Research in Astron. Astrophys. **2010** Vol. **10** No. **12**, 1275–1286
http://www.raa-journal.org *http://www.iop.org/journals/raa*

Research in Astronomy and Astrophysics

Non-similar collapse of singular isothermal spherical molecular cloud cores with nonzero initial velocities

Mohsen Nejad-Asghar

[1] Department of Physics, University of Mazandaran, Babolsar, Iran; *nejadasghar@umz.ac.ir*
[2] Research Institute for Astronomy and Astrophysics of Maragha, Maragha, Iran
[3] Department of Physics, Damghan University, Damghan, Iran

Received 2010 May 1; accepted 2010 July 1

Abstract Theoretically, stars formed from the collapse of cores in molecular clouds. Historically, the core had been assumed to be a singular isothermal sphere (SIS), and the collapse had been investigated in a self-similar manner. When the rotation and magnetic fields lead to non-symmetric collapse, a spheroidal shape may occur. Here, the result of the centrifugal force and magnetic field gradient is assumed to be in the normal direction to the rotational axis, and its components are supposed to be a fraction β of the local gravitational force. In this research, a collapsing SIS core is considered to find the importance that the parameter β plays in the oblateness of the mass shells, which are the crests of the expansion waves. We apply the Adomian decomposition method to solve the system of nonlinear partial differential equations because the collapse does not occur in a spherically symmetric and self-similar manner. In this way, we obtain a semi-analytical relation for the mass infall rate $\dot{M}$ of the shells in the envelope. Near the rotational axis, $\dot{M}$ decreases with the increase of the non-dimensional radius ξ, while a direct relation is observed between $\dot{M}$ and ξ in the equatorial regions. Also, the values of $\dot{M}$ in the polar regions are greater than their equatorial values, and this difference occurs more often at smaller values of ξ. Overall, the results show that before reaching the crest of the expansion wave, the visible shape of the molecular cloud cores can evolve into oblate spheroids. The ratio of major to minor axes of oblate cores increases when increasing the parameter β, and its value can approach the observed elongated shapes of cores in the maps of molecular clouds, such as those in Taurus and Perseus.

Key words: ISM: clouds — ISM: evolution — stars: formation – methods: analytical

1 INTRODUCTION

A great deal is now known about dense cores in molecular clouds that are the progenitors of protostars (e.g., di Francesco et al. 2007; Ward-Thompson et al. 2007). We know that, approximately, all cores in the maps of molecular clouds seem to be elongated rather than spherical. For example, we can refer to the recent work of Curtis & Richer (2010) for the two-dimensional ellipticity of cores in the Perseus molecular cloud, or to the old report of Myers et al. (1991) for the apparent elongated shapes of cores in the Taurus maps. Overall, the observations show that, on average, the ratios of the major to minor axes of cores vary between approximately 1.2 to 2 with a mean value of ≈ 1.6.

Determining the exact three-dimensional shape of a core from the apparent observations of the plane-of-the-sky is impossible; instead, statistical techniques have to be applied. Several studies have analyzed the observations in the context of a random distribution of inclinations to infer that cores are more nearly prolate than oblate (e.g., Curry 2002; Jones & Basu 2002). This prolate elongation of cores may be inferred to be a remnant of their origin in filaments (e.g., Hartmann 2002). Also, models with cores forming from turbulent flows predict random triaxial shapes with a slight preference for prolateness (Gammie et al. 2003; Li et al. 2004). Although some work indicates a preference for prolate cores, there are many studies that consistently favor oblate shapes (e.g., Jones et al. 2001; Goodwin et al. 2002; Tassis 2007; Offner & Krumholz 2009; Tassis et al. 2009). If strong magnetic fields are present then collapsing cores are expected to be oblate (e.g., Galli & Shu 1993; Basu & Ciolek 2004; Ciolek & Basu 2006), which could also be caused by strong rotational motion (e.g., Cassen & Moosman 1981; Terebey et al. 1984).

Historically, in primary theoretical models for the collapse of cores and formation of stars, an isothermal equation of state had been used, from which, as a consequence of subsonic communications between different parts of the cloud, an inverse square profile for density appeared (Bodenheimer & Sweigart 1968). Larson (1969) and Penston (1969) were the first to analyze this inverse square behavior of the density profile using the similarity method. In this case, which had been subsequently extended by Hunter (1977), one begins with a static cloud of constant density and follows the formation of the r^{-2} density profile. In the opposite case, Shu (1977) assumed that the density is initially in the inverse square profile of the singular isothermal sphere (SIS) core, and constructed the expansion wave collapse solution to suggest the inside-out collapse scenario. These two limiting solutions of Hunter and Shu may be described as fast and slow collapses, respectively, and the reality may be somewhere in between (McKee & Ostriker 2007). Since then, a lot of asymptotic solutions and global numerical simulations have been found and developed in which authors have approximately considered the effects of three important mechanisms: turbulence, rotation and magnetic fields (see, e.g., Hartmann 2009).

In this research, we return to the basic spherical collapse problem for polytropic spheres, which was used as the idealized SIS similarity solution by Shu (1977). In addition, we consider an initial inward flow, i.e. the conditions observed in some molecular cloud cores and used by Fatuzzo et al. (2004), to investigate its effects on the collapse of the SIS. The goal of this paper is to reexamine the gravitational collapse of the SIS with a focus on the oblateness of a core via the effect of rotation and magnetic fields. We suggest that the centrifugal force and magnetic field pressure lead to oblateness of the envelope of a molecular core before triggering the collapsing density wave. Since the collapse is not in a spherically symmetric manner, the similarity method cannot be used. Instead, we use the Adomian decomposition method (Adomain 1994) to semi-analytically solve the system of differential equations. For this purpose, the collapse of SIS using the Adomian method is given in Section 2. The non-similar collapse of SIS is investigated in Section 3 in which the oblateness of the core is also obtained. Finally, Section 4 is devoted to a summary and conclusions.

2 COLLAPSE OF SIS BY ADOMIAN METHOD

The initial density of an SIS is assumed to be in the form of an inverse square, and the collapse is assumed to be only in the radial direction. The mass continuity equation is

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 \rho u) = 0, \qquad (1)$$

where u is the radial velocity which follows the force equation,

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} = -\frac{a_{\rm s}^2}{\rho}\frac{\partial \rho}{\partial r} - g, \qquad (2)$$

$a_{\rm s}$ is the sound speed, and the gravitational acceleration g follows the Poisson's equation,

$$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 g) = 4\pi G\rho. \tag{3}$$

Choosing the sound speed as the unit of velocity, and $[t] = 1/\sqrt{4\pi G\rho_{\rm c}}$ as the unit of time where $\rho_{\rm c}$ is the density at the center of the core, the basic Equations (1)–(3) can be rewritten as follows

$$\frac{\partial\rho}{\partial t} + u\frac{\partial\rho}{\partial\xi} + \rho\frac{\partial u}{\partial\xi} + \frac{2}{\xi}\rho u = 0, \tag{4}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial\xi} + \frac{1}{\rho}\frac{\partial\rho}{\partial\xi} + g = 0, \tag{5}$$

$$\frac{1}{\xi^2}\frac{\partial}{\partial\xi}(\xi^2 g) = \rho, \tag{6}$$

where $\xi \equiv r/(a_{\rm s}[t])$ is the nondimensional radius and the density and gravitational acceleration are non-dimensionalized by $\rho_{\rm c}$ and $a_{\rm s}/[t]$, respectively.

If the cloud is in hydrostatic equilibrium ($u = 0$), Equations (4)–(6) lead to the Lane-Emden equation which is well-known in the theory of stellar structure (e.g., Chandrasekhar 1939). The Lane-Emden equation can be solved by the Adomian decomposition method which is found in Appendix A. The SIS is a special case of the Lane-Emden equation with inverse square density which does not conform to the boundary conditions. Here, we choose the initial density as $\rho(\xi, 0) = \Lambda/\xi^2$ where Λ is the overdensity parameter with $\Lambda = 2$ for hydrostatic equilibrium. The boundary condition for gravitational acceleration is $g(0, t) = 0$, and we assume that the initial velocity is inward so that $u(\xi, 0) = -u_\infty$, where u_∞ is constant.

As mentioned in Appendix A, for using the Adomian decomposition method, Equations (4)–(6) must be rewritten as

$$L_t\rho \;+\; N_1(\rho, u) = 0, \tag{7}$$

$$L_t u \;+\; g + N_2(\rho, u) = 0, \tag{8}$$

$$L_\xi g \;-\; \rho = 0, \tag{9}$$

where where $L_t(\circ) \equiv \frac{\partial(\circ)}{\partial t}$ and $L_\xi(\circ) \equiv \frac{1}{\xi^2}\frac{\partial}{\partial\xi}\left(\xi^2(\circ)\right)$ are operators, and

$$N_1(\rho, u) \;\equiv\; u\frac{\partial\rho}{\partial\xi} + \rho\frac{\partial u}{\partial\xi} + \frac{2}{\xi}\rho u, \tag{10}$$

$$N_2(\rho, u) \;\equiv\; u\frac{\partial u}{\partial\xi} + \frac{1}{\rho}\frac{\partial\rho}{\partial\xi}, \tag{11}$$

are the nonlinear terms of the differential equations which can be written by Adomian series $N_j(\rho, u) = \sum_{n=0}^{\infty} A_n^{(j)}$ for $j = 1, 2$, where

$$A_n^{(j)} = \frac{1}{n!}\lim_{\lambda\to 0}\left[\frac{\partial^n}{\partial\lambda^n}N_j\left(\sum_{i=0}^{n}\rho_i\lambda^i, \sum_{i=0}^{n}u_i\lambda^i\right)\right], \tag{12}$$

are the Adomian polynomials (Adomian 1994). In this way, the final solutions are given by the series $\rho = \sum_{n=0}^{\infty}\rho_n$, $u = \sum_{n=0}^{\infty}u_n$ and $g = \sum_{n=0}^{\infty}g_n$ where the terms can be obtained by the recurrence relations

$$\rho_{n+1} \;=\; -L_t^{-1}A_n^{(1)}, \tag{13}$$

$$u_{n+1} \;=\; -L_t^{-1}(g_n + A_n^{(2)}), \tag{14}$$

$$g_{n+1} \;=\; L_\xi^{-1}\rho_n, \tag{15}$$

and where $L_t^{-1}(\circ) \equiv \int(\circ)dt$ and $L_\xi^{-1}(\circ) \equiv \xi^{-2}\left(\int \xi^2(\circ)d\xi\right)$ are the integration operators and the $n = 0$ terms are from initial and boundary conditions: $\rho_0 \equiv \rho(\xi, 0)$, $u_0 \equiv u(\xi, 0)$ and $g_0 \equiv g(0, t)$.

Using mathematical software, such as Maple (see Appendix B), the density and inward velocity are obtained respectively as follows:

$$\rho = \frac{\Lambda}{\xi^2}\left[1 - \frac{(\Lambda-2)}{2}\frac{t^2}{\xi^2} + u_\infty\frac{2(\Lambda-2)}{3}\frac{t^3}{\xi^3} + \frac{(\Lambda-2)(6-2\Lambda+3u_\infty^2)}{4}\frac{t^4}{\xi^4} + O\left(\frac{t^5}{\xi^5}\right)\right] \quad (16)$$

$$u = -u_\infty - (\Lambda-2)\frac{t}{\xi}\left[1 - \frac{u_\infty}{2}\frac{t}{\xi} + \frac{(6-\Lambda+2u_\infty^2)}{6}\frac{t^2}{\xi^2} - \frac{(24-5\Lambda+3u_\infty^2)u_\infty}{12}\frac{t^3}{\xi^3}\right] + O\left(\frac{t^5}{\xi^5}\right). \quad (17)$$

In the case of $u_\infty = 0$, these results reduce to equation (19) of the well-known paper of Shu (1977) in which the similarity variable x is replaced by t/ξ (note that eqs. (45) and (46) of Fatuzzo et al. 2004 are mistyped). According to the convergence problem in the series of the Adomian decomposition method (as mentioned in Appendix A), Equations (16) and (17) are reliable in the range of $t/\xi < 1$. Thus, Equations (16) and (17) are acceptable only for the outer regions from the crest of the expansion wave (i.e., $\xi = t$), and the mass infall rate $\dot{M} \equiv 4\pi\xi^2\rho \mid u \mid$ of the shells in the envelope can be determined. For the inner regions of the expansion wave, the terms of series (16) and (17) are not convergent, thus, we must use some suitable method, such as the piecewise-adaptive decomposition method (e.g., Ramos 2009), to perform the analysis, which is beyond the scope of this paper.

3 NON-SIMILAR COLLAPSE OF SIS

The cores of molecular clouds rotate and the magnetic fields are also the non-negligible parts of this medium. In this section, we investigate the effects of these mechanisms on the dynamics of the collapsing SIS core. In this case, the infall velocity of matter in spherical coordinates has two components given by $\boldsymbol{v} = \hat{r}u + \hat{\theta}w$, so that the mass continuity equation is expressed as

$$\frac{\partial \rho}{\partial t} + \frac{1}{\xi^2}\frac{\partial}{\partial \xi}(\xi^2\rho u) + \frac{1}{\xi\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\rho w) = 0, \quad (18)$$

which is non-dimensionalized according to the units given in Section 2. Here, the effects of the centrifugal force and pressure of magnetic field, in the envelope of the core, are assumed to be in the normal direction of the rotational axis as follows

$$\boldsymbol{F}_{\rm mag+rot} = \hat{r}F_\xi\sin^2\theta + \hat{\theta}F_\theta\sin 2\theta, \quad (19)$$

where for simplicity, the components are assumed to be a fraction of the local gravitational force as $F_\xi = -\beta\partial\phi/\partial\xi$ and $F_\theta = -\beta\partial\phi/\partial\theta$, where the magnetic-rotational parameter $0 \leq \beta \leq 1$ indicates the importance of rotation and magnetic fields, and ϕ is the gravitational potential which follows the Poisson's equation,

$$\frac{1}{\xi^2}\frac{\partial}{\partial\xi}(\xi^2\frac{\partial\phi}{\partial\xi}) + \frac{1}{\xi^2\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\frac{\partial\phi}{\partial\theta}) = \rho. \quad (20)$$

In this way, the force equation has two components as follows

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial\xi} + \frac{w}{\xi}\frac{\partial u}{\partial\theta} = -\frac{1}{\rho}\frac{\partial\rho}{\partial\xi} - (1-\beta\sin^2\theta)\frac{\partial\phi}{\partial\xi}, \quad (21)$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial\xi} + \frac{w}{\xi}\frac{\partial w}{\partial\theta} = -\frac{1}{\rho\xi}\frac{\partial\rho}{\partial\theta} - (1-\beta\sin 2\theta)\frac{1}{\xi}\frac{\partial\phi}{\partial\theta}. \quad (22)$$

Since the components of gravitational force, $\partial\phi/\partial\xi$ and $\partial\phi/\partial\theta$, are zero at the center of the core, the boundary condition of the gravitational potential at $\xi = 0$ is assumed to be $\phi(0, t) = 0$.

At the beginning of collapse, the components of velocity are assumed to be $u(r,0) = -u_\infty$ and $w(r,0) = 0$, and the initial density is assumed to be the density of SIS: $\rho(\xi,0) = \Lambda/\xi^2$. We rewrite Equations (18), (20), (21) and (22) in the Adomian form as follows

$$L_t\rho + N_1(\rho,u,w) = 0, \tag{23}$$

$$L_{\xi\xi}\phi + \frac{1}{\xi^2}\frac{\partial^2\phi}{\partial\theta^2} + \frac{\cot\theta}{\xi^2}\frac{\partial\phi}{\partial\theta} - \rho = 0, \tag{24}$$

$$L_t u + (1-\beta\sin^2\theta)\frac{\partial\phi}{\partial\xi} + N_2(\rho,u,w) = 0, \tag{25}$$

$$L_t w + (1-\beta\sin 2\theta)\frac{1}{\xi}\frac{\partial\phi}{\partial\theta} + N_3(\rho,u,w) = 0, \tag{26}$$

respectively, where the nonlinear terms are

$$N_1(\rho,u,w) \equiv u\frac{\partial\rho}{\partial\xi} + \rho\frac{\partial u}{\partial\xi} + \frac{2}{\xi}\rho u + \frac{w}{\xi}\frac{\partial\rho}{\partial\theta} + \frac{\rho}{\xi}\frac{\partial w}{\partial\theta} + \frac{\cot\theta}{\xi}\rho w, \tag{27}$$

$$N_2(\rho,u,w) \equiv u\frac{\partial u}{\partial\xi} + \frac{w}{\xi}\frac{\partial u}{\partial\theta} + \frac{1}{\rho}\frac{\partial\rho}{\partial\xi}, \tag{28}$$

$$N_3(\rho,u,w) \equiv u\frac{\partial w}{\partial\xi} + \frac{w}{\xi}\frac{\partial w}{\partial\theta} + \frac{1}{\rho\xi}\frac{\partial\rho}{\partial\theta}. \tag{29}$$

In this way, by using the Adomian polynomials,

$$A_n^{(j)} = \frac{1}{n!}\lim_{\lambda\to 0}\left[\frac{\partial^n}{\partial\lambda^n}N_j\left(\sum_{i=0}^{n}\rho_i\lambda^i, \sum_{i=0}^{n}u_i\lambda^i, \sum_{i=0}^{n}w_i\lambda^i\right)\right], \tag{30}$$

for $j = 1,2,3$, we obtain the recurrence relations as follows

$$\rho_{n+1} = -L_t^{-1}A_n^{(1)}, \tag{31}$$

$$\phi_{n+1} = -L_{\xi\xi}^{-1}\left[\frac{1}{\xi^2}\frac{\partial^2\phi_n}{\partial\theta^2} + \frac{\cot\theta}{\xi^2}\frac{\partial\phi_n}{\partial\theta} - \rho_n\right], \tag{32}$$

$$u_{n+1} = -L_t^{-1}\left[(1-\beta\sin^2\theta)\frac{\partial\phi_n}{\partial\xi} + A_n^{(2)}\right], \tag{33}$$

$$w_{n+1} = -L_t^{-1}\left[(1-\beta\sin 2\theta)\frac{1}{\xi}\frac{\partial\phi_n}{\partial\theta} + A_n^{(3)}\right], \tag{34}$$

where the $n = 0$ terms are given by the initial and boundary conditions. Thus, we can obtain the terms of the series $\rho = \sum_{n=0}^{\infty}\rho_n$, $u = \sum_{n=0}^{\infty}u_n$, $w = \sum_{n=0}^{\infty}w_n$ and $\phi = \sum_{n=0}^{\infty}\phi_n$ with mathematical software such as Maple (see Appendix B). Here, we turn our attention to the mass infall rate, $\dot{M} \equiv 4\pi\xi^2\rho|u|$, in the envelope of the core ($\xi > t$) where the Adomian decomposition method is reliable. The result is

$$\dot{M} = 4\pi\Lambda u_\infty + 4\pi\Lambda(\Lambda - 2 - \Lambda\beta\sin^2\theta)\frac{t}{\xi} - 4\pi\Lambda(\Lambda - 2 - \Lambda\beta\sin^2\theta)u_\infty\frac{t^2}{\xi^2} + O\left(\frac{t^3}{\xi^3}\right), \tag{35}$$

whose values for $\Lambda = 2.1$, $u_\infty = 0.1$ and $\beta = 0.1$ at time $t = 1/6$ are shown in Figure 1.

Since the infall rates at the polar and equatorial regions are different, an initial spherical shell in the outer region of the expansion wave will be spheroidal after some time, as schematically shown

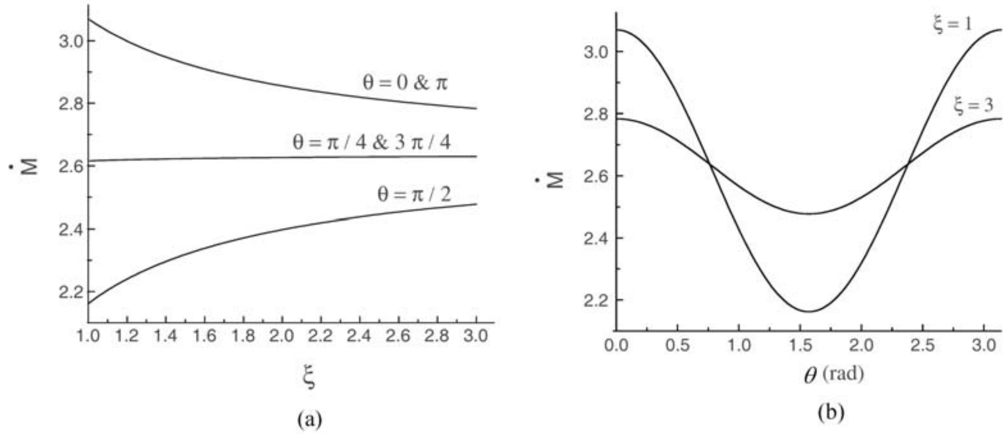

Fig. 1 Mass infall rate versus (a) nondimensional length ξ, and (b) polar angle θ, with $\Lambda = 2.1$, $u_\infty = 0.1$ and $\beta = 0.1$ at time $t = 1/6$.

in Figure 2. In the first order approximation of mass infall rate (i.e. to the order of t/ξ), the ratio of major to minor axes of the spheroid can be approximated as

$$\frac{b}{a} \equiv \frac{\xi - u_\infty t - \frac{1}{2}\ddot{M}_{(\theta=\pi/2)} t^2}{\xi - u_\infty t - \frac{1}{2}\ddot{M}_{(\theta=0)} t^2}. \tag{36}$$

Inserting $\ddot{M} \simeq 4\pi\Lambda(\Lambda - 2 - \Lambda\beta \sin^2\theta)/\xi$, obtained from Equation (35), we have

$$\frac{b}{a} = \frac{1 - u_\infty \frac{t}{\xi} - 2\pi\Lambda(\Lambda - 2 - \Lambda\beta)\frac{t^2}{\xi^2}}{1 - u_\infty \frac{t}{\xi} - 2\pi\Lambda(\Lambda - 2)\frac{t^2}{\xi^2}}, \tag{37}$$

which is depicted in Figure 3 at time $t = 1/6$.

4 SUMMARY AND CONCLUSIONS

Stars have been formed from the collapse of cores in molecular clouds. A basic standard model for the collapse of cores assumes an SIS, in which the collapse occurs inside-out accompanied by an expansion wave. We know that not only the molecular cores rotate, but also the magnetic fields affect their dynamics. If we assume that the effect of centrifugal force and magnetic pressure are in the normal direction to the rotational axis, we expect that the shape of the envelope of the core (regions outside of the crest of the expansion wave) will be modified into a spheroid. In this case where the collapse is in a non-symmetric manner, we used the Adomian decomposition method to solve the differential equations. In Appendix A, we solved the well-known Lane-Emden equation to find that the Adomian method is applicable. Then, in Section 2, the collapse of an SIS core is investigated. The results show that the Adomian decomposition method presents convergent solutions only for the regions outside of the crest of the expansion wave (i.e. envelope).

In Section 3, the non-similar collapse of an SIS core, which is affected by the centrifugal force and magnetic pressure, is solved by the Adomian decomposition method. The mass infall rate of the shells in the envelope of the core is obtained and depicted in Figure 1 with typical values of the parameters. According to Figure 1(a), the mass infall rate in the polar regions (near the rotational axis) decreases with an increase of radius, while in the equatorial regions, the mass infall rate in regions

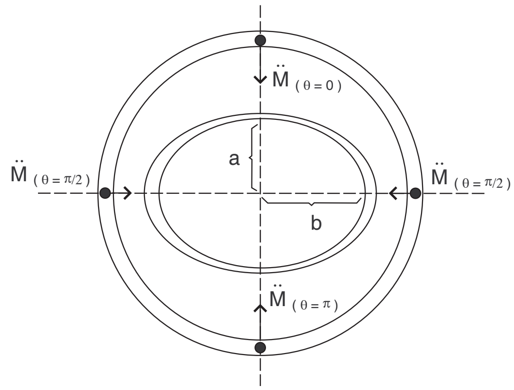

Fig. 2 Schematic diagram for mass infall rate which leads to the formation of a spheroidal shell from an initial spherical shell.

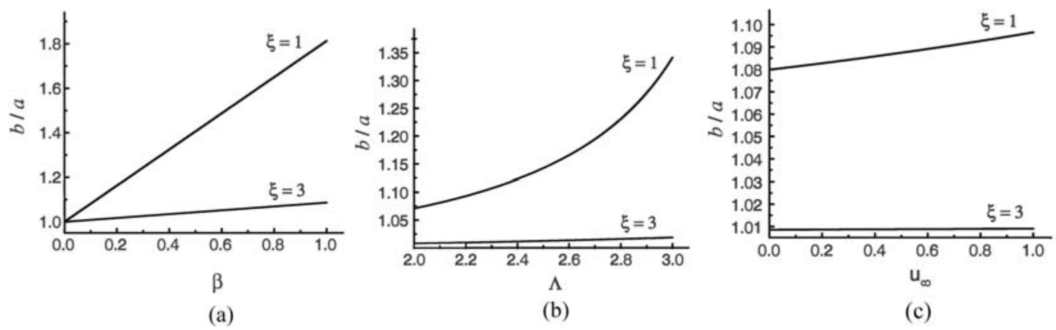

Fig. 3 Ratio of major to minor axes of spheroidal shells of a core at time $t = 1/6$, versus (a) magnetic-rotational parameter β with $\Lambda = 2.1$ and $u_\infty = 0.1$, (b) overdensity parameter Λ with $\beta = 0.1$ and $u_\infty = 0.1$, and (c) inward initial velocity u_∞ with $\Lambda = 2.1$ and $\beta = 0.1$.

far from the crest of the expansion wave is greater than that in regions near to it. Figure 1(b) shows that the mass infall rate in equatorial regions is less than that in polar regions, thus, before reaching the expansion wave, the shape of the envelope is converted to a spheroid as shown schematically in the Figure 2.

In this way, according to the mass infall rate, we determined the ratio of major to minor axes of the spheroid, and its values are shown in Figure 3 with typical values of the parameters. We see that the ratio of major to minor axes of the spheroid does not strongly depend on the initial inward velocity u_∞, but its value depends on the overdensity parameter Λ and magnetic-rotational parameter β. Thus, the results show that the magnetic field and rotation lead to a non-symmetric collapse so that before reaching the crest of the expansion wave, the apparent shape of the envelope will be converted to that of triaxial oblate spheroids. Also, the results show that the ratio of major to minor axes of the spheroids can reach values which are consistent with the observed maps of the cores in the molecular clouds, such as those in Taurus and Perseus.

Acknowledgements This work has been supported by the Research Institute for Astronomy and Astrophysics of Maragha (RIAAM).

Appendix A: SOLVING THE LANE-EMDEN EQUATION BY THE ADOMIAN DECOMPOSITION METHOD

A spherical polytropic gas in hydrostatic equilibrium satisfies the equation

$$\frac{1}{\rho}\frac{dp}{dr} = -\frac{GM(r)}{r^2}, \tag{A.1}$$

where $M(r)$ is the mass inside the radius r and the polytropic pressure is $p = \kappa\rho^{\Gamma}$, for which, in the isothermal case, we have $\kappa = a_{\rm s}^2$ and $\Gamma = 1$, where $a_{\rm s}$ is the isothermal sound speed. Substituting $\rho = \rho_{\rm c}\exp(-\psi)$ into Equation (A.1), we have

$$\frac{1}{\xi^2}\frac{d}{d\xi}\left[\xi^2 e^{(1-\Gamma)\psi}\frac{d\psi}{d\xi}\right] = e^{-\psi}, \tag{A.2}$$

where $\xi \equiv \left(\frac{4\pi G}{\kappa\Gamma\rho_{\rm c}^{\Gamma-2}}\right)^{1/2} r$ is the nondimensional radius. In the isothermal case, Equation (A.2) reduces to the well-known Lane-Emden equation. The boundary conditions are $\psi_{(\xi=0)} = 0$ and $(d\psi/d\xi)_{\xi=0} = 0$. Thus, Equation (A.2) can straightforwardly be solved by numerical methods such as fourth-order Runge-Kutta.

Here, we solve Equation (A.2) by the Adomian decomposition method, then we compare the result with the numerical method. For this purpose, we rewrite Equation (A.2) as follows

$$L_{\xi\xi}\psi + N(\psi) = 0, \tag{A.3}$$

where $L_{\xi\xi}(\circ) \equiv \frac{1}{\xi^2}\frac{d}{d\xi}\left(\xi^2\frac{d(\circ)}{d\xi}\right)$ is the operator, and nonlinear terms of the differential equation are assembled in the function $N(\psi) \equiv (1-\Gamma)\left(\frac{d\psi}{d\xi}\right)^2 - e^{(\Gamma-2)\psi}$. The basis of using the Adomian decomposition method is to replace the function ψ by a series $\psi = \sum_{n=0}^{\infty}\psi_n$, and the nonlinear term $N(\psi)$ by a Taylor expansion series $N(\psi) = \sum_{n=0}^{\infty} A_n$, where A_n are Adomian polynomials

$$\begin{aligned}
A_0 &= N(\psi_0),\\
A_1 &= \psi_1\left(\frac{d\psi}{d\xi}\right)_{\psi=\psi_0},\\
A_2 &= \psi_2\left(\frac{d\psi}{d\xi}\right)_{\psi=\psi_0} + \frac{1}{2!}\psi_1^2\left(\frac{d^2\psi}{d\xi^2}\right)_{\psi=\psi_0},
\end{aligned} \tag{A.4}$$

and so on (Adomian 1994), which can be expressed in the general form

$$A_n = \frac{1}{n!}\lim_{\lambda\to 0}\left[\frac{d^n}{d\lambda^n}N\left(\sum_{i=0}^{n}\psi_i\lambda^i\right)\right], \tag{A.5}$$

and where λ is only a dummy variable which is inserted to recover Equation (A.4).

In this way, Equation (A.2) leads to a recurrence relation

$$\psi_{n+1} = -L_{\xi\xi}^{-1}A_n, \tag{A.6}$$

where $L_{\xi\xi}^{-1}(\circ) \equiv \int \xi^{-2}\left(\int \xi^2(\circ)d\xi\right)d\xi$ is the integration operator and $\psi_0 \equiv \psi_{(\xi=0)}$. Thus, we can determine the ψ_ns with the help of any mathematical software, such as Maple (see Appendix B). The result is a series like

$$\psi = a_2\xi^2 + a_4\xi^4 + a_6\xi^6 + ..., \tag{A.7}$$

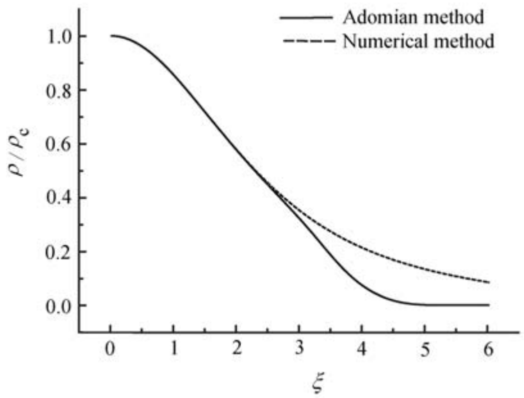

Fig. A.1 Ratio of density to the central density of an isothermal spherical core in hydrostatic equilibrium.

where the coefficients are

$$a_2 = +\frac{1}{3!},$$
$$a_4 = -\frac{1}{3}(8-5\Gamma)\frac{1}{5!},$$
$$a_6 = +\frac{2}{3}\left[4(2-\Gamma)^2 + 5(2-\Gamma)(1-\Gamma) + \frac{8}{3}(1-\Gamma)^2\right]\frac{1}{7!}, \tag{A.8}$$

and so on. In the isothermal case ($\Gamma = 1$), this result reduces to the result of Wazwaz (2001). In Figure A, the ratio of density to central density, $\frac{\rho}{\rho_c} = e^{-\psi}$, in the isothermal case, which is obtained from three terms of the series (A.8), is compared to the numerical results, which are obtained from the fourth-order Runge-Kutta method. As can be seen, the Adomian decomposition method gives reliable solutions for $\xi < 2$. This is because of using the Taylor expansion which destroys the convergence of the series at large ξ (Liao 2003; Singh et al. 2009). Substituting the appropriate values of typical molecular clouds, we have

$$\xi = \frac{r_{(\mathrm{pc})}}{0.03\mathrm{pc}\left(\frac{T}{10\mathrm{K}}\right)^{1/2}\left(\frac{n_c}{10^4\mathrm{cm}^{-3}}\right)^{-1/2}}, \tag{A.9}$$

where n_c is the number density at the center of the core. Thus, in the hydrostatic equilibrium of a typical molecular cloud core, the results, which are obtained by the Adomian decomposition method, are usable only for radii less than $\sim 0.06\mathrm{pc}$.

Appendix B: MAPLE PROGRAMS

– The Maple program for the Lane-Emden equation is as follows:

```
> psi[0]:= 0:
> n:= 0:
> N:= proc (psi) options operator, arrow;
      (1-Gamma)*(diff(psi,xi))^2-exp((Gamma-2)*psi)
     end proc:
> psi[1]:= -int(xi^(-2)*int( xi^2*N(psi[0]),xi),xi):
> for n from 1 to 7 do
    N:= proc (psi) options operator, arrow;
        (1-Gamma)*(diff(psi,xi))^2-exp((Gamma-2)*psi)
       end proc:
```

```
    psisum:= sum(psi[m]*lambda[n]^m,m=0..n):
    A[n]:= (diff(N(psisum),$(lambda[n],n)))/factorial(n):
    lambda[n]:= 0:
    psi[n+1]:= -int(xi^(-2)*int(xi^2*A[n],xi),xi):
  end do:
> psitotal:= collect(sum(psi[m],m=0..n),xi);
```

– The Maple program for the collapse of SIS is as follows:

```
> g[0]:= 0:
> rho[0]:= Lambda/xi^2:
> u[0]:= -u[infinity]:
> n:= 0:
> N1:= proc (rho,u) options operator, arrow;
        u*diff(rho,xi)+rho*diff(u,xi)+2*u*rho/xi
       end proc:
> N2:= proc (rho,u) options operator, arrow;
        u*diff(u,xi)+rho^(-1)*diff(rho,xi)
       end proc:
> g[1]:= xi^(-2)*int(xi^2*rho[0],xi):
> rho[1]:= -int(N1(rho[0],u[0]),t):
> u[1]:= -int(g[0]+N2(rho[0],u[0]),t):
> for n from 1 to 7 do
    N1:= proc (rho,u) options operator, arrow;
          u*diff(rho,xi)+rho*diff(u,xi)+2*u*rho/xi
         end proc:
    N2:= proc (rho,u) options operator, arrow;
          u*diff(u,xi)+rho^(-1)*diff(rho,xi)
         end proc:
    rhosum:= sum(rho[m]*lambda[n]^m,m=0..n):
    usum:= sum(u[m]*lambda[n]^m,m=0..n):
    A1[n]:= (diff(N1(rhosum,usum),$(lambda[n],n)))/factorial(n):
    A2[n]:= (diff(N2(rhosum,usum),$(lambda[n],n)))/factorial(n):
    lambda[n]:= 0:
    g[n+1]:= xi^(-2)*int(xi^2*rho[n],xi):
    rho[n+1]:= -int(A1[n],t):
    u[n+1]:= -int(g[n]+A2[n],t):
  end do:
> rhototal:= collect(sum(rho[m],m=0..n),t);
> utotal:= collect(sum(u[m],m=0..n),t);
> gtotal:= collect(sum(g[m],m=0..n),t);
```

– The Maple program for the non-similar collapse of SIS is as follows:

```
> Phi[0]:= 0:
> rho[0]:= Lambda/xi^2:
> u[0]:= -u[infinity]:
> w[0]:= 0:
> n:= 0:
> N1:= proc (rho,u,w) options operator, arrow;
        u*diff(rho,xi)+rho*diff(u,xi)+2*u*rho/xi+rho/xi
        *diff(w,theta)+w/xi*diff(rho,theta)+cot(theta)*rho*w/xi
       end proc:
> N2:= proc (rho,u,w) options operator, arrow;
        u*diff(u,xi)+w/xi*diff(u,theta)+rho^(-1)*diff(rho,xi)
       end proc:
> N3:= proc (rho,u,w) options operator, arrow;
        u*diff(w,xi)+w/xi*diff(w,theta)+rho^(-1)*diff(rho,theta)/xi
       end proc:
> Phi[1]:= -int(xi^(-2)*int(xi^2*(diff(Phi[0],theta,theta)/xi^2
           +cot(theta)*diff(Phi[0],theta)/xi^2-rho[0]),xi),xi):
> rho[1]:= -int(N1(rho[0],u[0],w[0]),t):
> u[1]:= -int((1-beta*(sin(theta))^2)*diff(Phi[0],xi)
         +N2(rho[0],u[0],w[0]),t):
```

```
> w[1]:= -int((1-beta*sin(2*theta))*diff(Phi[0],theta)/xi
         +N3(rho[0],u[0],w[0]),t):
> for n from 1 to 7 do
    N1:= proc (rho,u,w) options operator, arrow;
          u*diff(rho,xi)+rho*diff(u,xi)+2*u*rho/xi+rho/xi
          *diff(w,theta)+w/xi*diff(rho,theta)+cot(theta)*rho*w/xi
         end proc:
    N2:= proc (rho,u,w) options operator, arrow;
          u*diff(u,xi)+w/xi*diff(u,theta)+rho^(-1)*diff(rho,xi)
         end proc:
    N3:= proc (rho,u,w) options operator, arrow;
          u*diff(w,xi)+w/xi*diff(w,theta)+rho^(-1)*diff(rho,theta)/xi
         end proc:
    rhosum:=sum(rho[m]*lambda[n]^m,m=0..n):
    usum:=sum(u[m]*lambda[n]^m,m=0..n):
    wsum:=sum(w[m]*lambda[n]^m,m=0..n):
    A1[n]:= (diff(N1(rhosum,usum,wsum),$(lambda[n],n)))
            /factorial(n):
    A2[n]:= (diff(N2(rhosum,usum,wsum),$(lambda[n],n)))
            /factorial(n):
    A3[n]:= (diff(N3(rhosum,usum,wsum),$(lambda[n],n)))
            /factorial(n):
    A4[n]:= (diff(N4(rhosum,usum,wsum),$(lambda[n],n)))
            /factorial(n):
    lambda[n]:=0:
    Phi[n+1]:=-int(xi^(-2)*int(xi^2*(diff(Phi[n],theta,theta)
               /xi^2+cot(theta)*diff(Phi[n],theta)/xi^2-rho[n]),xi),xi):
    rho[n+1]:=-int(A1[n],t):
    u[n+1]:=-int((1-beta*(sin(theta))^2)*diff(Phi[n],xi)
            +A2[n],t):
    w[n+1]:=-int((1-beta*sin(2*theta))*diff(Phi[n],theta)/xi
            +A3[n],t):
  end do:
> rhototal:=collect(sum(rho[m],m=0..n),t):
> utotal:=collect(sum(u[m],m=0..n),t):
> wtotal:=collect(sum(w[m],m=0..n),t):
> phitotal:=collect(sum(Phi[m],m=0..n),t):
> mdot:=collect(4*Pi*xi^2*(rhototal)*(-utotal),t);
```

References

Adomian, G. 1994, Solving Frontier Problems of Physics: The Decomposition Method (Kluwer Academic Publishers)

Basu, S., & Ciolek, G. E. 2004, ApJ, 607, L39

Bodenheimer, P., & Sweigart, A. 1968, ApJ, 152, 515

Cassen, P., & Moosman, A. 1981, Icarus, 48, 353

Chandrasekhar, S. 1939, Introduction to the Study of Stellar Structure (Univ. of Chicago Press)

Ciolek, G. E., & Basu, S. 2006, ApJ, 652, 442

Curry, C. L. 2002, ApJ, 576, 849

Curtis, E. I., & Richer, J. S. 2010, MNRAS, 402, 603

di Francesco, J., Evans II, N. J., Caselli, P., et al. 2007, Protostars and Planets V, 17

Fatuzzo, M., Adams, F. C., & Myers, P. C. 2004, ApJ, 615, 813

Galli, D., & Shu, F. H. 1993, ApJ, 417, 243

Gammie, C. F., Lin, Y., Stone, J. M., & Ostriker, E. C. 2003, ApJ, 592, 203

Goodwin, S. P., Ward-Thompson, D., & Whitworth, A. P. 2002, MNRAS, 330, 769

Hartmann, L. 2002, ApJ, 578, 914

Hartmann, L. 2009, Accretion Processes in Star Formation (Cambridge Astrophys. Ser. 47) (Cambridge: Cambridge Univ. Press)
Hunter, C. 1977, ApJ, 218, 834
Jones, C. E., Basu, S., & Dubinski, J. 2001, ApJ, 551, 387
Jones, C. E., & Basu, S. 2002, ApJ, 569, 280
Larson, R. B. 1969, MNRAS, 145, 271
Li, P. S., Norman, M. L., Mac Low, M., & Heitsch, F. 2004, ApJ, 605, 800
Liao, S. 2003, Appl. Math. Comput., 142, 1
McKee, C. F., & Ostriker, E. C. 2007, ARA&A, 45, 565
Myers, P. C., Fuller, G. A., Goodman, A. A., & Benson, P. J. 1991, ApJ, 376, 561
Offner, S. S. R., & Krumholz, M. R. 2009, ApJ, 693, 914
Penston, M. V. 1969, MNRAS, 144, 425
Ramos, J. I. 2009, Chaos, Solitons & Fractals, 40, 1623
Shu, F. H. 1977, ApJ, 214, 488
Singh, O. P., Pandey, R. K., & Singh, V. K. 2009, Comput. Phys. Comm., 180, 1116
Tassis, K. 2007, MNRAS, 379, L50
Tassis, K. Dowell, C. D., Hildebrand, R. H., Kirby, L., & Vaillancourt, J. E. 2009, MNRAS, 399, 1681
Terebey, S., Shu, F. H., & Cassen, P. 1984, ApJ, 286, 529
Ward-Thompson, D., André, P., Crutcher, R., et al. 2007, Protostars and Planets V, 33
Wazwaz, A. M. 2001, Appl. Math. Comput., 118, 287