelope of a DA white dwarf can extend upward to the photosphere due to the convective overshooting, the hydrogen atmosphere will be mixed into the convective helium layer and the white dwarf will have the opportunity to change its apparent chemical composition, in other words, to transform into a DB star. In this section, we will discuss what happens when such a transformation occurs.

We assume that the convective mixing region includes the overshooting region and the convection zone, in which hydrogen and helium are homogeneously mixed. The mass of the mixing zone is denoted as  $M_{\text{mix}}$ , and the mass of hydrogen in the mixing zone is denoted as  $M_{\text{Hmix}}$ . The remainder is helium whose mass is equal to  $M_{\text{mix}} - M_{\text{Hmix}}$ . When the mass of helium exceeds the mass of hydrogen in the mixing zone, i.e.

$$M_{\text{mix}} - M_{\text{Hmix}} > M_{\text{Hmix}} , \quad (3)$$

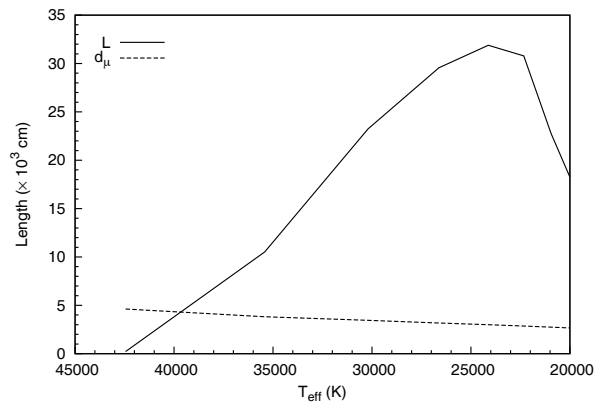
that is,

$$M_{\text{mix}} > 2M_{\text{Hmix}} , \quad (4)$$

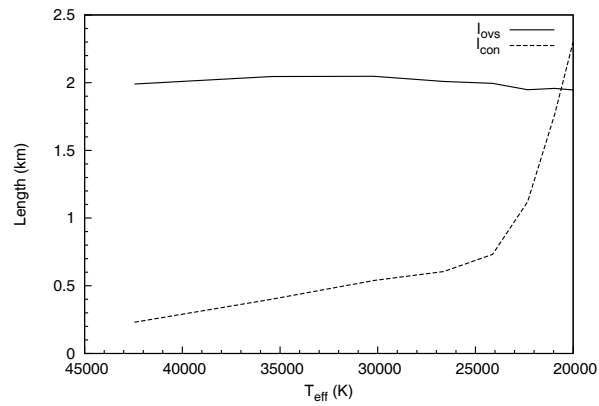
hydrogen will be overwhelmed by helium and we assume that the transformation of the spectral type will occur. We use  $M_{\text{mix}} = 2M_{\text{Hmix}}$  as a critical condition for a DA star evolving into a DB one. The effective temperature at this critical point is called the transition temperature.

### 3.3 Discussions of the Calculation Results

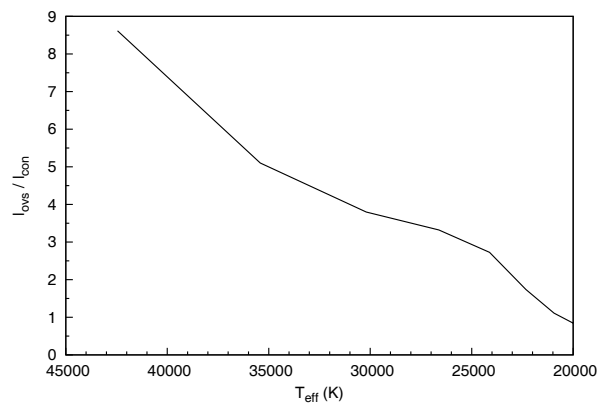
We have computed a series model of a DA white dwarf with  $M_{\text{H}} = 1.0 \times 10^{-15} M_{\odot}$ . The convection zone varies with the effective temperature as shown in Figure 9, in which we use  $\lg(1 - M_r/M)$  to indicate the location of the convection zone. The convective motion appears just below the H/He interface ( $\lg(1 - M_r/M) \approx -15$ ) and extends into the stellar interior when the white dwarf model



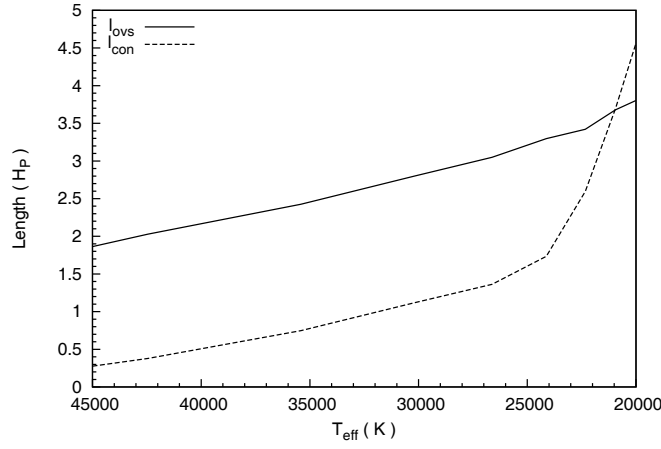
**Fig. 3**  $L$  and  $d_\mu$  versus  $T_{\text{eff}}$ .



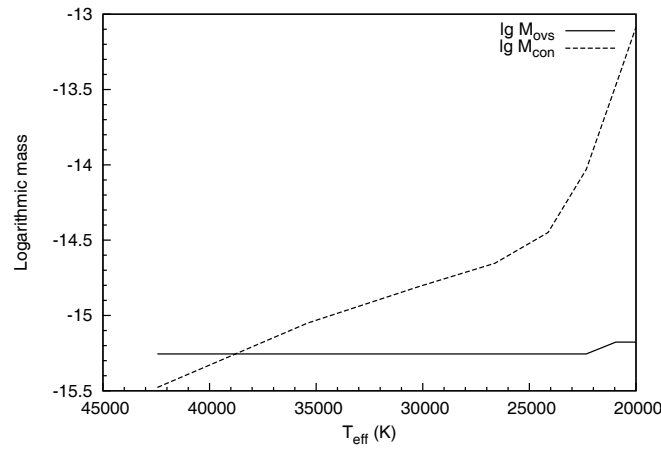
**Fig. 4** Evolution of the length of the overshooting region and that of the convection zone. In this case,  $l_{\text{ows}}$  is the length of the overshooting region and  $l_{\text{con}}$  the length of the convection zone.



**Fig. 5** Ratio of  $l_{\text{ows}}$  to  $l_{\text{con}}$  varies with  $T_{\text{eff}}$ .



**Fig. 6** Similar to Fig. 4, but the two lengths are expressed in units of  $H_P$ .

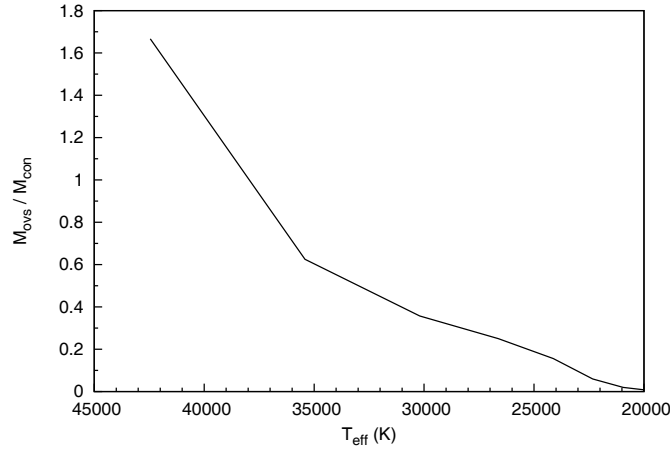


**Fig. 7** Masses of the overshooting region and convection zone vary with  $T_{\text{eff}}$ .  $M_{\text{ovs}}$  is the mass of the overshooting region and  $M_{\text{con}}$  is the mass of the convection zone.

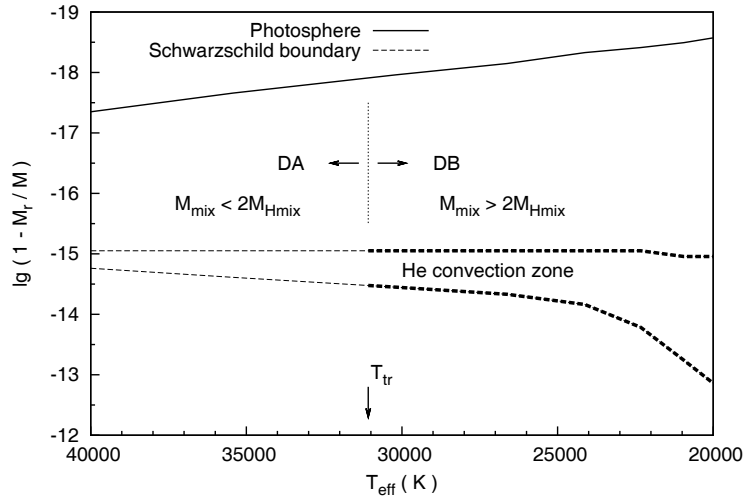
cools down. When the convective mixing zone becomes thick enough, Equation (4) is satisfied (indicated by the thick dashed line). It can be found that the transition temperature ( $T_{\text{tr}}$ ) of this model is about 31 000 K. If the convective overshooting can reach the photosphere at this temperature, we will observe that the white dwarf evolves into a DB star. According to the discussions in Section 3.1, the required length of the convective overshooting is about  $3H_P$ .

Other series of white dwarf evolution models that we have computed show that the thicker the hydrogen layer is, the lower the transition temperature will be. Table 2 lists the value of  $T_{\text{tr}}$  of our models with different  $M_H$  values. It can be noticed that the transition temperature of the model with  $M_H = 1.0 \times 10^{-14} M_\odot$  is below 20 000 K. It is expected, therefore, that models with hydrogen layers heavier than  $10^{-14} M_\odot$  must have  $T_{\text{tr}}$  lower than 18 000 K. So, a DA white dwarf with  $M_H > 10^{-15} M_\odot$  may have the opportunity to change its spectral type when it cools much below  $T_{\text{eff}} \sim 30\,000$  K. Moreover, greater  $M_H$  also requires stronger convective overshooting to bring helium to the stellar surface.





**Fig. 8** Ratio of  $M_{\text{ovs}}$  to  $M_{\text{con}}$  varies with  $T_{\text{eff}}$ .

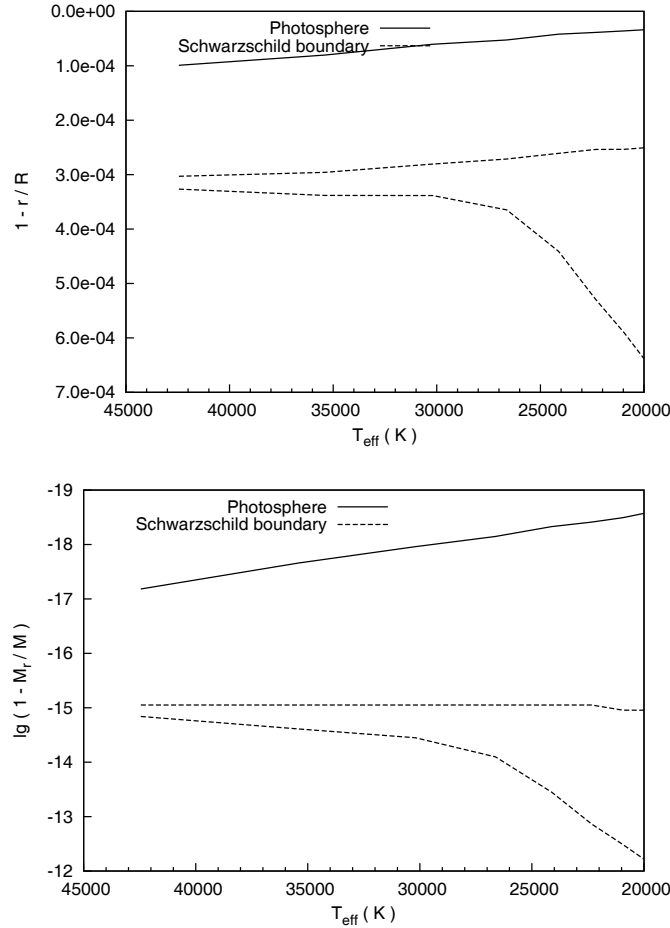


**Fig. 9** A schematic representation of the spectral evolution of a white dwarf model with  $M_{\text{H}} = 1.0 \times 10^{-15} M_{\odot}$ .

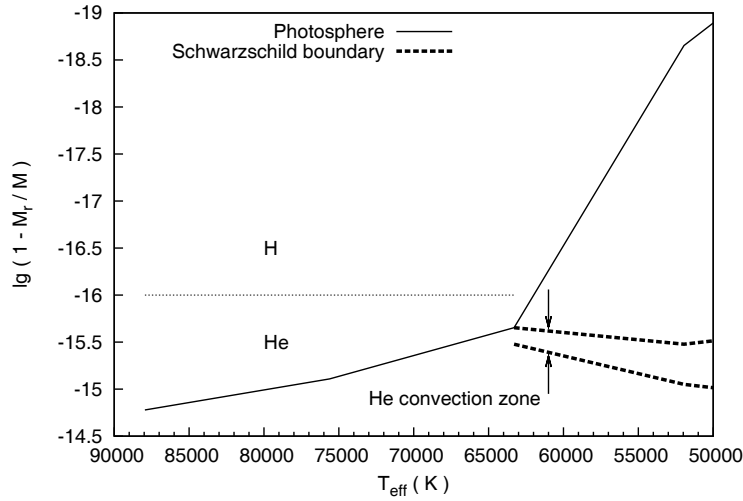
Furthermore, a more efficient convection can change the deepening of the inner boundary of the He convection during the white dwarf's evolution, and thus change the transition temperature. In the MLT (mixing-length theory), the mixing-length  $l$  represents the efficiency of convective heat transfer. We considered a series of models in which the mixing-length is set to  $2H_{\text{P}}$ , which is 2 times larger than before. Our calculations show that the deepening of the convection zone does occur at a higher effective temperature. As shown in Figure 10, the convection zone of a model with  $M_{\text{H}} = 10^{-15} M_{\odot}$  deepens at  $T_{\text{eff}} \sim 30\,000$  K (compared with Fig. 2). For this reason, Equation (4) will be satisfied earlier and it will lead to a change of  $T_{\text{tr}}$  (see Table 2). However, the variation of  $T_{\text{tr}}$  is relatively small, especially around  $30\,000$  K. We therefore believe that the efficiency of the convection will not significantly affect the results.

**Table 2** Transition Temperatures for Different White Dwarf Models

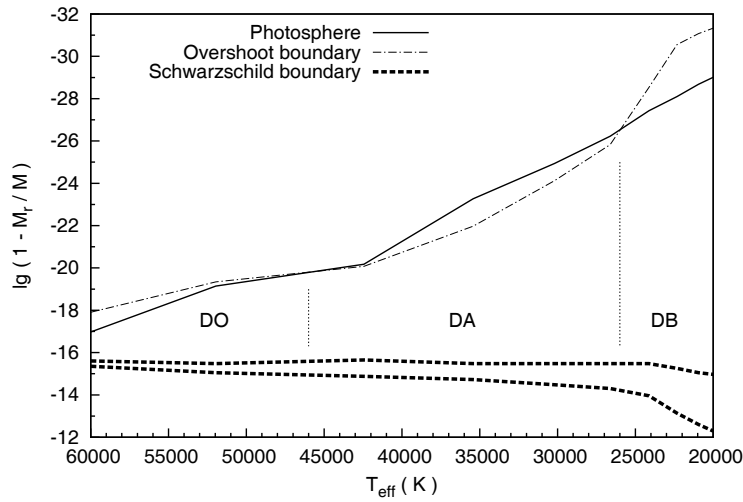
$M_{\text{H}} (M_{\odot})$ ( $\times 10^{-15}$ )	$T_{\text{tr}} \text{ (K)}$		$M_{\text{H}} (M_{\odot})$ ( $\times 10^{-15}$ )	$T_{\text{tr}} \text{ (K)}$	
	$l = H_{\text{P}}$	$l = 2H_{\text{P}}$		$l = H_{\text{P}}$	$l = 2H_{\text{P}}$
1.0	31 084	31 084	1.8	25 191	26 222
1.1	29 349	29 769	2.0	24 001	26 209
1.2	27 920	29 006	3.0	22 112	24 004
1.4	27 509	28 456	4.0	20 785	22 799
1.5	27 257	28 240	5.0	19 966	22 477
1.6	26 360	27 466	10.0	18 460	19 802

**Fig. 10** Similar to Fig. 2, but  $l = 2H_{\text{P}}$ .

The above discussions imply that DB white dwarfs are likely born from DA white dwarfs because of the convective mixing. Is it possible that a DO white dwarf can evolve into a DA star? Our calculations show that if a DA white dwarf has a sufficiently thin hydrogen layer of  $M_{\text{H}} \sim 10^{-16} M_{\odot}$ , this transformation is possible. As shown in Figure 11, during the early time of the evolution, the model's  $T_{\text{eff}}$  is very high and hydrogen is completely ionized in its atmosphere.



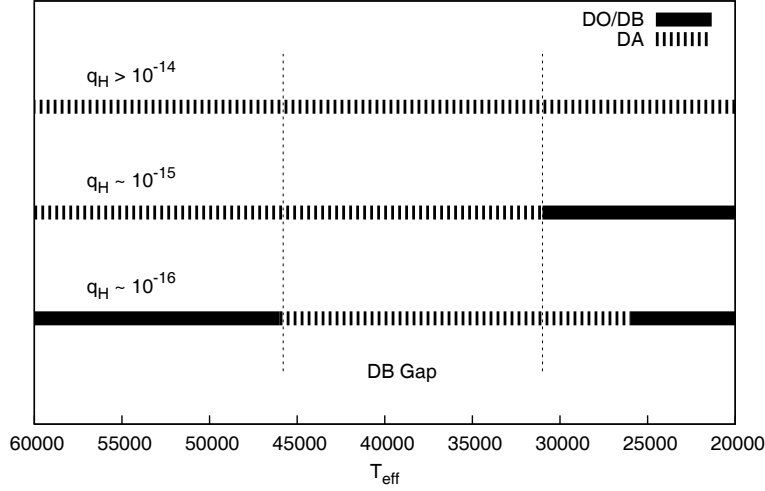
**Fig. 11** A schematic representation of the formation of a DO white dwarf from a DA star with  $M_H \sim 10^{-16} M_\odot$ .



**Fig. 12** A schematic representation of the spectral evolution of a white dwarf model with  $M_H \sim 10^{-16} M_\odot$ . The dash-dotted line corresponds to the boundary of the overshooting region.

The photosphere lies deep in the helium layer and thus helium is visible, resulting in the white dwarf appearing as a DO star. When the white dwarf model cools down to  $T_{\text{eff}} \sim 63\,000$  K, the photosphere rises to the hydrogen layer, but at the same time convection appears in the He II/III ionization zone. Because of the thin hydrogen layer, the convective overshooting can possibly reach the photosphere and Equation (4) can easily be satisfied. The convective motion in the helium layer will dilute the hydrogen layer, so the white dwarf is still a DO star.

When the white dwarf cools down below  $60\,000$  K, as shown in Figure 12, the location of the photosphere will quickly move up towards the stellar surface. The extension of the convective mo-



**Fig. 13** A schematic representation of the DB gap, where  $q_H = M_H/M_\odot$ .

tion cannot go so far as to reach the photosphere. Therefore, the convective mixing of helium is invisible and hydrogen will re-accumulate in the atmosphere, making the DO white dwarf transform into a DA star. This result is similar to Shibahashi's assumption (Shibahashi 2005, 2007). According to the observational data, there are no DO white dwarfs below  $T_{\text{eff}} \sim 45\,000$  K. This fact allows us to adjust the overshooting length to let our model change its spectral type at  $T_{\text{eff}} \sim 45\,000$  K. We can reasonably assume that the overshooting length is proportional to the dimension of the convection zone. In practice, we choose the overshooting length to be the length of the convection zone (being approximately equal to  $H_P$ ) plus  $0.375H_P$ . Our numerical result shows that at an effective temperature ( $T_{\text{eff}} \sim 26\,000$  K) the convection zone has become thick enough that the convective overshooting can again reach above the photosphere. Then, the hydrogen layer will be mixed with the helium layer and the white dwarf will become a DB star.

There is another possibility for DO white dwarfs transforming into DA stars in the literature. It is probable that the progenitors of DO white dwarfs have undergone a born-again phase and burnt most of their hydrogen envelope (Althaus et al. 2005). The mass loss due to stellar wind during the hot white dwarf stage is likely to eject the superficial hydrogen and prevent the gravitational settling (Unglaub & Bues 2000). When the white dwarfs cool down to  $T_{\text{eff}} \sim 45\,000$  K, hydrogen previously left in the internal layer is able to float to the stellar surface and DA white dwarfs are formed.

#### 4 CONCLUSIONS

From the above investigations, we have found that the DB gap could be explained as a consequence of the convective mixing in white dwarfs. DA white dwarfs with  $M_H/M_\odot \sim 10^{-16}$  have opportunities to transform into DO white dwarfs at  $T_{\text{eff}} \gtrsim 46\,000$  K or DB white dwarfs at  $T_{\text{eff}} \lesssim 26\,000$  K, respectively. DA white dwarfs with  $M_H/M_\odot \sim 10^{-15}$  can transform into DB stars below  $T_{\text{eff}} \sim 31\,000$  K. White dwarfs with  $M_H$  greater than  $10^{-14} M_\odot$  always appear as DA stars at  $T_{\text{eff}} \gtrsim 18\,000$  K. It is obvious that in the effective temperature range between  $46\,000 \lesssim T_{\text{eff}} \lesssim 31\,000$  K almost all of white dwarfs have the DA spectral type, as shown in Figure 13. This scenario coincides well with observations. We can also estimate that the hydrogen

mass is  $M_{\text{H}}/M_{\odot} \sim 10^{-16}$  for the DO white dwarfs and it is  $M_{\text{H}}/M_{\odot} \sim 10^{-15}$  for the hot DB white dwarfs ( $T_{\text{eff}} > 20\,000$  K).

Based on our numerical results, convective overshooting plays a crucial role in the formation of the so-called DB gap, through the convective mixing effect. It allows the convective motion to penetrate into the hydrogen layer and makes helium in the deep stellar interior become observable on the stellar photosphere. The overshooting length is an important parameter of the model. According to our results, the overshooting length should be proportional to the thickness of the convection zone, which gives better agreement between the model results and observations.

The hydrogen mass  $M_{\text{H}}$  is another important parameter, which is used as a criterion for deciding when helium is dominant in the atmosphere of the white dwarf. It decisively determines the critical effective temperature for the white dwarf to change its spectral type.

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