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GRB jet beaming angle statistics [*]

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Abstract Existing theory and models suggest that a Type I (merger) GRB should have a larger jet beaming angle than a Type II (collapsar) GRB, but so far no statistical evidence is available to support this suggestion. In this paper, we obtain a sample of 37 beaming angles and calculate the probability that this is true. A correction is also devised to account for the scarcity of Type I GRBs in our sample. The probability is calculated to be 83% without the correction and 71% with it.

Key words: gamma rays: bursts — ISM: jets and outflows — methods: statistical

1 INTRODUCTION

There are two intrinsically different phenomena that give rise to gamma-ray bursts (GRBs). One is the merging of two compact objects, such as a neutron star and a black hole; the other is the core collapse of a massive star ("collapsar"), such as the birth of a Type Ib/c Supernova. According to a classification scheme developed recently (Zhang 2007; Zhang et al. 2009), a GRB obtained from the former channel is defined as a Type I GRB, while one from the latter would be classified as a Type II GRB. This classification scheme, though more intrinsic than the classical short/hard vs. long/soft categories, is not easy to carry out at present, since many different criteria need to be applied in order to determine the progenitor of a GRB, few of which are decisive. One of these criteria is that Type I GRBs are usually short/hard ($T_{90} \leq 2$ s), while Type II GRBs are usually long/soft ($T_{90} > 2$ s). This criterion, though supported by many individual cases, is not decisive, because any short GRB could theoretically have been a long GRB had it occurred at a high enough redshift. Of the criteria that are decisive, such as whether or not a GRB emanates gravitational radiation, most are impractical and the remainder are not applicable to a great majority of GRBs at the present stage.

GRBs eject their energy in the form of jets. These jets have already been modeled, and their beaming angles (also called opening angles) can be calculated from observed physical data (Sari et al. 1999). The size of the beaming angle, according to the models, is highly dependent on the nature of the GRB progenitor: theoretically, Type II GRBs should have smaller beaming angles than Type I GRBs, due to the collimation effect of the stellar wind hugging collapsars. However, this dependence has not yet been supported statistically, possibly due to the exceptionally small sample of Type I GRB jet beaming angles available. Hence, this paper specifically seeks to find statistical evidence of this theoretical relationship.

This paper is divided into 5 sections, of which this introduction is the first. In the second, we will list the relevant observational data and use them to obtain the jet beaming angles of 37 GRBs. The third section will cover data reduction and statistical analysis. The results and discussion of the analysis will be presented in the fourth section, and the entire paper will be summarized in the fifth section, which is the conclusion.

2 COLLECTED DATA

According to the literature (Sari et al. 1999; Frail et al. 2001), the jet beaming angle of a GRB can be obtained by Equation (1):

$$\theta_j = 0.057 \left(\frac{t_j}{1\,\mathrm{d}}\right)^{\frac{3}{8}} \left(\frac{1+z}{2}\right)^{-\frac{3}{8}} \left(\frac{E_{\mathrm{iso}}(\gamma)}{10^{53}\,\mathrm{erg}}\right)^{-\frac{1}{8}} \left(\frac{\eta_\gamma}{0.2}\right)^{\frac{1}{8}} \left(\frac{n}{0.1\,\mathrm{cm}^{-3}}\right)^{\frac{1}{8}}, \tag{1}$$

where θ_j is the jet beaming angle mentioned above, t_j is the time of the jet break, z is the redshift at which the GRB took place, $E_{\mathrm{iso}}(\gamma)$ is the isotropic energy released by the burst, η_γ is the gamma-ray radiative efficiency of the jet and n is the density of the interstellar medium. In this case, t_j, z and $E_{\mathrm{iso}}(\gamma)$ are observable parameters, shown in Table 1. However, η_γ and n are not observable, but, thankfully, they do not have a very significant effect on the value of θ_j. Therefore, we set $\eta_\gamma = 0.2$ and $n = 0.1\,\mathrm{cm}^{-3}$. The jet beaming angles obtained from these data are presented in Table 1 and in Figure 1. In Figure 1, the bars filled in black, stripes and grey correspond to Sample I, Sample II and unclassified jet beaming angles (defined below), respectively.
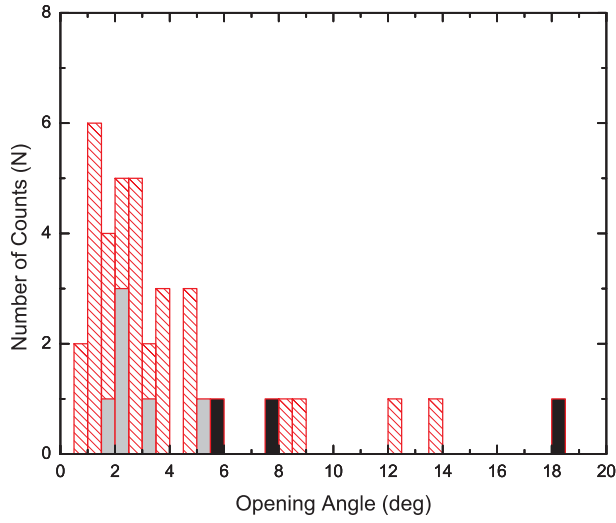
Fig. 1 Jet beaming angles: a complete sample. Sample I is shown with black filling, while Sample II is shown with striped filling. Filled in grey are those GRBs that have not been positively identified as either Sample I or Sample II. The horizontal axis is θ_j in degrees, while the vertical axis is the number of opening angles that have the specified magnitude.

3 DATA REDUCTION

As shown in Table 1, we have only managed to confirm 2 Type I bursts and 13 Type II bursts among the GRBs whose parameters we obtained.

Of the remaining GRBs, one is identified to be a short burst, while 21 of the others are known to be long. According to experience, short GRBs are usually of Type I origin, while long bursts are usually of Type II. Therefore, we incorporate the short GRBs into the Type I sample, and the long GRBs into the Type II sample. It is hoped that, statistically, this will serve the purpose of giving a larger sample while maintaining sample integrity.

Table 1 Data from Observations and Derived Data

GRB	E_{iso}	t_j	z	Category	$\theta_j(°)$
050709	0.00069[1]	10[1]	0.16[1]	Type I[2]	18.1957
060614	0.021[2]	1.3[4]	0.13[4]	Type I[2]	5.57885
970508	0.061[2]	25[4]	0.835[4]	Type II[2]	12.3366
980703	0.72[2]	2.49[4]	0.966[4]	Type II[2]	3.71802
990123	22.9[2]	1.8[4]	1.6[4]	Type II[2]	1.92367
990712	0.0672[5]	1.6[5]	0.433[5]	Type II[2]	4.77012
000418	0.75137[6]	25[6]	0.1181[6]	Type II[2]	8.54109
000926	2.71[2]	2.03[4]	2.307[4]	Type II[2]	2.40104
011211	0.67234[6]	1.77[6]	2.14[6]	Type II[2]	2.76815
020405	1[2]	2.74[4]	0.689[4]	Type II[2]	3.91555
020813	6.6[2]	0.46[4]	1.254[4]	Type II[2]	1.42145
000328	3.61[5]	0.8[5]	1.52[5]	Type II[2]	1.80902
030329	0.166[5]	0.5[5]	0.169[5]	Type II[2]	2.97281
041006	0.83[5]	0.16[5]	0.716[5]	Type II[2]	1.37314
050525A	0.25[2]	0.16[4]	0.606[4]	Type II[2]	1.63548
051221A	0.024[7]	4.1[4]	0.5465[4]	short[8]	7.50341
010921	0.13611[6]	33[6]	0.4509[6]	long[9]	13.5235
020124	2.15[5]	3[5]	3.198[5]	long[9]	2.61651
021004	0.55601[6]	7.6[6]	2.332[6]	long[9]	4.78799
030226	0.67[5]	0.84[5]	1.986[5]	long[9]	2.13395
030429	0.173[5]	1.77[5]	2.656[5]	long[9]	3.09816
050315	0.49[10]	2.6[4]	1.95[4]	long[11]	3.40529
050318	0.22[3]	0.12[4]	1.44[4]	long[11]	1.27531
050401	5.323[12]	0.06[12]	2.9[12]	long[11]	0.55384
050416A	0.0083[5]	1[5]	0.653[5]	long[11]	4.92335
050820A	9.74[3]	3.99[4]	2.61[4]	long[11]	2.55110
060124	4.1[3]	0.61[4]	2.3[4]	long[11]	1.45364
060206	0.43[3]	0.82[4]	4.05[4]	long[11]	1.83552
060210	4.15[13]	2.16[4]	3.91[4]	long[11]	2.00912
060526	0.26[3]	0.98[4]	3.21[4]	long[11]	2.23733
060605	0.25[14]	0.27[14]	3.773[14]	long[11]	1.32273
060814	0.7[3]	0.79[4]	0.84[4]	long[11]	2.48693
061121	2.25[3]	0.28[4]	1.31[4]	long[11]	1.33753
061126	1.06[15]	25.7[15]	1.1588[15]	long[11]	8.20785
070125	10.6[16]	3.8[16]	1.547[16]	long[16]	2.82483
071010A	0.036[17]	1[17]	0.98[17]	long[11]	3.83010
080319B	13[18]	0.032[18]	0.937[18]	long[11]	0.50876
970828	2.1982[6]	2.2[6]	0.9578[6]		3.09196
990510	1.76349[6]	1.2[6]	1.6187[6]		2.27045
990705	2.55952[6]	1[6]	0.8424[6]		2.30918
991216	5.35369[6]	1.2[6]	1.02[6]		2.17822
000301C	0.43749[6]	7.3[6]	2.0335[6]		5.03377
010222	8.57841[6]	0.93[6]	1.4768[6]		1.72899

[1] Fox et al. (2005); [2] Zhang et al. (2009); [3] Amati et al. (2008); [4] Liang et al. (2008); [5] Ghirlanda et al. (2007); [6] Bloom et al. (2003); [7] Soderberg et al. (2006); [8] Sakamoto et al. (2008); [9] Pélangeon et al. (2008); [10] Amati (2006); [11] Evans et al. (2009); [12] Kamble et al. (2009); [13] Ghirlanda et al. (2008); [14] Ferrero et al. (2009); [15] Perley et al. (2008); [16] Chandra et al. (2008); [17] Covino et al. (2008); [18] Racusin & Burrows (2008).

Six bursts, namely GRB 970828, GRB 990510, GRB 990705, GRB 991216, GRB 000301C and GRB 010222, cannot be identified as either Type I/II or long/short bursts, and, therefore, cannot be used in the statistics. These bursts are represented by the columns filled in grey in Figure 1.

Thus, we obtain a "Type I + short" sample, noted as "Sample I" for the rest of the paper, and a "Type II + long" sample, hereby noted as "Sample II." Their jet beaming angles are presented in Figure 1 (black and striped filling respectively). A primary contributing factor to the scarcity of Sample I data is the fact that t_j is hard to determine in short bursts, rendering Equation (1) unapplicable. Of the five bursts known to be of Type I origin (Zhang et al. 2009), we obtained the opening angles for two of them (see Table 1 for details). Of the remaining three (GRB 050509B, GRB 050724 and GRB 061006), the jet break time of GRB 050509B and GRB 061006 cannot be found, while only a lower limit constraint could be placed on the jet break time of GRB 050724. The problem of whether lower limit constraints could be used in the sample is discussed further in Section 4. Of the short/hard bursts listed in Zhang et al. (2009), the opening angle of only one could be ascertained. Since GRB 080503, the latest short/hard burst mentioned in Zhang et al. (2009), five bursts (GRB 080702A, GRB 080905A, GRB 080919, GRB 081024 and GRB 090510) have been observed according to GCN reports that fall indisputably in the short/hard category. However, no jet break time has been found for any of them.

From Figure 1, it should be quite obvious that Sample I data have a greater arithmetic mean in comparison to Sample II data. This is indeed the case, as Sample I has a mean value of 10.42°, while the mean of Sample II is only 3.42°. This is supportive of the statement that Type I GRBs have a larger beaming angle than Type II GRBs. However, the fact that there are 4 of the Sample II data that are larger than 2 of the 3 Sample II data leaves plenty of room for argument. Therefore, statistical analysis based on the data that lead to a quantitative result on the validity of the statement above is required. In order to obtain such a quantitative result, statistical fitting must be carried out. Many papers consider the Gaussian distribution under similar circumstances (for instance: Zhang & Meszaros 2002), and therefore we will fit the data using the Gaussian distribution. Whether opening angles really do follow Gaussian distributions is indeed rather problematic: this topic will be discussed further below.

Assuming that elements taken from Samples I and II are random variables that have Gaussian distributions ($N(\mu, \sigma^2)$), we take their probability densities to be

$$N(\theta_j) = \frac{1}{\sigma_i \sqrt{2\pi}} \exp\left[-\frac{1}{2} \left(\frac{\theta_j - \mu_i}{\sigma_i} \right)^2 \right], \quad (i = 1, 2), \tag{2}$$

respectively. Here, $i = 1$ corresponds to Sample I, while $i = 2$ corresponds to Sample II. Also, μ_i and σ_i are the means and standard variations of the two samples respectively, and are hereby defined mathematically as $\mu_i = \sum_k X_{ki}$ and $\sigma_i^2 = \sum_k (\mu_i - X_{ki})^2$, where X_{ki} is the "kth" datum from Sample i.

Taking the sample mean and sample standard deviation as μ and σ respectively for both samples ($\mu_1 = 10.42$, $\mu_2 = 3.42$, $\sigma_1^2 = 46.24$, $\sigma_2^2 = 9.07$), we obtain best-fit curves as shown in Figures 2 and 3, where they have been superimposed on their respective sample distributions for comparison. For the rest of this paper, these best-fit distributions for Sample I and Sample II will be termed Fit I and Fit II respectively. A Kolmogorov-Smirnov test has been preformed for Fit II, resulting in $D_{34} = 0.125$, in other words, an acceptance level of $1 - \alpha \approx 65\%$. Fit I is expected to have a very low acceptance level, which we did not calculate but are certain that it is (possibly much) smaller than 30%, which is one of the reasons why we decided to devise the correction below.
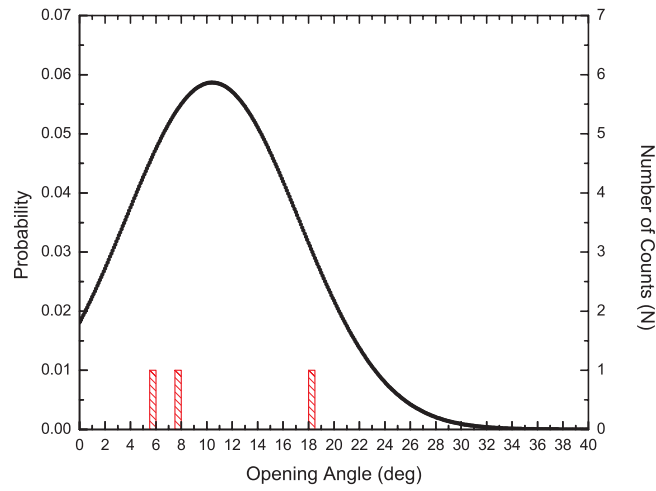
Fig. 2 Sample I best fit probability distribution curve superimposed on the Sample I bar graph.
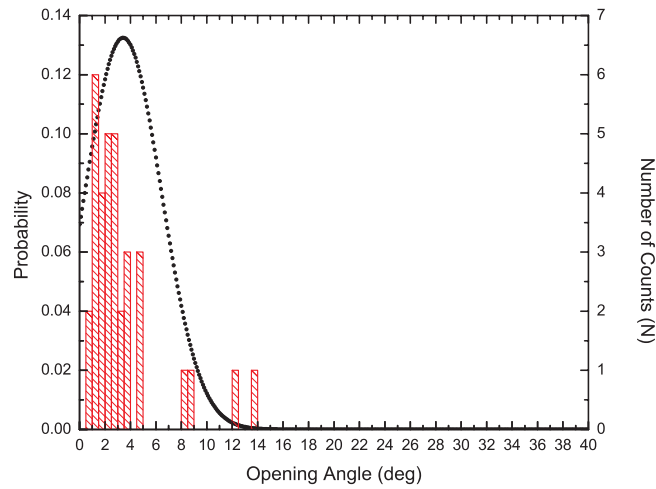
Fig. 3 Sample II best fit probability distribution curve superimposed on the Sample II bar graph.

Next, we calculate the probability that a random variable from Fit I (θ_{j1}) is larger than a random variable from Fit II (θ_{j2}). The random variable $\theta_{j1} - \theta_{j2}$ should have a distribution of

$$N(\theta_{j1} - \theta_{j2}) = \frac{1}{\sqrt{\sigma_1^2 + \sigma_2^2}\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{(\theta_{j1} - \theta_{j2}) - (\mu_1 - \mu_2)}{\sqrt{\sigma_1^2 + \sigma_2^2}}\right)^2\right], \tag{3}$$

This distribution is another normal distribution with a mean of $(\mu_1 - \mu_2)$ and a variance of $(\sigma_1^2 + \sigma_2^2)$, as shown in Figure 4. Thus,

$$P(\theta_{j1} > \theta_{j2}) = P(\theta_{j1} - \theta_{j2} > 0) = \int_0^{+\infty} N(\theta_{j1} - \theta_{j2})d(\theta_{j1} - \theta_{j2}),$$

i.e.

$$P_1(\theta_{j1} > \theta_{j2}) = \int_0^{+\infty} \frac{1}{\sqrt{\sigma_1^2 + \sigma_2^2}\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x - (\mu_1 - \mu_2)}{\sqrt{\sigma_1^2 + \sigma_2^2}}\right)^2\right] dx. \qquad (4)$$

Calculating this by means of a Fortran program, we obtain

$$P_1(\theta_{j1} > \theta_{j2}) = 0.83. \qquad (5)$$

The probability shown in Equation (5) is the probability that a Type I jet angle is larger than a Type II jet beaming angle, but only if Type I and II GRB beaming angles have a distribution strictly the same as Fit I and Fit II, respectively. In the case of Fit II, 34 samples have been taken to make the fit, therefore, the difference is considered negligible. However, for Fit I, which was made with only three samples, the effects of inconsistency must be considered.

According to Equation (4), $P(\theta_{j1} > \theta_{j2})$ can be calculated once μ_1 and σ_1^2 are given as constants. However, the μ_1 and σ_1^2 in this equation have probability distributions of their own, which are reliant on the consistency of Fit I. They are

$$t(N - 1) \sim \frac{(\overline{X} - \mu_1)\sqrt{N}}{S_X}, \qquad (6)$$

and

$$\chi^2(N - 1) \sim \frac{(N - 1)S_X^2}{\sigma_1^2}, \qquad (7)$$

respectively, where $N = 3$, the number of Sample I elements. Here, t and χ^2 correspond to a t distribution and a χ^2 distribution respectively, $\overline{X}$ is the sample mean, S_X is the sample variance; μ_1 and σ_1 are the theoretical mean and standard variation of Sample I respectively (shown here as variables).

Integrating Equation (4) over these probability distributions, taking $\frac{(\overline{X} - \mu_1)\sqrt{N}}{S_X}$ and $\frac{(N-1)S_X^2}{\sigma_1^2}$ to be y and z respectively, we write $P_2(\theta_{j1} > \theta_{j2}) = \int_0^{+\infty} \chi^2(z, 2) \int_{-\infty}^{+\infty} t(y, 2) \cdot P_1 dydz$; note that P_1, though given as a constant in Equation (5), is dependent on $(\mu_1 - \mu_2)$ and $(\sigma_1^2 + \sigma_2^2)$, which are not constants for the purpose of the following calculations. Substituting the right hand side of Equation (4) for $P_1(y, z)$, and equivalent terms in y and z for μ_1 and σ_1, respectively, we obtain

$$P_2(\theta_{j1} > \theta_{j2}) = \int_0^{+\infty} \chi^2(z, 2) \int_{-\infty}^{+\infty} t(y, 2) \cdot P_1(y, z) dydz$$

$$= \int_0^{+\infty} \chi^2(z, 2) \int_{-\infty}^{+\infty} t(y, 2) \int_0^{+\infty} \frac{1}{\sqrt{g(z)+\sigma_2^2}\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x - (f(y) - \mu_2)}{\sqrt{g(z)+\sigma_2^2}}\right)^2\right] dxdydz, \qquad (8)$$

where $t(y, 2) = \frac{1}{2\sqrt{2}}\left(1 + \frac{y^2}{2}\right)^{-\frac{3}{2}}$, $\chi^2(z, 2) = \frac{1}{2}\exp(-\frac{z}{2})$, $f(y) = \mu_1 = \overline{X} - \frac{S_X}{\sqrt{3}}y$, and $g(z) = \sigma_1^2 = (N - 1)S_X^2/z$.

Again using a Fortran program, we calculate this final result to be

$$P_2(\theta_{j1} > \theta_{j2}) = 0.71. \qquad (9)$$
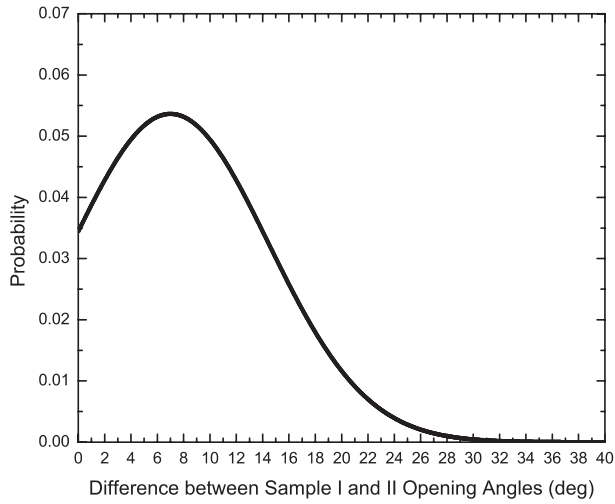
Fig. 4 Distribution of random variable $\theta_{j1} - \theta_{j2}$. This is the probability distribution of the value of the difference between a random variable from Fit I and another from Fit II.

4 DISCUSSION

Theoretically, it would have been more logical to perform a similar correction on Sample II in conjunction with the one used on Sample I. However, this would lead to 5-dimensional integration, which would be far more than what our PC could manage in a short period of time (the 3-D integration for P_2 took 10 min at double precision, with a step length of 0.03 for x, y and z). Instead, we performed the 3-D integration correction on Samples I and II separately. Taking into account the first 4 digits after the decimal point, P_1 is calculated to be 0.8272. This merely decreases to 0.8254 after application of the correction to Sample II, while it decreases to $P_2 = 0.7118$ after application of the correction to Sample I. From these numbers, we find that neglecting the Sample II correction induces an error of only a magnitude of 10^{-3} (i.e. the third digit after the decimal point), and thus find it safe to retain the first 2 digits without performing the correction to Sample II. Hence, this yielded the number of digits used in the presentation of our results (Eqs. (5) and (9)).

With an acceptance level of only 65% for even Fit II, it is indeed debatable whether a Gaussian fit is appropriate for the samples. However, due to the fact that an established fit for opening angles does not exist, a Gaussian fit seems to be the natural choice. Further research may experiment with other parametrical fits that may yield better acceptance levels.

It has been proposed that GRBs, which would fall into the Sample I category but have only minimum constraints for their jet break times (such as GRB 050724), could be used in the statistics as well. In doing so, a minimum value could be calculated for both P_1 and P_2, on the basis of a larger sample. However, this could lead to complexities, since there are GRBs which would also fall into the Sample II category that only have minimum constraints for their jet break times. In this paper, we wish to avoid such complexities, and therefore use only exact data.

The reader might want to notice that there are several problems that have not been taken into account. Firstly, all data on $E_{\rm iso}(\gamma)$, z and t_j in this paper have been collected from the literature. While this should cause no problems for z and $E_{\rm iso}(\gamma)$, it could be somewhat problematic for t_j, since the time of the jet break is different for different bands at which the specific observation was made. Throughout this paper, this matter has been treated indiscriminately. Also, the data, as shown in Table 1, are presented with a certain deviation in the literature. In other words, they are not precise.

Even more prominent is the problem that if we take Fit I to be exactly the distribution of Type I GRB jet beaming angles, a significant number of Type I jet beaming angles will be smaller than zero. These problems, throughout the paper, have been treated as insignificant details, hereby submitted to future scrutiny by the reader.

5 CONCLUSIONS

In this paper, we have obtained a Sample I of 3 jet beaming angles and a Sample II of 34 jet beaming angles. These two samples are expected to be representative of Type I and Type II GRBs respectively. After that, normal (Gaussian) distributions have been fitted to the samples, resulting in Fit I and Fit II. Taking these fits to be representative of Type I and Type II GRBs, we then proceed to calculate the probability that a random variable taken from Fit I is larger than another taken from Fit II, thereby deriving the probability that a Type I GRB has a larger opening angle than a Type II GRB. Taking into account the uncertainty caused by the very small Sample I, we then devise a correction for the probability stated above, and reach a probability that is much lower but still significantly high nevertheless.

If we take Sample I to be a sample perfectly representative of Type I GRBs, then it could be said, with an 83% degree of confidence, that Type I GRBs have larger jet beaming angles than Type II GRBs. However, this 83% drops to 71% once we take into account the uncertainty caused by our small sample in Sample I. In either case, it could be justifiably concluded, from our current samples, that Type I GRBs generally have a larger beaming angle in comparison to Type II GRBs.

Our results are supportive of current models and theory. Type I GRBs in general have larger beaming angles in comparison to Type II GRBs, with a fairly high degree of certainty, though by far not high enough to become a decisive criterion for whether a GRB is of Type I or Type II origin (so far).

It is possible that with a larger Sample I, higher levels of certainty could be reached. Therefore, further observations that yield data concerning Sample I jet beaming angles (preferably Type I GRB jet beaming angles) are required for progress. It is also possible that with a large enough Sample I, the correction methods used in this paper (i.e. the correction for Sample I) will become obsolete; however, given the current rate at which Sample I opening angles are being derived, it seems unlikely that this would be the case in the near future. Lastly, it will be desirable to find a parametrical fit which could be supported theoretically, and which yields a higher level of acceptance than the Gaussian.

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