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On the radiation problem of high mass stars

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Abstract A massive star is defined as one with mass greater than $\sim 8-10 M_{\odot}$. Central to the on-going debate on how these objects [massive stars] come into being is the so-called Radiation Problem. For nearly forty years, it has been argued that the radiation field emanating from massive stars is high enough to cause a global reversal of direct radial in-fall of material onto the nascent star. We argue that only in the case of a non-spinning isolated star does the gravitational field of the nascent star overcome the radiation field. An isolated non-spinning star is a non-spinning star without any circumstellar material around it, and the gravitational field beyond its surface is described exactly by Newton's inverse square law. The supposed fact that massive stars have a gravitational field that is much stronger than their radiation field is drawn from the analysis of an isolated massive star. In this case the gravitational field is much stronger than the radiation field. This conclusion has been erroneously extended to the case of massive stars enshrouded in gas and dust. We find that, for the case of a nonspinning gravitating body where we take into consideration the circumstellar material, at ~ 8–10 \mathcal{M}_{\odot} , the radiation field will not reverse the radial in-fall of matter, but rather a stalemate between the radiation and gravitational field will be achieved, i.e. the infall is halted but not reversed. This picture is very different from the common picture that is projected and accepted in the popular literature where at $\sim 8-10 M_{\odot}$, all the circumstellar material, from the surface of the star right up to the edge of the molecular core, is expected to be swept away by the radiation field. We argue that massive stars should be able to start their normal stellar processes if the molecular core from which they form has some rotation, because a rotating core exhibits an Azimuthally Symmetric Gravitational Field which causes there to be an accretion disk and along this equatorial disk. The radiation field cannot be much stronger than the gravitational field, hence this equatorial accretion disk becomes the channel via which the nascent massive star accretes all of its material.

Key words: stars: circumstellar matter — stars: formation — radiative transfer

1 INTRODUCTION

According to current and prevailing wisdom, it is bona-fide scientific knowledge that our current understanding of massive star formation is lacking. This is due to the existing theoretical and observational dichotomy. In the gestation period of a star's life, its mass will grow via the in-falling envelope (i.e., circumstellar material) and also through the formation of an accretion disk lying along

the plane of its equator. As far as our theoretical understanding is concerned, this works well for stars less than about $8-10 \mathcal{M}_{\odot}$. In the literature, it is said that the problem of massive stars ($\mathcal{M}_{star} > 8-10 \mathcal{M}_{\odot}$) arises because as the central protostar's mass grows, so does the radiation pressure from it, and at about $8-10 \mathcal{M}_{\odot}$, the star's radiation pressure becomes powerful enough to halt any further infall of matter onto the protostar (Larson & Starfield 1971; Kahn 1974; Yorke & Kr ügel 1977; Wolfire & Cassinelli 1987; Palla & Stahler 1993; Yorke 2002; Yorke & Sonnhalter 2002). So the problem is: how does the star continue to accumulate more mass beyond the $8-10 \mathcal{M}_{\odot}$ limit? If the radiation field really did reverse any further in-fall of matter and protostars exclusively accumulated mass via direct radial in-fall of matter onto the nascent star and also via the accretion disk, this would set a mass upper limit of $8-10 \mathcal{M}_{\odot}$ for any star in the Universe. Unfortunately (or maybe fortunately) this is not what we observe. It therefore means that some process(es) responsible for the formation of stars beyond the $8-10 \mathcal{M}_{\odot}$ limit must be at work. A solution to the problem must be sought because observations dictate that it exists.

If this is the case, i.e. the radiation problem really did exist as stated above, and our physics was complete viz gravitation and radiation transport, then, the solution to the conundrum would be to seek a star formation model that overcomes the radiation pressure problem while at the same time allowing for the star to form (accumulate all of its mass) before it exhausts its nuclear fuel. Two such (competing) models have been set-forth:(1) the Accelerated Accretion Model (AAM) (Yorke 2002, 2004) and, (2) the Coalescence Model (CM) (Bonnell et al. 1998, 2001, 2004, 2006, 2007; Bonnell & Bate 2002).

The latter scenario, the CM, is born out of the observational fact that massive stars are generally found in the centers of dense clusters (see e.g. Hillenbrand 1997; Clarke et al. 2000). In these dense environments, the probability of collision of proto-stellar objects is significant, leading to the CM. This model easily by-passes the radiation pressure problem and, despite the fact that not a single observation to date has confirmed it (directly or indirectly), it [CM] appears¹ to be the most natural mechanism by which massive stars form given the said observational fact about massive stars and their preferential environment.

The AAM is just a scaled up version of the accepted accretion paradigm applicable to Low Mass Stars (LMSs). This accretion takes place via the accretion disk and, for the reason mentioned above that the accretion mechanism must be such that it allows for the star to form before it exhausts its nuclear fuel, the accretion cannot take place at the same steady rate as in the case of LMSs ($\mathcal{M} \leq 3 \mathcal{M}_{\odot}$) but must be accelerated and significantly higher. While there exist many examples of massive stars surrounded by accretion disks, one of the chief obstacles in verifying this paradigm is that examples of HMSs tend to be relatively distant (> 1 kpc), deeply embedded, and confused with other emission sources (see e.g. Matthews et al. 2007). Additionally, HMSs evolve rapidly, and by the time an unobstructed view of the young star emerges, the disk and outflow structures may have been destroyed. Consequently, observations to date have been unable to probe the 10–100 AU spatial scales over which outflows from the accretion disks are expected to be launched and collimated (e.g. Matthews et al. 2007).

The other alternative, which is less pursued, would be to seek a physical mechanism that overcomes the radiation pressure problem as has been conducted by the authors Krumholz et al. (2009). These authors (Krumholz et al. 2009) believe that the radiation problem does not exist, because radiation-driven bubbles that block accreting gas are subject to Rayleigh-Taylor instability which occurs anytime a dense, heavy fluid is being accelerated by lighter fluid, for example, when a cloud receives a shock, or when a fluid of a certain density floats above a fluid of lesser density, such as dense oil floating on water. The Rayleigh-Taylor instabilities allow fingers of dense gas to break into the evacuated bubbles and reach the stellar surface while, in addition, outflows from massive stars create optically thin cavities in the accreting envelope. These channel radiation away from the bulk

¹ This relies on the assumption that our understanding of gravitation and radiation transport is complete.

of the gas and reduce the radiation pressure it experiences. In this case, the radiation pressure feedback is not the dominant factor in setting the final size of massive stars and accretion will proceed, albeit at much higher rates. Amongst others, the model by the authors Krumholz et al. (2009) is ad hoc rather than natural, in that *Nature* has to make a special arrangement or must configure herself in such a way that massive stars have a way of starting their normal stellar processes. Does there not exist a smooth and natural way to bring massive stars into existence?

In this paper, we redefine the radiation problem (for the spherically symmetric case) and we do this via a subtle and overlooked assumption made in the analysis leading to the radiation problem: that the surroundings of the protostar are a vacuum (see e.g. Yorke 2002; Yorke & Sonnhalter 2002; Zinnecker & Yorke 2007); surely, this is clearly not true. The researchers Yorke 2002; Yorke & Sonnhalter 2002; Zinnecker & Yorke 2007; among others, hold the view that from a theoretical stand-point, the radiation field is stronger than the gravitational field for massive stars, hence the in-fall process of material must be reversed; but this conclusion has been reached, as will be shown in the next section after comparing the gravitational field strength at point r of a star in empty space to its radiation field strength at point r. In practice, stars are found embedded inside a significant mass of gas and dust. The radiation problem is arguably the most important problem of all in the study of the formation of stars, thus, it is important to make sure that this problem is clearly defined and understood.

Having taken into consideration the circumstellar material, we find that at $\sim 8-10 M_{\odot}$, the radiation field will not reverse the radial in-fall of matter but rather a stalemate between the radiation and gravitational fields will be achieved, where in-fall is halted but not reversed. Certainly, this picture is not at all congruent (or somewhere near there) to the common picture that is accepted in the popular literature where at $\sim 8-10 M_{\odot}$, all the circumstellar material, from the surface of the star right up to the edge of the molecular cloud core, is expected to be swept away by the powerful radiation field. This finding is not a complete but rather a partial solution to the radiation problem in that beyond the $8-10 M_{\odot}$ limit, the nascent star will not accrete any further. Under this model, its mass will stay at this value each time it accretes from the stagnant and frozen envelope once its mass drops below this $8-10 M_{\odot}$ limit. A very important point to note is that this is for a spherically symmetric gravitational setting where the gravitational field only has the radial dependence and is exactly described by Newton's inverse square law.

In a different paper, Nyambuya (2010a), an Azimuthally Symmetric Theory of Gravitation (ASTG) was set-up and thereby a thesis was set-forth to the effect that (1) for a non-spinning star, its gravitational field is spherically symmetric, so it is only depends on the radial distance from the central body; (2) for a spinning gravitating body, the gravitational field of the body in question is azimuthally symmetric, that is to say it is dependent on the radial distance from the central body and as-well the azimuthal angle. In a follow-up paper of Nyambuya (2010b), it has been shown that the ASTG predicts (1) that bipolar outflows may very well be a purely gravitational phenomenon and also; (2) that along the spin-equator of a spinning gravitating body, gravity will channel matter onto the spinning nascent star via the spin-equatorial disk without radiation having to reverse this inflow, thus allowing stars beyond the critical mass $8 - 10 M_{\odot}$ to come into existence.

If the ASTG proves itself, then the present paper together with Nyambuya (2010a,b) comprise (in our view) a solution to the radiation problem. Given that the solution to this problem has been sought via sophisticated computer simulations and lengthy numerical solutions, and additionally, given the simplicity and naïvety of the present approach which seeks to further our understanding of this problem; perhaps this paper presents not only my misunderstanding of the problem, but also of the approach to the problem. However, more on the optimistic side of things, I believe the radiation problem as discussed herein has been understood and that the approach is mathematically and physically legitimate, so much that we hold the objective view of seeking a solution to this problem, which makes this paper something worthwhile.

2 THE RADIATION PROBLEM

Following Yorke (2002), for direct radial accretion and accretion via the disk to occur onto the nascent star, it is required that the Newtonian gravitational force, $G\mathcal{M}_{\text{star}}(t)/r^2$, at a point distance r from the star of mass $\mathcal{M}_{\text{star}}$ and luminosity $\mathcal{L}_{\text{star}}(t)$ at any time t, must exceed the radiation force $\kappa_{\text{eff}}\mathcal{L}_{\text{star}}(t)/4\pi cr^2$, i.e.

$$\frac{G\mathcal{M}_{\rm star}(t)}{r^2} > \frac{\kappa_{\rm eff}\mathcal{L}_{\rm star}(t)}{4\pi cr^2},\tag{1}$$

where $c = 2.99792458 \times 10^8 \text{ ms}^{-1}$ is the speed of light in a vacuum, $G = 6.667 \times 10^{-11} \text{ kg}^{-1} \text{ m}^3 \text{ s}^{-2}$ is Newton's universal constant of gravitation, κ_{eff} is the effective opacity which is the measure of the gas' state of being opaque or a measure of the gas' imperviousness to light rays and is measured in m² kg⁻¹. This analysis by Yorke (2002), which is also reproduced in Zinnecker & Yorke (2007), is a standard and well accepted analysis that assumes spherical symmetry and, at the same time, it does not take into account the nascent star's circumstellar material. On the other hand, star formation is not a truly spherically symmetric phenomenon (see e.g. reviews by Zinnecker & Yorke 2007; McKee & Ostriker 2007) but this simple calculation suffices in so far as defining the curtain-region of 8–10 \mathcal{M}_{\odot} when radiation pressure is expected to become a significant player in the star's formation. What will be done in this paper is simple: to perform the same calculation albeit with the circumstellar material taken into account. In the penultimate part of this section, we shall make our case based on the above statements.

Now, this calculation by Yorke (2002) and Zinnecker & Yorke (2007), proceeds as follows: the inequality (1), sets a maximum condition for the accretion of material, namely $\kappa_{\text{eff}} < 4\pi c G \mathcal{M}_{\text{star}}(t) / \mathcal{L}_{\text{star}}(t)$, and evaluating this we obtain

$$\kappa_{\rm eff} < 1.30 \times 10^4 \left(\frac{\mathcal{M}_{\rm star}(t)}{\mathcal{M}_{\odot}}\right) \left(\frac{\mathcal{L}_{\rm star}(t)}{\mathcal{L}_{\odot}}\right)^{-1},$$
(2)

where $\mathcal{M}_{\text{star}}(t)$ and $\mathcal{L}_{\text{star}}(t)$ are in solar units. Given that $\mathcal{L}_{\text{star}}(t) = \mathcal{L}_{\odot} \left(\mathcal{M}_{\text{star}}(t) / \mathcal{M}_{\odot} \right)^3$, this implies that

$$\kappa_{\rm eff} < 1.30 \times 10^4 \left(\frac{\mathcal{M}_{\rm star}(t)}{\mathcal{M}_{\odot}}\right)^{-2} \Rightarrow \left(\frac{\mathcal{M}_{\rm star}}{\mathcal{M}_{\odot}}\right) > \left(\frac{1.30 \times 10^4}{\kappa_{\rm eff}}\right)^{1/2}.$$
 (3)

Now, given that the dusty ISM's averaged opacity is measured to be about $20.0 \text{ m}^2 \text{ kg}^{-1}$ (Yorke 2002) and using this (as an estimate to setting the minimum critical mass, see Yorke 2002; Zinnecker & Yorke 2007), we find that this sets a minimum upper mass limit for stars of about $10 M_{\odot}$ for gravitation to dominate the scene before radiation does. It is clear here that the opacity of the molecular cloud material is what sets the critical mass, thus a cloud of lower opacity will have a higher critical mass. It is expected that the opacity inside the cloud will be lower than in the ISM. In adopting the value $\kappa_{\text{eff}} = 20.0 \text{ m}^2 \text{ kg}^{-1}$ (see Yorke 2002; Zinnecker & Yorke 2007), this was done only to set a minimum lower bound for massive stars. Dust and gas opacities are significantly frequency-dependent and one has to take this into account for a more rigid constraint of a minimum mass for when the radiation field is expected to overcome the gravitational field.

As can be found in Yorke (2002), the AAM finds some of its ground around the alteration of the opacity. For example, if the opacity inside the gas cloud is significantly lower than the ISM value, then accretion can proceed via the AAM. To reduce the opacity inside the gas and dust cloud, the AAM posits as one of its options that optical and Ultra-Violet (UV) radiation inside the accreting material is shifted from the optical/UV into the far Infrared (IR) and also that the opacity may be lower than the ISM value because the opacity will be reduced by the accretion of optically thick material in the blobs of the accretion disk. This reduces the opacity or else we need to find a physical

mechanism that reduces the opacity to values lower than the ISM, which is a viable solution to the radiation problem. The above mechanism to reduce the opacity is rather ad hoc and dependent on the environment.

Now that we have presented the radiation problem as it is commonly understood, we are ready to make our case by inspecting (1). Clearly and without any doubt, the left hand side of this inequality is the gravitational field intensity for a gravitating body in empty space while the right hand side is the radiation field of this same star in empty space. From this, clearly, we are actually comparing the radiation and gravitational field intensity of a star in empty space, whereas in their real setting in *Nature*, stars are found which can be heavily enshrouded by gas and dust. Clearly, the conclusions that one finds from (1) such as that, at about $8-10 M_{\odot}$, the radiation field of the nascent star is powerful enough to not only halt but reverse the in-fall of material onto the nascent star; this cannot be extended to the scenario where a star is submerged in gas and dust, and it is erroneous to do so. Clearly, at this very simplistic, naive and fundamental level, there is a need to redefine the radiation problem by including in the left hand side of (1), the circumstellar material. Wolfire & Cassinelli (1987) among others, have performed this calculation where they have taken into account the circumstellar material and reached similar conclusions (as e.g. those of Yorke 2002). We reach a different conclusion to that of Wolfire & Cassinelli (1987) because, unlike those researchers, we use the observational fact that molecular clouds and molecular cores are found exhibiting a well behaved density profile $\rho \propto r^{-\alpha_{\rho}}$, and from this, we calculate a general mass distribution ($\mathcal{M} \propto r^{-\alpha}$). We use this to compare the gravitational and radiation field strengths at point r and from there draw our interesting conclusions.

3 RADIATION AND THE CIRCUMSTELLAR MATERIAL

Neglecting thermal and magnetic effects, turbulence and any other forces (as will be shown later in this section, these forces do not change the essence of our argument, hence we do not need to worry about them here) and considering only the gravitational and radiation field from the nascent star, we assume here that a star is formed from a gravitationally bound system of material enclosed in a volume space of radius $\mathcal{R}_{core}(t)$ and we shall call this system of material the core and further assume that this core shall have a constant total mass \mathcal{M}_{core} at all times. Now, as long as the material enclosed in the sphere of radius $r < \mathcal{R}_{core}(t)$ is such that

$$\frac{G\mathcal{M}(r,t)}{r^2} > \frac{\kappa_{\text{eff}}\mathcal{L}_{\text{star}}(t)}{4\pi cr^2},\tag{4}$$

then radiation pressure will not exceed the gravitational force in the region $r < \mathcal{R}_{core}(t)$, thus direct radial in-fall is expected to continue in that region. If $\mathcal{M}_{csl}(r,t)$ is the mass of the circumstellar material at time t enclosed in the region stretching from the surface of the star to the radius r, then, $\mathcal{M}(r,t) = \mathcal{M}_{csl}(r,t) + \mathcal{M}_{star}(t)$. Hence, the difference between Equations (4) and (1) is that in Equation (4) we have included the circumstellar material. This is not the whole story.

Now, Equation (4) can be written differently as

$$\mathcal{M}(r,t) > \frac{\kappa_{\text{eff}} \mathcal{L}_{\text{star}}(t)}{4\pi G c},\tag{5}$$

which basically says that as long as the amount of matter enclosed in the region of sphere radius r satisfies the above condition, the radiation force will not exceed the gravitational force in that region of radius r. In fact, Equation (5) is the Eddington limit applied to the region of radius r. This is identical to equation (10) in Wolfire & Cassinelli (1987). In their work, Wolfire & Cassinelli (1987) solve numerically the radiative transfer problem to determine the effective opacity at the outer edge of the massive star forming core and, from this, they determine the limits of grain-sizes that are needed for the formation of massive stars. Wolfire & Cassinelli (1987)'s approach is a typical approach used to probe the conditions necessary for massive stars to form.

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Our approach is very different from that of Wolfire & Cassinelli (1987) and most typical approaches used to study the radiation problem where sophisticated computer simulations and numerical solutions are used. Ours is a simple and naïve approach needing no computer simulations nor numerical codes. We shall insert $\mathcal{M}(r,t) = \mathcal{M}_{csl}(r,t) + \mathcal{M}_{star}(t)$ into Equation (4) and after rearranging, one obtains

$$\mathcal{M}_{\rm csl}(r,t) > \left[\frac{\kappa_{\rm eff}\mathcal{L}_{\rm star}(t)}{4\pi c G \mathcal{M}_{\rm star}(t)} - 1\right] \mathcal{M}_{\rm star}(t) = \left[\left(\frac{\mathcal{M}_{\rm star}(t)}{10 \,\mathcal{M}_{\odot}}\right)^2 - 1\right] \mathcal{M}_{\rm star}(t). \tag{6}$$

Our main thrust is to seek values of r in the above inequality that satisfy it. We shall do this by finding a form for $\mathcal{M}_{csl}(r, t)$.

Before doing this, let us apply Equation (5) to the entire core, that is $r = \mathcal{R}_{\text{core}}$. This must give us the condition when the star's radiation field is strong enough to sweep away all the circumstellar material from the surface of the star right up to the outer edge of the core. In doing so, one finds that the star's luminosity should be such that

$$\mathcal{M}_{\rm core} > \frac{\kappa_{\rm eff} \mathcal{L}_{\rm star}(t)}{4\pi G c}.$$
 (7)

In making this calculation, we have made the tacit and fundamental assumption that the star's mass will continue to increase until the star reaches a critical luminosity determined by the mass of the core. Let us denote this critical luminosity by \mathcal{L}_{core}^* . From the above, it follows that

$$\mathcal{L}_{\rm core}^* = \frac{4\pi c G \mathcal{M}_{\rm core}}{\kappa_{\rm eff}}.$$
(8)

With this definition, then for the radiation field to globally overcome the gravitational field, the nascent star's luminosity must exceed the critical luminosity of the core, i.e.

$$\mathcal{L}_{\text{star}}(t) > \mathcal{L}_{\text{core}}^*.$$
(9)

Now, knowing the mass-luminosity relationship of stars is given by $\mathcal{L}_{star}(t) = \mathcal{L}_{\odot} (\mathcal{M}(t)/\mathcal{M}_{\odot})^3$, then the critical condition $\mathcal{L}_{star}(t) = \mathcal{L}_{core}^*$ will occur when

$$\left(\frac{\mathcal{M}_{\text{star}}}{\mathcal{M}_{\odot}}\right) = \left(\frac{\kappa_{\text{eff}}\mathcal{L}_{\odot}}{4\pi G \mathcal{M}_{\odot} c}\right)^{-1/3} \left(\frac{\mathcal{M}_{\text{core}}}{\mathcal{M}_{\odot}}\right)^{1/3}.$$
(10)

Given this and taking $\kappa_{\text{eff}} = 20.0 \text{ m}^2 \text{ kg}^{-1}$ and then plugging this and the other relevant values, such as G, c, etc., into the above, we are lead to

$$\left(\frac{\mathcal{M}_{\max}}{\mathcal{M}_{\odot}}\right) = \left(\frac{\mathcal{M}_{\text{core}}}{10\,\mathcal{M}_{\odot}}\right)^{1/3}.$$
(11)

where we have set $\mathcal{M}_{star} = \mathcal{M}_{max}$. As we already said, using $\kappa_{eff} = 20.0 \text{ m}^2 \text{ kg}^{-1}$ gives us the minimum lower bound. What this means is that the mass of the core from which a star is formed may very well be crucial in deciding the final mass of the star because the mass of the core determines the time when global in-fall reversal will occur.

From this simplistic and rather naïve calculation, we can estimate the efficiency of the core

$$\xi_{\rm core} = \left(\frac{\mathcal{M}_{\rm star}}{\mathcal{M}_{\rm core}}\right) = 0.10 \left(\frac{\mathcal{M}_{\rm core}}{10 \,\mathcal{M}_{\odot}}\right)^{-2/3},\tag{12}$$

thus a $100 \mathcal{M}_{\odot}$ core will (according to the above) form a star at an efficiency rate of about 2% and it will produce a star of mass $2 \mathcal{M}_{\odot}$. A $10 \mathcal{M}_{\odot}$ star will be produced by a core of mass $10^4 \mathcal{M}_{\odot}$ at

an efficiency rate of about 0.1%. A $10^4 M_{\odot}$ core is basically a fully-fledged molecular cloud. The production of this $10M_{\odot}$ star is based on the assumption that the rest of the material $(10^4 M_{\odot} - 10 M_{\odot} = 9.99 \times 10^3 M_{\odot})$ will not form stars. In reality, some of the material in this $10^4 M_{\odot}$ core will form many other stars. Furthermore, a $100 M_{\odot}$ star will form in a GMC of mass about $10^7 M_{\odot}$. The above deductions, that high mass stars will need to form in clouds of mass $\geq 10^4 M_{\odot}$, resonate with the observational fact that massive stars are not found in isolation (e.g. Hillenbrand 1997; Clarke et al. 2000) since the other material will form stars.

Relationship (11) is interesting because of its similarity to Larson's (1982) empirical discovery. With a handful of data, Larson (1982) was the first to note that the maximum stellar mass of a given population of stars is related to the total mass of the parent cloud from which the stellar population has been born. That is to say, if \mathcal{M}_{cl} is the mass of a molecular cloud and \mathcal{M}_{max} is the maximum stellar mass of the population, then

$$\mathcal{M}_{\rm max} = \left(\frac{\mathcal{M}_{\rm cl}}{\mathcal{M}_0}\right)^{\alpha_{\rm L}},\tag{13}$$

where $\mathcal{M}_0 = 13.2 \mathcal{M}_{\odot}$ and $\alpha_L = 0.430$. This law was obtained from a sample of molecular clouds whose masses are in the range $1.30 \leq \log_{10} (\mathcal{M}/\mathcal{M}_{\odot}) \leq 5.50$. Larson's Law is thought to be a result of statistical sampling but we are not persuaded to think that this is the case; such a coincidence is, in our opinion and understanding, too good to be true. We believe Larson's Law is *Nature*'s subtle message to researchers; it is telling us something about the underlying dynamics of star formation. This said, could the relationship (11) be related to Larson's result? The indices of Larson's relation and relationship (11) have a deviation of about 33% and the constant \mathcal{M}_0 has a similar deviation of about 33%. Could Larson's fitting procedure be "tuned" to conform to relationship (11), and if so, does that mean that Larson's relationship finds an explanation from this behavior?

Perhaps the deviation of our relation from that of Larson may well be that our result is derived from an ideal situation where we have not considered other forces, such as the magnetic and thermal forces, etc., but also we have considered star formation as a spherically symmetric process, which it is not, and this may also be a source of correction to this result in order to bring it into agreement with Larson's result. Let us represent all these other forces by F_{other} (e.g. magnetic, turbulence, viscosity, etc). Clearly these forces will not aid gravity in its endeavor to squeeze all the material to a single point but rather aid the radiation pressure in opposing this. Given this, we must write Inequality (4) as

$$\frac{G\mathcal{M}(r,t)}{r^2} > \frac{\kappa_{\rm eff}\mathcal{L}_{\rm star}(t)}{4\pi cr^2} + \frac{|\boldsymbol{F}_{\rm other}|}{m},\tag{14}$$

where m is the average mass of the molecular species of the material constituting the cloud. The above can be written in the form

$$\mathcal{L}_{\text{star}}(t) < \frac{4\pi c G \left(\mathcal{M}(r,t) - r^2 |\boldsymbol{F}_{\text{other}}| / m \right)}{\kappa_{\text{eff}}},\tag{15}$$

and writing $\mathcal{M}'(r,t) = r^2 |\boldsymbol{F}_{other}|/m$, we will have

$$\mathcal{L}_{\text{star}}(t) < \frac{4\pi c G \left[\mathcal{M}(r,t) - \mathcal{M}'(r,t)\right]}{\kappa_{\text{eff}}}.$$
(16)

From this, it is clear that the other forces will act in a manner so as to reduce the critical luminosity of the core. Thus our result (11), when compared to natural reality where these other forces are present, is expected to show that a deviation from the real observations must occur. As stated in the opening of this section, the inclusion of the magnetic and thermal forces, etc., will not change the essence of our argument, hence the above argument justifies why we did not have to worry about these other

forces because the essence of our result still stands. The situation is only critical when these other forces become significant in comparison to the gravitational force.

In the succeeding section, we compute the mass distribution function and then show that one arrives at the same result as Equation (5). Additionally and more importantly, we are able to compute the boundaries where the radiation field will be strong enough to overcome the gravitational field. Among other interesting outcomes, we shall see that the radiation field will create a cavity inside the star forming core and that this cavity grows with time in proportion to the radiation field of the nascent star.

4 MASS DISTRIBUTION FUNCTION

First, we compute the enclosed mass $\mathcal{M}(r, t)$. We know that stellar systems such as molecular clouds and cores are found to exhibit radial density profiles given by

$$\rho(r,t) = \rho_0(t) \left(\frac{r_0(t)}{r}\right)^{\alpha_{\rho}} \tag{17}$$

where $\rho_0(t)$ and $r_0(t)$ are time dependent normalization constants and α_ρ is the density index. In order to make sense of this density profile (17), we have to calculate these normalization constants. In its bare form, the power law (17) as it stands implies an infinite density at r = 0. In general, power laws have this property. Obviously, one has to deal with this. The usual or typical way is to impose a minimum value for r, say $r = r_{\min} = r_0(t)$, and assign a density there. Here, this minimum radius has been made time dependent for the sole reason that if the cloud is undergoing free fall as in the case in star formation regions, this quantity will respond dynamically to this, so it will be time dependent.

Now, for a radially dependent density profile, the mass distribution is calculated from the integral

$$\mathcal{M}(r,t) = \int_{r_{\min}}^{r} 4\pi r^2 \rho(r,t) dr.$$
 (18)

Inserting the density function (17) into the above integral and then evaluating the resultant integral, we are led to

$$\mathcal{M}(r,t) = \left(\frac{4\pi\rho_0(t)r_{\min}^{\alpha_{\rho}}(t)}{3-\alpha_{\rho}}\right) \left(r^{3-\alpha_{\rho}} - r_{\min}^{3-\alpha_{\rho}}(t)\right),\tag{19}$$

and this formula does not apply to the case $\alpha_{\rho} = 3$ and it is valid for $0 \le \alpha_{\rho} < 3$. The case $\alpha_{\rho} = 3$ is described by a special MDF which is $\mathcal{M}(r,t) = \left[4\pi\rho_0(t)r_{\min}^3(t)\right]\ln(r/r_{\min}(t))$. We shall not consider this case as it will not change the essence of our argument.

Now we have to normalize the MDF by imposing some boundary conditions. The usual or traditional boundary condition is to set $\mathcal{M}(r_{\min}, t) = 0$ and this in fact means that there will be a cavity of radius $r_{\min}(t)$ in the cloud. What we shall do next is different from this traditional normalization. We shall set $\mathcal{M}(r_{\min}, t) = \mathcal{M}_{\text{star}}$ where $\mathcal{M}_{\text{star}}$ is the mass of the central star, hence $r_{\min}(t) = \mathcal{R}_{\text{star}}(t)$. Thus what we have done is to place the nascent star in the cavity, which means we must write our MDF as

$$\mathcal{M}(r,t) = \left(\frac{4\pi\rho_0(t)\mathcal{R}_{\mathrm{star}}^{\alpha_{\rho}}(t)}{3-\alpha_{\rho}}\right) \left(r^{3-\alpha_{\rho}} - \mathcal{R}_{\mathrm{star}}^{3-\alpha_{\rho}}(t)\right) + \mathcal{M}_{\mathrm{star}}(t),\tag{20}$$

and this applies for $\mathcal{R}_{\text{star}}(t) \leq r \leq \mathcal{R}_{\text{core}}(t)$.

Now, if the mass enclosed inside the core remains constant throughout, then we must have, at $r = \mathcal{R}_{core}(t)$, the boundary condition $\mathcal{M}(\mathcal{R}_{core}, t) = \mathcal{M}_{core}$. We know that the sum total of all

the circumstellar material at any time is given by $\mathcal{M}_{csl}(t) = \mathcal{M}_{core} - \mathcal{M}_{star}(t)$. Combining all the information, we will have

$$\frac{4\pi\rho_0(t)r_0^{\alpha_\rho}(t)}{3-\alpha_\rho} = \frac{\mathcal{M}_{\rm csl}(t)}{\mathcal{R}_{\rm core}^{3-\alpha_\rho}(t) - \mathcal{R}_{\rm star}^{3-\alpha_\rho}(t)},\tag{21}$$

and this means the MDF can now be written as

Circumstellar Material in Region Radius r

$$\mathcal{M}(r,t) = \mathcal{M}_{csl}(t) \left(\frac{r^{3-\alpha_{\rho}} - \mathcal{R}_{star}^{3-\alpha_{\rho}}(t)}{\mathcal{R}_{core}^{3-\alpha_{\rho}}(t) - \mathcal{R}_{star}^{3-\alpha_{\rho}}(t)} \right) + \frac{\mathcal{M}_{ass of the nascent star}}{\mathcal{M}_{star}(t)} \quad \text{for} \quad r \ge \mathcal{R}_{star}(t).$$
(22)

We shall take this as the final form of our mass distribution function. If the reader accepts this, then what follows is a straight forward exercise and leads to what we believe is a significant step forward in the resolution of the radiation problem. The reader may want to query that we have overstretched our boundary limits by making the assumption that the MDF be continuous from the surface of the star right up to the edge of the core. In that event, we need to make this point clear and reach an accord.

First, let us consider a serene molecular core long before a star begins to form at the center. We know that the density is not a fundamental physical quantity but a physical quantity derived from two fundamental physical quantities which are mass and volume, i.e., density=mass/volume. We must note that this is defined for (volume> 0). We shall assume that this core exhibits the density profile $\rho \propto r^{-\alpha_{\rho}}$. This fact that $\rho \propto r^{-\alpha_{\rho}}$, when combined with the fact that density is not a fundamental physical quantity but a quantity derived from two fundamental quantities, suggests that at any given time the mass must be distributed in proportion to the radius, i.e., $\mathcal{M}(r,t) \propto r^{\alpha}$. The radial dependence of the density is an indicator that that mass has a radial dependence. The relationship $\mathcal{M}(r,t) \propto r^{\alpha}$ means we must have $\mathcal{M}(r,t) = ar^{\alpha} + b$ where (a, b) are constants. We expect that $\mathcal{M}(0,t) = 0$. If this is to hold (as it must), then (b = 0) and $(\alpha \ge 0)$. We also expect the condition $\mathcal{M}(\mathcal{R}_{core},t) = \mathcal{M}_{core}$ to hold. If this is to hold (as it must), then we will have $a = \mathcal{M}_{core}/\mathcal{R}_{core}^{\alpha}(t)$, hence $\mathcal{M}(r,t) = \mathcal{M}_{core}(r/\mathcal{R}_{core}(t))^{\alpha}$. From the definition of density this means

$$\rho(r,t) = \left(\frac{3\mathcal{M}_{\text{core}}}{4\pi\mathcal{R}_{\text{core}}^{\alpha}(t)}\right) r^{\alpha-3} \quad \text{for} \quad r > 0.$$
(23)

Now, if the density profile is to fall off as r increases, as is the case in *Nature*, then $(\alpha - 3 \le 0)$ which implies $(\alpha \le 3)$. Combining this with $(\alpha \ge 0)$ we will have $(0 \le \alpha \le 3)$. Comparing this with the profile $(\rho \propto r^{-\alpha_{\rho}})$, we have: $(-\alpha_{\rho} = \alpha - 3)$ and substituting this into $(0 \le \alpha \le 3)$, one obtains $(0 \le 3 - \alpha_{\rho} \le 3)$. From $(3 - \alpha_{\rho} \le 3)$, we have $(\alpha_{\rho} \ge 0)$, and from $(0 \le 3 - \alpha_{\rho})$, we have $(\alpha_{\rho} \le 3)$, hence $(0 \le \alpha_{\rho} \le 3)$.

Now, in this serene molecular cloud, a small lamp begins to form; let this lamp have a radius $\mathcal{R}_{\text{lamp}}(t)$ and mass $\mathcal{M}_{\text{lamp}}$. I shall pose a question: do we expect this lamp to cause any fundamental changes to the mass distribution $\mathcal{M}(r,t) = ar^{\alpha} + b$? I think not. If this is the case, then our mass distribution must now be defined up to the radius of the lamp, $\mathcal{M}(\mathcal{R}_{\text{lamp}},t) = \mathcal{M}_{\text{lamp}}$ and this condition leads to: $b = \mathcal{M}_{\text{lamp}} - a\mathcal{R}_{\text{lamp}}^{\alpha}$, thus

$$\mathcal{M}(r,t) = a(r^{3-\alpha_{\rho}} - \mathcal{R}_{\text{lamp}}^{3-\alpha_{\rho}}) + \mathcal{M}_{\text{lamp}} \quad \text{for} \quad r \ge \mathcal{R}_{\text{lamp}}(t),$$
(24)

where we have substituted $\alpha = 3 - \alpha_{\rho}$. Now inserting the condition that $\mathcal{M}(\mathcal{R}_{core}, t) = \mathcal{M}_{core}$, we will have

$$a = \left(\frac{\mathcal{M}(r,t) - \mathcal{M}_{\text{lamp}}}{\mathcal{R}_{\text{core}}^{3-\alpha_{\rho}}(t) - \mathcal{R}_{\text{lamp}}^{3-\alpha_{\rho}}}\right) \quad \text{for} \quad r \ge \mathcal{R}_{\text{lamp}}(t),$$
(25)

and putting all this together we will have

$$\mathcal{M}(r,t) = \mathcal{M}_{\rm csl}(t) \left(\frac{r^{3-\alpha_{\rho}} - \mathcal{R}_{\rm lamp}^{3-\alpha_{\rho}}}{\mathcal{R}_{\rm core}^{3-\alpha_{\rho}}(t) - \mathcal{R}_{\rm lamp}^{3-\alpha_{\rho}}} \right) + \mathcal{M}_{\rm lamp} \quad \text{for} \quad r \ge \mathcal{R}_{\rm lamp}(t), \tag{26}$$

where $\mathcal{M}_{csl}(t) = \mathcal{M}_{core} - \mathcal{M}_{lamp}(t)$. Comparison of the above with Equation (22) shows that the lamp in the above formula is the star in Equation (22).

We are certain that the reader will have no problem with Equation (26) because the lamp does not disrupt the mass distribution since it has no radiation. Hence, we would expect a continuous distribution of mass right up to the surface of the lamp as material will be flowing into the lamp. However, this same lamp is a protostar and at some point it must switch on to become a star. At this moment, assuming the correctness of the thesis that at $8-10 M_{\odot}$, the radiation field begins to push material away from the nascent star, we could from logic expect that the mass distribution must be continuous up till that time when disruption starts at $8-10 M_{\odot}$. During the time when the lamp's (or protostar's) mass is in the range $0 \le M_{lamp}(t) < 8-10 M_{\odot}$, the MDF Equation (26) must hold. From this, we have just justified the formula (22) for the mass range: $0 \le M_{star}(t) < 8-10 M_{\odot}$. When the radiation field begins to be significant, we shall have to check and revise this formula.

Now, from the MDF Equation (22), the gravitational field intensity, at any given time t and at any given point r inside the core from the surface of the star, will be given by

$$\boldsymbol{g}(r,t) = \overbrace{-\left(\frac{G\mathcal{M}_{csl}(t)}{r^2}\right) \left(\frac{r^{3-\alpha_{\rho}} - \mathcal{R}_{star}^{3-\alpha_{\rho}}(t)}{\mathcal{R}_{core}^{3-\alpha_{\rho}}(t) - \mathcal{R}_{star}^{3-\alpha_{\rho}}(t)}\right) \hat{\boldsymbol{r}} - \overbrace{\left(\frac{G\mathcal{M}_{star}(t)}{r^2}\right) \hat{\boldsymbol{r}}}^{\text{Star's Gravitation}}.$$
 (27)

Clearly, we have been able to separate the gravitation due to the star from that due to the circumstellar material.

Now, from the above, the inequality (4) becomes

$$\left(\frac{G\mathcal{M}_{\rm csl}(t)}{r^2}\right) \left(\frac{r^{3-\alpha_{\rho}} - \mathcal{R}_{\rm star}^{3-\alpha_{\rho}}(t)}{\mathcal{R}_{\rm core}^{3-\alpha_{\rho}}(t) - \mathcal{R}_{\rm star}^{3-\alpha_{\rho}}(t)}\right) + \left(\frac{G\mathcal{M}_{\rm star}(t)}{r^2}\right) > \frac{\kappa_{\rm eff}\mathcal{L}_{\rm star}(t)}{4\pi r^2 c},\tag{28}$$

where the first term on the left hand-side of Equation (28) is clearly the gravitational field intensity of the circumstellar material and the second term is the gravitational field of the nascent star.

5 RADIATION CAVITY

The inequality (5) gives us a condition that must be met before the radiation field is powerful enough that it can push away (all) the circumstellar material inside the shell of radius r. Beyond this radius, the radiation field is not at all powerful enough to overcome the gravitational field. Unfortunately, one cannot deduce this radius r from Equation (5). The inequality (28), as does (5) and (28), tells us the conditions to be met before the radiation field is powerful enough to halt in-fall. In addition to this, Equation (28) yields more information than Equation (5) because in Equation (28) we have quantified the MDF for the circumstellar material and this allows us to compute the region r where the radiation field is much stronger than the gravitational field. From Equation (28), we deduce that the radiation field will create a cavity in the star forming core; in this cavity, the radiation field is much stronger than the gravitational field, thus there will be a radiation cavity with no material but only radiation, hence the term. To see that Equation (28) describes a cavity, we simply have to write Equation (28) with r as the subject of the formula; after doing so, one arrives at

$$r > \left[\frac{\left(\kappa_{\rm eff}\mathcal{L}_{\rm star}(t) - 4\pi c G \mathcal{M}_{\rm star}(t)\right) \left(\mathcal{R}_{\rm core}^{3-\alpha_{\rho}}(t) - \mathcal{R}_{\rm star}^{3-\alpha_{\rho}}(t)\right)}{4\pi c G \mathcal{M}_{\rm csl}(t)} + \mathcal{R}_{\rm star}^{3-\alpha_{\rho}}(t)\right]^{\frac{1}{3-\alpha_{\rho}}} = \mathcal{R}_{\rm cav}(t),$$
(29)

where $\mathcal{R}_{cav}(t)$ is the radius of the cavity. Now that there is a cavity, let us pause so that we can revise the MDF. Clearly, in the case where there are outflows, this must be given by

$$\mathcal{M}(r,t) = \mathcal{M}_{\rm csl}(t) \left[\frac{r^{3-\alpha_{\rho}} - \mathcal{R}_{\rm cav}^{3-\alpha_{\rho}}}{\mathcal{R}_{\rm core}^{3-\alpha_{\rho}}(t) - \mathcal{R}_{\rm cav}^{3-\alpha_{\rho}}} \right] + \mathcal{M}_{*}(t) \quad \text{for} \quad r \ge \mathcal{R}_{\rm cav}(t), \tag{30}$$

where $\mathcal{M}_{csl}(t) = \mathcal{M}_{core} - \mathcal{M}_{*}(t)$ and $\mathcal{M}_{*}(t) = \mathcal{M}_{star}(t) + \mathcal{M}_{disk}(t) + \mathcal{M}_{outf}(t)$; $\mathcal{M}_{disk}(t)$ is the disk mass inside the cavity at time t and \mathcal{M}_{outf} the bipolar outflow contained in the cavity at time t.

Now, what this inequality (29) is "saying" is that, at any given moment when the star has surpassed the critical mass (8–10 M_{\odot}), there will exist a region $r < \mathcal{R}_{cav}(t)$ where the radiation field will reverse the radially in-falling material and in the region $r > \mathcal{R}_{cav}(t)$, for material therein, the radiation field has not reached a state where it exceeds the gravitational field. Hence, in-fall reversal in that region has not been achieved. This region [i.e. $r < \mathcal{R}_{cav}(t)$] grows with time, thus the radiation field slowly and gradually pushes the material further and further away from the nascent star until $\mathcal{R}_{cav}(t) = \mathcal{R}_{cl}$ where radial in-fall is completely halted. This will occur when the star has reached the critical core luminosity \mathcal{L}^*_{core} . The condition when the critical core luminosity has been shown earlier leads to (12) which is a Larson-like relation, i.e. Equation (13). *Ipso facto*, this strongly suggests that Larson's Law may not be a result of statistical sampling but a statement about and a fossil record of the battle of forces between gravitation and the radiation field.

By saying that the nascent massive star will create a cavity, we have made a tacit and fundamental assumption that its mass will continue to grow soon after the cavity begins to form and that its mass will, thereafter, continue to grow while in the cavity. However, how can this be since the cavity separates the nascent star from the circumstellar matter? The nascent star now does not have a channel to feed its mass, so there can be no growth in its mass unless there exists a channel via which its mass feeds. At this juncture, we direct the reader to the papers (Nyambuya 2010a,b).

In Nyambuya (2010a), as already said in the introductory section, we set-up the ASTG such that the thesis was advanced to the effect (1) that, for a non-spinning star, its gravitational field is spherically symmetric (to be specific, it only depends on the radial distance from the central body); (2) that, for a spinning gravitating body, the gravitational field of the body in question is azimuthally symmetric, i.e., it depends on the radial distance (r) from the central body and the azimuthal angle (θ). In a follow-up paper, (Nyambuya 2010b), we showed that the ASTG predicts (1) that bipolar outflows may very well be a purely gravitational phenomenon (i.e., a repulsive gravitational phenomenon) and also; (2) that along the spin-equator (defined in Nyambuya 2010b) of a spinning gravitating body, gravity will channel matter onto the spinning nascent star via the accretion disk (lying along the spin-equator), thus allowing stars beyond the critical mass 8–10 \mathcal{M}_{\odot} to form and begin their stellar processes. It should be said that accretion disks can also be formed by a number of different mechanisms other than an Azimuthally Symmetric Gravitational Field (ASGF).

The accretion of matter beyond the $8-10 \mathcal{M}_{\odot}$ limit must only be possible for a spinning star because it possesses the ASGF that is needed to continue the channeling of matter onto the star via the accretion disk – see the illustration in Figure 1. For a non-spinning core, the nascent star's accretion cannot proceed beyond $8-10 \mathcal{M}_{\odot}$. It is halted because the moment the radiation field tries to create a cavity, when the (non-spinning) star's mass is $8-10 \mathcal{M}_{\odot}$, the (non-spinning) star that very moment becomes separated from the surrounding circumstellar material. This means the (non-spinning) star's mass feeds. Should the (non-spinning) star's mass fall below $8-10 \mathcal{M}_{\odot}$, the circumstellar material will fall onto the nascent (non-spinning) star until its mass is restored to its previous value of $8-10 \mathcal{M}_{\odot}$. This means the star's mass for a non-spinning star stays at $8-10 \mathcal{M}_{\odot}$. As explained in the above paragraphs, this scenario is different for a spinning star because the ASGF (which comes about due to the spin of the nascent star) allows matter to continue accreting via the equatorial disk. The accretion disk will exist inside the radiation cavity and this



Fig. 1 An illustration of the cavity inside the core, and an accretion disk. For a non-spinning core at ~ 8–10 \mathcal{M}_{\odot} , the nascent stars's accretion is halted (and importantly, in-fall is not reversed but only halted) because when the radiation field tries to create a cavity, in which process, the star is separated from its accretion source which is the circumstellar material. This means that the star's mass accretion is halted because its mass can no longer grow since there exists no other channel(s) via which its mass feeds. Should the star's mass fall below ~ 8–10 \mathcal{M}_{\odot} , the circumstellar material will fall onto the nascent star until its mass is restored to its previous value of ~ 8–10 \mathcal{M}_{\odot} . In order for the radiation field to start pushing the circumstellar material, its mass must exceed ~ 8–10 \mathcal{M}_{\odot} . Since there is no way to do this, in-fall is only halted and not reversed. Hence, the star's mass for a non-spinning star stays at ~ 8–10 \mathcal{M}_{\odot} . As noted in Nyambuya (2010b), this scenario is different for a spinning star because the ASGF (which comes about due to the spin of the nascent star) allows matter to continue accreting via the equatorial disk inside the cavity as illustrated above. The accretion disk will exist inside the radiation cavity and, according to the azimuthally symmetric theory of gravitation (Nyambuya 2010b), this disk should channel mass onto the nascent star right-up to the surface of the star without radiation hindrances.

disk should, according to the ASTG (Nyambuya 2010b), channel mass right up to the surface of the star without radiation hindrances. The scenario just presented is completely different from that projected in much of the wider literature where, at $8-10 M_{\odot}$, suddenly the radiation is so powerful that it reverses any further in-fall. It is bona-fide knowledge that star formation is not a spherically symmetric process and, from the above, it follows that stars beyond the $8-10 M_{\odot}$ limit must, with no hindrance, form the radiation field and the only limit to their existence is if the gravitationally bound core has enough mass to form them.

6 DISCUSSION AND CONCLUSIONS

This contribution coupled with Nyambuya (2010b) seems to strongly point to the possibility that the radiation problem of massive stars may not exist as previously thought. In the present paper, we find that beginning at the time when $\mathcal{M}_{star}(t) \simeq 8-10 \mathcal{M}_{\odot}$, the radiation field will create a cavity inside the star forming core and the circumstellar material inside the region $\mathcal{R}_{cav}(t) < r \leq \mathcal{R}_{core}(t)$ is going to be pushed gradually (in particular, not blown away) as the radiation field from the star grows until a point is reached when the cavity is the size of the core itself. At this point, a complete

in-fall reversal is attained. If the radiation field of the star is to grow, its mass must grow, thus, the cavity must not prevent accretion of mass onto the nascent star and this is possible for a spinning massive star. Once the cavity is created, the mass of the nascent will, for a spinning massive star, feed via the accretion disk and this disk is not affected by the radiation field. By saying the disk is not affected by the radiation field as it pushes the other material on the disk is not going to be pushed away by the radiation field as it pushes the other material away because the azimuthally symmetric gravitational field of the star is powerful enough along this plane to overcome the radiation field. It has been shown or argued in Nyambuya (2010b) that this must be the case.

The ASGF is only possible for a spinning star; since all known stars are spinning, every star should, according to the ASTG, have the potential to grow to higher masses. This means that massive stars should start their stellar processes because of their spin, which brings about the much needed ASGF. A non-spinning star will have no ASGF, hence there will be no disk around it to channel material once the radiation field begins to take its toll. In this case of a non-spinning star, once the star has reached the critical mass $\sim 8-10 M_{\odot}$, its mass cannot grow any further because at the moment it tries to grow, the star and the circumstellar material become separated due to the radiation field which, in this case, is stronger than the gravitational field. In this event, any further growth in the mass of a non-spinning star will stay constant at $\sim 8-10 M_{\odot}$ because, the moment it falls slightly below $\sim 8-10 M_{\odot}$, gravity becomes more powerful, thus accreting only enough mass to restore it to its previous value of $\sim 8-10 M_{\odot}$. In this case, we have an "eternal" stalemate between the gravitational and radiation fields.

An important and subtle difference between the present work and that of other researchers (Larson & Starfield 1971; Kahn 1974; Yorke & Krügel 1977; Wolfire & Cassinelli 1987; Palla & Stahler 1993; Yorke 2002; Yorke & Sonnhalter 2002) is that we have seized on the observational fact that molecular clouds and cores are found to exhibit well defined density profiles. From this, we computed the MDF which enabled us to exactly find the physical boundaries where the gravitational field is expected to be much stronger than the radiation field once the star exceeds the critical mass. Additionally and more importantly is that from Nyambuya (2010b), we have been able to argue that, even after the cavity has been created, mass will be channeled on to the star via the accretion disk. Without the ideas presented in Nyambuya (2010b), we would have been stuck because we were going to find ourselves without a means to justify how the mass accretion continues once the cavity has been created.

Importantly, we have pointed out a real problem in Yorke (2002), Yorke & Sonnhalter (2002) and Zinnecker & Yorke (2007), namely that these researchers have neglected the treatment of the circumstellar material in their theoretical arguments leading to their definition of the radiation problem because they used Newton's inverse square law which clearly only applies to a non-rotating mass in empty space, so the inequality (4) applies only for a star in empty space. In empty space, it is correct to say that the radiation field for a star of mass 10 M_{\odot} and beyond will exceed the gravitational field everywhere in space beyond the nascent star's surface, but the same is not true for a star submerged in a pool of gas, which is the case for the stars that we observe.

Another important outcome is that it appears Larson's Laws may well be a signature and fossil record of the battle of forces between the radiation and gravitational fields. At present, it is thought of as being a result of statistical sampling. Thus, the present results cause us to start rethinking this view. We are not persuaded to think that this is a result of statistical sampling. This view finds support from Weidner et al. (2009)'s most recent and exciting work. In this work, these researchers present a thorough literature study of the most-massive stars in several young star clusters in order to assess whether or not star clusters are populated from the stellar initial mass function (IMF) by random sampling over the mass range $(0.01 M_{\odot} \leq M_{star} \leq 150 M_{\odot})$ without being constrained by the cluster mass. Their data reveal a partition of the sample into lowest mass objects ($M_{cl} \leq 100 M_{\odot}$), moderate mass clusters ($100 M_{\odot} \leq M_{cl} \leq 1000 M_{\odot}$) and rich clusters above ($M_{cl} \geq 1000 M_{\odot}$)

where \mathcal{M}_{cl} is the mass of the molecular cloud. Their statistical tests of this data set reveal that the hypothesis of random sampling is highly unlikely, thus strongly suggesting that there exists some well defined physical cause.

In closing, allow us to say that we do not claim to have solved the radiation problem, but merely believe that what we have presented herein, together with the papers (Nyambuya 2010a,b), is work that may very well be a significant step forward in the endeavor to resolve this massive star formation riddle.

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