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Bipolar outflows as a repulsive gravitational phenomenon

— Azimuthally Symmetric Theory of Gravitation (II)

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Abstract This paper is part of a series on the Azimuthally Symmetric Theory of Gravitation (ASTG). This theory is built on Laplace-Poisson's well known equation and it has been shown that the ASTG is capable of explaining, from a purely classical physics standpoint, the precession of the perihelion of solar planets as a consequence of the azimuthal symmetry emerging from the spin of the Sun. This symmetry has and must have an influence on the emergent gravitational field. We show herein that the emergent equations from the ASTG, under some critical conditions determined by the spin, do possess repulsive gravitational fields in the polar regions of the gravitating body in question. This places the ASTG on an interesting pedestal to infer the origins of outflows as a repulsive gravitational phenomenon. Outflows are a ubiquitous phenomenon found in star forming systems and their true origin is a question yet to be settled. Given the current thinking on their origin, the direction that the present paper takes is nothing short of an asymptotic break from conventional wisdom; at the very least, it is a complete paradigm shift because gravitation is not at all associated with this process, but rather it is thought to be an all-attractive force that only tries to squash matter together onto a single point. Additionally, we show that the emergent Azimuthally Symmetric Gravitational Field from the ASTG strongly suggests a solution to the supposed Radiation Problem that is thought to be faced by massive stars in their process of formation. That is, at ~ 8–$10\,\mathcal{M}_{\odot}$, radiation from the nascent star is expected to halt the accretion of matter. We show that in-falling material will fall onto the equatorial disk and from there, this material will be channeled onto the forming star via the equatorial plane, thus accretion of mass continues well past the value of ~ 8–$10\,\mathcal{M}_{\odot}$, albeit via the disk. Along the equatorial plane, the net force (with the radiation force included) on any material there-on right up to the surface of the star is directed toward the forming star, hence accretion of mass by the nascent star is un-hampered.

Key words: stars: formation — stars: mass-loss — stars: winds, outflows — ISM: jets and outflows

1 INTRODUCTION

Champagne like bipolar molecular outflows are an unexpected natural phenomenon that graces the star formation podium. Bipolar molecular outflows are the most spectacular physical phenomenon intimately associated with newly formed stars. Studies of bipolar outflows reveal that they (bipolar outflows) are ubiquitous toward High Mass Star (HMS) forming regions. These outflows in HMS forming regions are far more massive and energetic than those found to be associated with Low Mass Star (LMS) forming regions (see e.g. Shepherd & Churchwell 1996a; Shepherd & Churchwell 1996b; Zhang et al. 2001; Zhang et al. 2005; Beuther et al. 2002). Obviously, this points to a correlation between the mass of the star and the outflow itself. Independent studies have established the existence of such a correlation. The mass outflow rate $\dot{\mathcal{M}}_{\rm out}$ has been shown to be related to the bolometric luminosity $\mathcal{L}$ by the relationship: $\dot{\mathcal{M}}_{\rm out} \propto \mathcal{L}_{\rm star}^{0.60}$, and this is for stars in the luminosity range: $0.30\mathcal{L}_{\odot} \leq \mathcal{L}_{\rm star} \leq 10^5\mathcal{L}_{\odot}$ (we shall use the term luminosity to mean bolometric luminosity). Another curious property of outflows is that the mass-flow rate, $\dot{\mathcal{M}}_{\rm out}$, is related to the speed of the molecular outflow $\dot{\mathcal{M}}_{\rm out} \propto V_{\rm out}^{-\gamma}$ where $\gamma \sim 1.80$ and $V_{\rm out}$ is the speed of the outflow. How and why outflows come to exhibit these properties is an interesting question that is not part of the present paper. However, we shall show that these relationships do emerge from our proposed ASTG Outflow Model. In the present article, we simply want to show that an outflow model emerges from the ASTG model. We set herein the mathematical foundations for such a model. Once we have a fully-fledged mathematical model, we shall move on to building a numerical model (i.e. computer code). Once this computer code is available, an endeavour to answer the above and other questions surrounding the nature of outflows will be made.

Pertaining to their association with star formation activity, it is believed that molecular outflows are a necessary part of the star formation process because their existence may explain the apparent angular momentum imbalance. It is well known that the amount of initial angular momentum in a typical star-forming molecular cloud core is several orders of magnitude too large to account for the observed angular momentum found in formed or forming stars (see e.g. Larson 2003). The sacrosanct Law of Conservation of angular momentum informs us that this angular momentum cannot just disappear into the oblivion of interstellar spacetime. So, the question is where this angular momentum goes. It is here that outflows are thought to come to the rescue as they can act as a possible agent that carries away the excess angular momentum. This angular momentum, if it were to remain as part of the nascent star would tear the star apart via the strong centrifugal forces. This however does not explain why they exist and how they come to exist, but simply posits them as a vehicle needed to explain the mystery of "The Missing Angular Momentum Problem" in star forming systems and the existence of stars in their intact and compact form as stable fiery balls of gas.

In the existing literature, viz the question of why and how molecular outflows exist, there are generally four proposed leading models that endeavour to explain the aforesaid. These four major proposals are

Wind Driven Outflow Model: In this model, a wide-angle radial wind blows into the stratified surrounding ambient material, forming a thin swept-up shell that can be identified as the outflow shell (see Shu et al. 1991; Li & Shu 1996; Matzner & McKee 1999).

Jet Driven Bow Shocks Model: In this model, a highly collimated jet propagates into the surrounding ambient material producing a thin outflow shell around the jet (see Raga & Cabrit 1993a; Masson & Chernin 1993).

Jet Driven Turbulent Outflow Model: In this model, Kelvin-Helmholtz instabilities along the jet and/or an environmental boundary lead to the formation of a turbulent viscous mixing layer, through which the molecular cloud gas is entrained (see Cantó & Raga 1991; Raga et al. 1993b; Stahler 1994; Lizano & Giovanardi 1995; Cantó et al. 2003).

Circulation Flows Model: In this model, the molecular outflow is not entrained by an underlying wind jet but is rather formed by in-falling matter that is deflected away from the protostar in the central torus of high magneto-hydrodynamic pressure through a quadrupolar circulation pattern around the protostar and is accelerated above escape speeds by local heating (see Fiege & Henriksen 1996).

All these *ad hoc* models and some that are not mentioned here explain outflows as a feedback effect. The endeavor of the work presented in this paper is to make an alternative suggestion albeit a complete, if not a radical, departure from the already existing models briefly mentioned above. Our model flows naturally from the Laplace-Poison equation, namely from the ASTG laid down in Nyambuya (2010a) (hereafter Paper I). This model is new and has never before appeared in the literature. Because we are at the stage of setting this model, we see no need to go into the details of the existing models as this would lead to an unnecessary digression, confusion, and an uncalled-for lengthy paper.

Our model is a complete departure from the already existing models; because of all the agents that could lead to outflows, gravitation is not even considered to be a possible agent because it is thought of as, or assumed to be, an all-attractive force. Actually, the idea of a gravitating body, such as a star, producing a repulsive gravitational field is at the very least unthinkable. Contrary to this, we show here that an azimuthally symmetric gravitational system does *in-principle* give rise to a bipolar repulsive gravitational field and this, in our view, clearly suggests that these regions of repulsive gravitation are possibly the actual driving force of the bipolar molecular outflows. We also see that the ASTG provides a neat solution (possibly and very strongly so) to the so-called Radiation Problem thought to bedevil and bewilder the formation of HMSs (see Larson & Starrfield 1971; Kahn 1974; Bonnell et al. 1998; Bonnell & Bate 2002; Palla & Stahler 1993) and as well the observed *Ring of Masers* (Bartkiewicz et al. 2008, 2009).

We need to reiterate this so as to make it clear to our reader, that the work presented in this paper is meant to lay down the mathematical foundations of the outflow model emergent from the ASTG. It is not a comparative study of this outflow model with those currently in existence. We believe we have to put effort into retiring these ideas and only worry about their plausibility, i.e. whether or not they correspond with experience and only thereafter make a literature wide comparative study. Given that this model flows naturally from a well accepted equation (the Poisson-Laplace equation), against the probability of all unlikelihood, this model should have a bearing with reality. If it does not have a bearing with reality, then, at the very least, it needs to be investigated since this solution of the Poisson-Laplace equation has not been explored anywhere in the literature [1].

Also, we should say that as we build this model, we are doing this with expediency, that is, watchful of what experience dictates; at the end of the day, if our efforts are to bear any fruits, our model must correspond with reality. This literature wide comparative study is expected to be done once a mathematical model of our proposed outflow model is in full-swing. This mathematical model is expected to form part of the future works where, only after that, it would make sense then to embark on this literature wide comparative study. How does one compare a baby human-being to a human-embryo? It does not make sense, does it? Should not the baby be born first and only thereafter a comparative study be conducted of this baby with those babies already in existence? We hope the reader concurs with us that this is perhaps the best way to set into motion a new idea amid a plethora of ideas that champion a similar if not the same endeavour.

Further, we need to say this, that, as already stated above, the direction that the present paper takes is nothing short of an asymptotic break from conventional wisdom; at the very least, it is a complete paradigm shift as gravitation is not at all associated let alone considered to have anything to do with the out-pouring of matter but is thought to be an all-attractive force that tries only to squash matter together into a single point. Because of this reason, that, the present is "nothing short of an asymptotic break from conventional wisdom" and that "at the very least, it is a complete paradigm shift," we strongly believe that this is enough to warrant the reader's attention to this seemingly seminal theoretical discovery.

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[1] In our exhaustive survey of the accessible literature, we have not come across a treatment of the Poisson-Laplace equation as is done in the present, hence our proclamation that this solution of the Poisson-Laplace equation is the first such one.

The synopsis of this paper is as follows. In the subsequent section, we present the theory to be used in setting up the proposed ASTG Outflow Model. In Section 3, we revisit the persistent problem of the ASTG model, that of "The ASTG's Undetermined Parameter Problem." Therein, we present what we believe may be a solution to this problem. As to what these parameters really may be, this is still an open question subject to debate. In Section 4, we present the main findings of the present paper, that is the repulsive bipolar gravitational field and therein we argue that this field fits the description of outflows. We present this for both the empty and non-empty space solutions of the Poisson-Laplace equation. In Section 5, we look at the anatomy of the outflow model, i.e. the switching on and off of outflows, the nature of the repulsive polar field, the emergent shock rings and the collimation factor of these outflows. In Section 6, we show that the ASTG model posits what strongly appears to be a perdurable solution to the so-called Radiation Problem that is thought to be faced by massive stars during their formation process. Lastly, in Section 7, we give a general discussion and make conclusions that can be drawn from this paper.

Lastly, it is important that we mention here in the penultimate of this introductory section that this paper is fundamental in nature and, because of this, we shall seek to begin whatever argument we seek to rise from the soils of its very basic and fundamental level. This is done so that we are at the same level of understanding with the reader. With the aforesaid approach, if at any point we have made mistakes, it would be easy to know and understand where and how we have made these mistakes.

2 THEORY

Newton's Law of universal gravitation can be written in a more general and condensed form as Poisson's Law, i.e.

$$\boldsymbol{\nabla}^2\Phi = 4\pi G\rho, \tag{1}$$

where ρ is the density of matter and $G = 6.667 \times 10^{-11}\mathrm{kg}^{-1}\mathrm{ms}^{-2}$ is Newton's universal constant of gravitation and the operator $\boldsymbol{\nabla}^2$ written for a spherical coordinate system (see Fig. 1 for the coordinate setup) is given by

$$\boldsymbol{\nabla}^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}, \tag{2}$$

where the symbols have their usual meanings. For a spherically symmetric setting, the solution to Poisson's equation outside the vacuum space (where $\rho = 0$) of a central gravitating body of mass $\mathcal{M}_{\mathrm{star}}$ is given by the traditional inverse distance Newtonian gravitational potential which is given by

$$\Phi(r) = -\frac{G\mathcal{M}_{\mathrm{star}}}{r}, \tag{3}$$

where r is the radial distance from the center of the gravitating body. The Poisson equation for the case $(\rho = 0)$ is known as the Laplace equation. The Poisson equation is an extension of the Laplace equation. Because of this, we shall generally refer to the Poisson equation as the Poisson-Laplace equation. In the case where there is material surrounding this central mass, that is $\mathcal{M} = \mathcal{M}(r)$, where

$$\mathcal{M}(r) = \int_0^r\int_0^{2\pi}\int_0^{2\pi} r^2\rho(r,\theta,\varphi)\sin\theta d\theta d\varphi dr, \tag{4}$$

we must, in Equation (3), make the replacement: $\mathcal{M}_{\mathrm{star}} \longmapsto \mathcal{M}(r)$. As already argued in Paper I, if the gravitating body in question is spinning, we ought to consider an Azimuthally Symmetric Gravitational Field (ASGF). Thus, we shall solve the azimuthally symmetric setting of Equation (1) for both cases of empty and non-empty space and show from these solutions that Poisson's equation

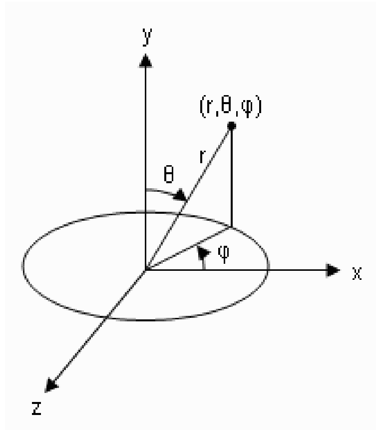

Fig. 1 Generic spherical coordinate system, with the radial coordinate denoted by r, the zenith (the angle from the North Pole, the co-latitude) denoted by θ, and the azimuth (the angle in the equatorial plane, the longitude) by φ.

entails a repulsive bipolar gravitational field. We shall assume that if one has the empty space solution, to obtain the non-empty space solutions, one has to make the replacement: $\mathcal{M}_{\,\mathrm{star}} \longmapsto \mathcal{M}(r)$, just as is done in Newtonian gravitation. This is a leaf that we shall take from spherically symmetric Newtonian gravitation into the ASTG model.

2.1 Empty Space Solutions

As already argued in Paper I, for a scenario or setting that exhibits azimuthal symmetry, such as a spinning gravitating body like the Sun and also the stars that populate the heavens (where the unexpected and spectacular champagne like bipolar molecular outflows are the observed), we must have: $\Phi = \Phi(r, \theta)$. In Paper I, the Poisson equation for empty space has been "solved" for a spinning gravitating system and the solution to it is

$$\Phi(r,\theta) = -\sum_{\ell=0}^{\infty}\left[\lambda_\ell c^2\left(\frac{G\mathcal{M}_{\mathrm{star}}}{rc^2}\right)^{\ell+1}P_\ell(\cos\theta)\right], \tag{5}$$

where λ_ℓ is an infinite set of dimensionless parameters with $\lambda_0 = 1$ and the rest of the parameters λ_ℓ for $(\ell > 1)$ generally take values different from unity. In Paper I, a suggestion as to what these parameters may be has been made. In Section 3, we go further and suggest a form for these parameters. This suggestion, if correct, puts the ASTG on a pedestal to make predictions without first seeking these values (i.e. the λ_ℓ's) from observations. We will show that embedded in Equation (5) is a solution that is such that the polar regions of the gravitating central body will exhibit a repulsive gravitational field. It is this repulsive gravitational field that we shall propose as the driving force causing the emergence of outflows. However, we must bear in mind that outflows are seen in regions in which the central gravitating body is found in the immurement of ambient circumstellar material, thus we must, for the azimuthally symmetric case (where the central gravitating body is spinning), solve the Poisson-Laplace equation for the setting $(\rho \neq 0)$.

2.2 Non-Empty Space Solutions

Clearly, in the event that $(\rho \neq 0)$ for the azimuthally symmetric case, we must have $\rho = \rho(r, \theta)$. In Paper I, an argument has been advanced in support of this claim that: $\Phi(r, \theta) \Rightarrow \rho(r, \theta)$. Taking this as given, the question we wish to answer is: what form does $\Phi(r, \theta)$ take for a given mass distribution $\rho(r, \theta)$? Or the converse, what form does $\rho(r, \theta)$ take for a given $\Phi(r, \theta)$? It is reasonable and most logical to assume that the gravitational field is what influences the distribution of mass and not the other way around. Taking this as the case, then we must have $\rho(r, \theta) = \rho(\Phi)$, i.e. the distribution of the matter in any mass distribution must be a function of the gravitational field. We find that the form for $\rho(r, \theta)$ that meets the requirement: $\rho(r, \theta) = \rho(\Phi)$, and most importantly the requirement that to obtain the non-empty space solution from the empty space solution one simply makes the replacement: $\mathcal{M}_{\rm star} \longmapsto \mathcal{M}(r)$, is

$$\rho(r, \theta) = -\frac{1}{4\pi G}\left(\frac{2}{r}\frac{\partial}{\partial r} - \frac{1}{r^2}\right)\frac{\partial \Phi(r, \theta)}{\partial \theta}. \tag{6}$$

How did we arrive at this? We have to answer this question. To make life very easy for us to arrive at the answer, we shall write Poisson's equation in rectangular coordinates, i.e.

$$\left(\sum_{j=1}^{3}\frac{\partial^2}{\partial x_j^2}\right)\Phi(x, y, z) = 4\pi G\rho(x, y, z), \tag{7}$$

where $x_1 = x, x_2 = y, x_3 = z$. Now suppose we had a function $F(x, y, z)$ such that

$$\left(\sum_{j=1}^{3}\frac{\partial}{\partial x_j}\right)^2 F(x, y, z) = 0. \tag{8}$$

This equation can be written as

$$\left(\sum_{j=1}^{3}\frac{\partial^2}{\partial x_j^2}\right)F(x, y, z) = -\left(\sum_{j}^{3}\sum_{i\neq j}^{3}\frac{\partial^2}{\partial x_i \partial x_j}\right)F(x, y, z). \tag{9}$$

Now, if and only if the gravitational potential did satisfy Equation (7), then comparison of Equation (7) with Equation (9) requires the identification: $\Phi(x, y, z,) \equiv F(x, y, z)$, and as well the identification

$$\rho(x, y, z) = -\frac{1}{4\pi G}\left(\sum_{j}^{3}\sum_{i\neq j}^{3}\frac{\partial^2}{\partial x_i \partial x_j}\right)\Phi(x, y, z). \tag{10}$$

What this means is that the non-linear terms of Equation (7) come about because of the presence of matter. Now, if we transform to spherical coordinates, it is understood as to why and how we came to the choice of ρ given in Equation (6). At the end of the day, what this means is that for whatever form of Φ we choose, the density ρ will have to conform and prefigure to this setting of the gravitational field via Equation (10). Only after accepting Equation (10) do we have the mathematical legitimacy to choose to maintain the form Equation (5) which we found for the case of empty space such that in the place of $\mathcal{M}_{\rm star}$ we can now put $\mathcal{M}(r)$, hence in the case where a central gravitating condensation of mass is in the immurement of ambient circumstellar material, we must have

$$\Phi(r, \theta) = -\sum_{\ell=0}^{\infty}\lambda_\ell c^2\left(\frac{G\mathcal{M}(r)}{rc^2}\right)^{\ell+1}P_\ell(\cos\theta), \tag{11}$$

where $\mathcal{M}(r)$ is given in Equation (4). We believe that this answers the question "What form does $\rho(r,\theta)$ take for a given $\Phi(r,\theta)$?" and at the same time we have justified Equation (6) viz how we have come to it. Importantly, it should be noted that the observed radial density profile is maintained by the choice Equation (10), i.e. $\rho(r) = \int_0^r \int_0^{2\pi} r^2\rho(r,\theta)\sin\theta d\theta dr \propto r^{-\alpha_\rho}$. It is also important to state clearly that all the above implies that the gravitational field is what influences the distribution of matter. This, in our view, resonates both with logic and intuition. We shall demonstrate the assertion that $\rho(r) = \int_0^r \int_0^{2\pi} r^2\rho(r,\theta)\sin\theta d\theta dr \propto r^{-\alpha_\rho}$. We know that

$$\int_0^r \int_0^{2\pi} r^2\rho(r,\theta)\sin\theta d\theta dr = \int_0^r r^2\left(\int_0^{2\pi}\rho(r,\theta)\sin\theta d\theta\right)dr = 4\pi\int_0^r r^2\rho(r)dr, \tag{12}$$

which means

$$\rho(r) = \frac{1}{4\pi}\int_0^{2\pi}\rho(r,\theta)\sin\theta d\theta. \tag{13}$$

Our claim is that if $\rho(r,\theta)$ is given by Equation (6) such that $\Phi(r,\theta)$ is given by Equation (11), where $\mathcal{M}(r)$ in Equation (11) is such that $\mathcal{M}(r) \propto r^{\alpha}$ for some constant α, then

$$\rho(r) = \frac{1}{4\pi}\int_0^{2\pi}\rho(r,\theta)\sin\theta d\theta \propto r^{\alpha_\rho}, \tag{14}$$

where α_ρ is some constant. We know that

$$\frac{1}{4\pi}\int_0^{2\pi}\rho(r,\theta)\sin\theta d\theta = -\frac{1}{16\pi^2 G}\int_0^{2\pi}\left(\frac{2}{r}\frac{\partial}{\partial r} - \frac{1}{r^2}\right)\frac{\partial\Phi(r,\theta)}{\partial\theta}\sin\theta d\theta. \tag{15}$$

We have substituted $\rho(r,\theta)$ in (6) into the above. This simplifies to

$$\frac{1}{4\pi}\int_0^{2\pi}\rho(r,\theta)\sin\theta d\theta = -\frac{1}{16\pi^2 G}\left(\frac{2}{r}\frac{\partial}{\partial r} - \frac{1}{r^2}\right)\int_0^{2\pi}\frac{\partial\Phi(r,\theta)}{\partial\theta}\sin\theta d\theta. \tag{16}$$

From Equation (11), we know that

$$\frac{\partial\Phi(r,\theta)}{\partial\theta} = \sum_{\ell=0}^{\infty}\lambda_\ell c^2\left(\frac{G\mathcal{M}(r)}{rc^2}\right)^{\ell+1}\sin\theta\frac{\partial P_\ell(\cos\theta)}{\partial(\cos\theta)}, \tag{17}$$

and this implies

$$\frac{1}{4\pi}\int_0^{2\pi}\rho(r,\theta)\sin\theta d\theta = -\frac{c^2}{16\pi^2 G}\left(\frac{2}{r}\frac{\partial}{\partial r} - \frac{1}{r^2}\right)\sum_{\ell=0}^{\infty}\lambda_\ell\int_0^{2\pi}\left(\frac{G\mathcal{M}(r)}{rc^2}\right)^{\ell+1}\sin^2\theta\frac{\partial P_\ell(\cos\theta)}{\partial(\cos\theta)}d\theta, \tag{18}$$

which simplifies to

$$\frac{1}{4\pi}\int_0^{2\pi}\rho(r,\theta)\sin\theta d\theta = -\frac{c^2}{16\pi^2 G}\left(\frac{2}{r}\frac{\partial}{\partial r} - \frac{1}{r^2}\right)\sum_{\ell=0}^{\infty}\lambda_\ell\left(\frac{G\mathcal{M}(r)}{rc^2}\right)^{\ell+1}\overbrace{\int_0^{2\pi}\sin^2\theta\frac{dP_\ell(\cos\theta)}{d(\cos\theta)}d\theta}^{\text{Let this be: } \mathrm{I}_\ell(\theta)}, \tag{19}$$

where $I_\ell(\theta)$ is as defined above. It should not be difficult to see that $I_0(\theta) = 0$, $I_1(\theta) = 1$ and that $I_\ell(\theta) \equiv 0$ for all $\ell \geq 2$. From this, it follows that

$$\rho(r) = \frac{1}{4\pi}\int_0^{2\pi}\rho(r,\theta)\sin\theta d\theta = \left(\frac{\lambda_1 c^2}{16\pi^2 G}\right)\left(\frac{2}{r}\frac{\partial}{\partial r} - \frac{1}{r^2}\right)\left(\frac{G\mathcal{M}(r)}{rc^2}\right)^2. \tag{20}$$

Now, if $\mathcal{M}(r) \propto r^{\alpha}$ this means $\mathcal{M}(r) = kr^{\alpha}$ for some adjustable constant k. Plugging this into the above, one obtains

$$\rho(r) = \left[\frac{(2\alpha - 1)\,\lambda_1 c^2}{16\pi^2 G}\right]\left(\frac{Gk}{c^2}\right)^2 r^{2\alpha-4}. \tag{21}$$

This verifies[2] our claim in Equation (14). As already said, all the above implies that the gravitational field is what influences the distribution of matter. Co-joining this result with the result $(0 \leq \alpha_{\rho} < 3)$ in Nyambuya (2010b) (hereafter Paper II), it follows that $(0.5 \leq \alpha < 2)$. Further, a deduction to be made from the above result is that the spin does control the mass distribution via the term λ_1.

3 THE UNDETERMINED CONSTANTS λ_ℓ

Again, as already stated in Paper I, one of the drawbacks of the ASTG is that it is heavily dependent on observations, for the values of λ_ℓ have to be determined from observations. Without knowledge of the $\lambda'_\ell s$, one is unable to produce the hard numbers required to make any numerical quantifications. Clearly, a theory incapable of making any numerical quantifications is, in the physical realm, useless. To avert this, already in Paper I and as well in this paper an effort to solve this problem has been made. In Paper I, a *reasonable suggestion* was made to the effect that

$$\lambda_\ell = \left[\frac{(-1)^{\ell+1}}{(\ell^\ell)!\,(\ell^\ell)}\right]\lambda_1. \tag{22}$$

This suggestion meets the intuitive requirements stated in Paper I. If these λ's are to be given by Equation (22), then there is just one unknown parameter and this parameter is λ_1. The question is what this depends on. We strongly believe that λ_1 is dependent on the spin angular frequency and the radius of the gravitating body in question and our reasons are as follows.

The ASTG will shortly be shown to be able to explain outflows as a gravitational phenomenon. Pertaining to their association with star formation activity, it is believed that molecular outflows are a necessary part of the star formation process because their existence may explain the apparent angular momentum imbalance. It is well known that the amount of initial angular momentum in a typical star-forming cloud core is several orders of magnitude too large to account for the observed angular momentum found in formed or forming stars (see e.g. Larson 2003). The sacrosanct Law of Conservation of angular momentum informs us that this angular momentum cannot just disappear into the oblivion of interstellar spacetime. So, the question is where this angular momentum goes. It is here that outflows are thought to come to the rescue as they can act as a possible agent that carries away the excess angular momentum. Whether or not this assertion is true or may have a bearing with reality, no one really knows.

This angular momentum, if it were to remain as part of the nascent star, would, via the strong centrifugal forces (the centrifugal acceleration is given by: $a_{\mathrm{c}} = \omega_{\mathrm{star}}^2 \mathcal{R}_{\mathrm{star}}$), tear the star apart. This however does not explain why they (outflows) exist and how they come to exist but simply posits them as a vehicle needed to explain the mystery of “The Missing Angular Momentum Problem” in star forming systems and the existence of stars in their intact and compact form as fiery balls of gas.

In this paper, guided more by intuition than anything else, it was drawn from the tacit thesis “that outflows possibly save the star from the detrimental centrifugal forces,” the suggestion that $\lambda_1 \propto (a_c)^{\zeta_0}$ where ζ_0 is a pure constant that must be universal, that is, it must be the same for all

[2] Under the prescribed conditions, $\mathcal{M}(r) \propto r^{\alpha}$ leads to $\rho(r) \propto r^{2\alpha-4}$. While $\mathcal{M}(r) = \int_0^r \int_0^{2\pi} \rho(r,\theta)\sin\theta d\theta dr$, the basic definition $\mathcal{M}(r) = 4\pi r^3 \rho(r)/3$ must hold too, since $\mathcal{M}(r)$ is the amount of mass enclosed in the volume of a sphere of radius r and $\rho(r)$ is the mass-density of material in this volume sphere. These two definitions must lead to identical formulas. If this is to be so, then one is led to the conclusion that $\alpha = 1$, and this means $\mathcal{M}(r) \propto r$ and $\rho(r) \propto r^{-2}$. In the face of observations, the later result is very interesting since MCs seem to favor this density profile.

spinning gravitating systems. This suggestion, if correct, leads us to

$$\lambda_\ell = \left(\frac{(-1)^{\ell+1}}{(\ell^\ell)!\,(\ell^\ell)}\right)\left(\frac{a_{\rm c}}{a_*}\right)^{\zeta_0}. \tag{23}$$

Knowing the solar values of λ_1 and as well the value of ζ_0 leads to $a_* = \omega_\odot^2 \mathcal{R}_\odot (\lambda_1^\odot)^{-\frac{1}{\zeta_0}}$. As will be demonstrated soon, the term λ_1 controls outflows. Given that λ_1 controls outflows and that outflows possibly aid the star in shedding excess spin angular momentum, the best choice [3] for this parameter is one that leads to these outflows responding to the spin of the star and as well the centrifugal forces generated by this spin in such a way that the star is able to shed this excess spin angular momentum. So, what led to this proposal $\lambda_1 \propto (a_{\rm c})^{\zeta_0}$ is the aforesaid. Now, we shall revise this suggestion by advancing what we believe is a better argument.

If outflows are there to save the nascent star from the ruthlessness of the centrifugal forces, then it is logical to imagine that at the moment the centrifugal forces are about to rip the star apart, outflows will switch-on, thus shedding off this excess spin angular momentum. The centrifugal forces have their maximum toll on the equatorial surface of the star, hence if the centrifugal forces are to rip the nascent star apart, this would start at the equator of the nascent star. The centrifugal force on the surface of the star acting on a particle of mass m is $F_{\rm c} = m\omega_{\rm star}^2 \mathcal{R}_{\rm star} = m a_{\rm c}$ and the gravitational force on the same particle is $F_{\rm g} = G\mathcal{M}m/\mathcal{R}_{\rm star}^2 = m g_{\rm star}$. Now let us define the quotient $\mathcal{Q} = F_{\rm c}/F_{\rm g} = a_{\rm c}/g_{\rm star}$. If the particle is put on the surface of the star, then we will have $F_{\rm c} - F_{\rm g} < 0 \Rightarrow \mathcal{Q} < 1$; if the particle were to fly off the surface, we will have $F_{\rm c} - F_{\rm g} > 0 \Rightarrow \mathcal{Q} > 1$. The critical condition before the star begins to be torn apart is $F_{\rm c} - F_{\rm g} = 0 \Rightarrow \mathcal{Q} = 1$. All the above can be summarized as

$$\mathcal{Q}\begin{cases} < 1 & \text{no outflow activity,} \\ = 1 & \text{critical condition,} \\ > 1 & \text{outflow activity.} \end{cases} \tag{24}$$

Let us call this quotient the Outflow Control Quotient (OCQ). Clearly, the OCQ determines the necessary conditions for outflows to switch on. Given this, and as well the thinking that λ_1 controls outflows, the suggestion is clear that $\lambda_1 \propto \mathcal{Q}^{\zeta_0}$. If this is correct, then

$$\lambda_1 = \zeta \mathcal{Q}^{\zeta_0}. \tag{25}$$

We shall take this as our proposal for λ_1 and this means we must determine (ζ, ζ_0). From the above, it follows that

$$\frac{\lambda_1^\oplus}{\lambda_1^\odot} = \left(\frac{\mathcal{Q}_\oplus}{\mathcal{Q}_\odot}\right)^{\zeta_0}, \tag{26}$$

where $\mathcal{Q}_\oplus = a_{\rm c}^\oplus / g_\oplus$ and $a_{\rm c}^\oplus$ is the centripetal acceleration generated by the Earth's spin at the equator and $g_\oplus$ is the gravitational field strength at the Earth's equator. Likewise, $\mathcal{Q}_\odot = a_{\rm c}^\odot / g_\odot$ is the solar outflow quotient where $a_{\rm c}^\odot$ is the centripetal acceleration generated by the Sun's spin at the solar equator and $g_\odot$ is the gravitational field strength at the solar equator. Given that ($\omega_\oplus = 7.27 \times 10^{-5}$ Hz and $\omega_\odot = 2.04 \times 10^{-5}$ Hz), ($\mathcal{R}_\oplus = 6.40 \times 10^6$ m and $\mathcal{R}_\odot = 6.96 \times 10^8$ m) and ($g_\oplus = 9.80\,{\rm ms}^{-2}$ and $g_\odot = 27.9 g_\oplus$), from these data, it follows that

$$\frac{\mathcal{Q}_\oplus}{\mathcal{Q}_\odot} = 169. \tag{27}$$

Now, we did show that depending on how one interprets the flyby equation, one obtains two values of $\lambda_1^\oplus$, i.e. $\lambda_1^\oplus = (2.00 \pm 0.80) \times 10^3$ and $\lambda_1^\oplus = (1.50 \pm 0.70) \times 10^4$. If the spin of the Earth

[3] We speak of "choice" here as though the decision is ours on what this parameter must be. No, the decision was long made by Nature, as ours is to find out what choice Nature has made. That said, we should say that this "choice" is made with expediency, i.e., this choice, which is based on intuition, is to be measured against experience.

Table 1 ζ_0, ζ Values for the Two Different Values of $\lambda_1^{\oplus}$

$\lambda_1^{\oplus}$ (10^3)	$\lambda_1^{\odot}$	ζ_0	$\zeta_{\odot}$ (10^5)	$\zeta_{\oplus}$ (10^5)
2.00 ± 0.80	21.00 ± 4.00	0.90 ± 0.10	13.00 ± 6.00	5.00 ± 3.00
15.00 ± 7.00	21.00 ± 4.00	1.30 ± 0.10	500 ± 400	400 ± 200

is significantly variable during the course of its orbit around the Sun, we will have $\lambda_1^{\oplus} = (2.00 \pm 0.80) \times 10^3$ and if the spin is not significantly variable, then, $\lambda_1^{\oplus} = (1.50 \pm 0.70) \times 10^4$. If $\lambda_1^{\oplus} = (2.00 \pm 0.80) \times 10^3$, then

$$\frac{\lambda_1^{\oplus}}{\lambda_1^{\odot}} = \frac{15000 \pm 7000}{21.00 \pm 4.00} = 800 \pm 500, \tag{28}$$

and from this it follows that $800 \pm 500 = 169.19^{\zeta_0}$, hence $\zeta_0 = 1.30 \pm 0.10$. If $\lambda_1^{\oplus} = (1.50 \pm 0.70) \times 10^4$, then

$$\frac{\lambda_1^{\oplus}}{\lambda_1^{\odot}} = \frac{2000 \pm 800}{21.00 \pm 4.00} = 100 \pm 60, \tag{29}$$

and from this it follows that $100 \pm 60 = 169.19^{\zeta_0}$, hence $\zeta_0 = 0.90 \pm 0.10$.

If $\zeta_{\oplus}$ and $\zeta_{\odot}$ are the ζ-values for the Earth and the Sun respectively, then for both $\lambda_1^{\oplus} = 15000 \pm 7000$ and $\lambda_1^{\oplus} = 2000 \pm 800$, we will have $\zeta_{\oplus} = (3.40 \pm 2.70) \times 10^7$ and $\zeta_{\odot} = (8.00 \pm 4.00) \times 10^{10}$. Table 1 is a self explanatory summary of all the above calculations. The mean value of ζ for the case $\lambda_1^{\oplus} = (2.00 \pm 0.80) \times 10^3$ is $\zeta = (8.00 \pm 1.00) \times 10^5$ and for the case $\lambda_1^{\oplus} = (1.50 \pm 0.70) \times 10^4$ is $\zeta = (4.00 \pm 2.00) \times 10^7$. These mean values have been obtained by taking the values of $\zeta_{\oplus}$ and $\zeta_{\odot}$ where they intersect in their error margins.

As argued in this paper, the value $\lambda_1^{\oplus} = (2.00 \pm 0.80) \times 10^3$ has been obtained from the assumption that the spin of the Earth varies widely during the course of its orbit around the Sun. This is not supported by observations, thus we are not persuaded to take-up/recommend this value of $\lambda_1^{\oplus} = (2.00 \pm 0.80) \times 10^3$. Also, as argued in this paper, the value $\lambda_1^{\oplus} = (1.50 \pm 0.70) \times 10^4$ is obtained from the assumption that the spin of the Earth does not vary widely during the course of its orbit. Thus, we shall adopt the values of (ζ_0, ζ) that conform with $\lambda_1^{\oplus} = (1.50 \pm 0.70) \times 10^4$ and $\lambda_1^{\odot} = 21.0 \pm 0.40$, hence

$$\lambda_1 = (4.00 \pm 2.00) \times 10^7 \left(\frac{a_{\rm c}}{g_{\rm star}}\right)^{1.30\pm0.10}. \tag{30}$$

Obviously, the greatest criticism against this result is that it is obtained from just two data points. To obtain something more reliable, one needs more data points. This is something that a future study must handle; at present, we simply want to set-up the mathematical model from the little available data and when data become available, amendments will accordingly be made. While we have used the minimal possible number of data points, one thing that can be deduced from these data is that this result points to a correlation as proposed in Equation (25), otherwise, if there was no correlation as proposed, the values of (ζ_0, ζ) obtained from the two values of λ_1 do not vary widely, as is expected if the proposed relationship (25) did not hold at all.

4 OUTFLOWS AS A GRAVITATIONAL PHENOMENON

We shall look into the empty and non-empty space solution of the Poisson-Laplace equation and show that both these solutions exhibit a repulsive bipolar gravitational field and that this repulsive gravitational field is controlled by the parameter λ_1.

4.1 Non-Empty Space Solutions

Now, if one accepts what has been presented thus far, as will be shown in this section, it follows that outflows may well be a gravitational phenomenon. First, from the previous section, it follows that we must take the ASTG only up to second order, i.e.

$$\Phi = -\frac{G\mathcal{M}(r)}{r}\left[1 + \frac{\lambda_1 G\mathcal{M}(r)\cos\theta}{rc^2} + \lambda_2\left(\frac{G\mathcal{M}(r)}{rc^2}\right)^2\left(\frac{3\cos^2\theta - 1}{2}\right)\right]. \tag{31}$$

We know that with the gravitational field intensity $\boldsymbol{g}(r,\theta) = -\boldsymbol{\nabla}\Phi(r,\theta) = g_r(r,\theta)\hat{\boldsymbol{r}} + g_\theta(r,\theta)\hat{\boldsymbol{\theta}}$, this means

$$g_r = g_{\mathrm{N}}\left[\overbrace{1 + \frac{2\lambda_1 G\mathcal{M}(r)\cos\theta}{rc^2}}^{\mathrm{Term I}} + \overbrace{3\lambda_2\left(\frac{G\mathcal{M}(r)}{rc^2}\right)^2\left(\frac{3\cos^2\theta - 1}{2}\right)}^{\mathrm{Term II}}\right], \tag{32}$$

where $g_{\mathrm{N}} = -G\mathcal{M}(r)/r^2$ is the Newtonian gravitational field intensity and

$$g_\theta = g_{\mathrm{N}} r^2 \sin\theta\left[\frac{\lambda_1 G\mathcal{M}(r)}{rc^2} + 9\lambda_2\left(\frac{G\mathcal{M}(r)}{rc^2}\right)^2\cos\theta\right]. \tag{33}$$

For gravitation to be exclusively attractive (as is expected), we must have $[g_r(r,\theta) > 0]$ and $[g_\theta(r,\theta) > 0]$. From Equations (32) and (33), it is clear that regions of exclusively repulsive gravitation will exist and these will occur in the region where $[g_r(r,\theta) < 0]$ and $[g_\theta(r,\theta) < 0]$. This region where gravity is exclusively repulsive is the region where it is not attractive, so it is the negated region of the region of attractive gravitation [i.e. $[g_r(r,\theta) > 0]$ and $[g_\theta(r,\theta) > 0]$. Let us start by treating the case $[g_r(r,\theta) < 0]$. From Equation (32), if $[g_r(r,\theta) < 0]$, then (Term I < 0) and (Term II < 0). The condition (Term I < 0) implies

$$r < -\lambda_1\left(\frac{2G\mathcal{M}(r)}{c^2}\right)\cos\theta = \lambda_1\left(\frac{2G\mathcal{M}(r)}{c^2}\right)\cos\theta, \tag{34}$$

(NB: $\cos\theta \equiv -\cos\theta$) and if one were to take r such that it only assumes positive values, then Equation (34) must be written in the equivalent form

$$r < \lambda_1\left(\frac{2G\mathcal{M}(r)}{c^2}\right)|\cos\theta|, \tag{35}$$

where the bars $|\ \ldots\ |$ represent the absolute value. We have to explain this, i.e. why we concealed the negative sign in Equation (34) and inserted the absolute value operator in Equation (35). From Equation (34), it is seen that this inequality includes negative values of r and to avoid any confusion as to what these negative values of r really mean; this needs to be explained for failure to do so or failure by the reader to understand this means they certainly will be unable to agree with the outflow "picture" laid down here. This explanation is important in order to understand the morphology of the outflow and as well the ASGF.

For a moment, imagine a flat Euclidean plane and on this plane let O, A and P be distinct and separate points on this plane with O and A being fixed and P a variable point. In polar coordinates, as in the present case, a point P is characterized by two numbers: the distance $(r \geq 0)$ to the fixed pole or origin O, and the angle θ the line OP makes with the fixed reference line OA. The angle θ is only defined up to a multiple of $360°$ (or 2π rad, in radians). This is the conventional definition.

Sometimes it is convenient as in the present case to relax the condition $(r \geq 0)$ and allow r to be assigned a negative value such that the point (r, θ) and $(-r, \theta + 180^\circ)$ represent the same-point, hence whenever we have $(-r, \theta)$, this must be replaced by $(r, \theta - 180^\circ)$. It is easier for us to always think of r as always being positive. To achieve this, given the fact that $(-r, \theta) \equiv (r, \theta - 180^\circ)$, we must write Equation (34) as has been done in Equation (35), hence Equation (35) finds justification. This explanation can be found in any good mathematics textbook that deals extensively with polar coordinates. Hereafter, whenever a similar scenario arises where negative values of r emerge, we will automatically and without notification assume that $(-r, \theta)$ is $(r, \theta - 180^\circ)$ and this will come with the introduction of the absolute value sign as has been done in Equation (35).

Now, proceeding from where we left off, as has already been explained at the beginning of this section, we have to substitute the Mass Distribution Function (MDF) $\mathcal{M}(r)$ into Equation (35) and having done so, we would have to make r the subject. It has been argued in equation (24) of Nyambuya (2010b) (hereafter Paper II), that for an MC that exhibits a density profile, $\rho(r) \propto r^{-\alpha_\rho}$ where α_ρ is the density index, the MDF is given by

$$\mathcal{M}(r) = \overbrace{\mathcal{M}_{\rm csl}\left(\frac{r^{3-\alpha_\rho} - \mathcal{R}_{\rm star}^{3-\alpha_\rho}}{\mathcal{R}_{\rm core}^{3-\alpha_\rho} - \mathcal{R}_{\rm star}^{3-\alpha_\rho}}\right)}^{\textbf{Circumstellar Mass Inside Region of Radius r}} + \overbrace{\mathcal{M}_{\rm star}}^{\textbf{Nascent Star's Mass}} \qquad \text{for} \quad r \geq \mathcal{R}_{\rm star}, \tag{36}$$

where $\mathcal{M}_{\rm csl}$ is the total mass of the circumstellar material at any given time, $\mathcal{R}_{\rm star}$ is the radius of the nascent star at any given time, and $\mathcal{R}_{\rm core}$ is the radius at any given time of the gravitationally bound core from which the star is forming.

Now, substituting the MDF (given above) into Equation (35) and thereafter making r the subject of the formula would lead to a horribly complicated inequality that would require the use of the Newton-Ralphson approach to solve. Since ours in the present is only a qualitative analysis, we can make some very realistic simplifying assumptions that can make our life much easier. If the spatial extent of the star is small compared to that of the core, i.e. $(\mathcal{R}_{\rm star} \ll \mathcal{R}_{\rm core} \Rightarrow \mathcal{R}_{\rm core}^{3-\alpha_\rho} - \mathcal{R}_{\rm star}^{3-\alpha_\rho} \simeq \mathcal{R}_{\rm core}^{3-\alpha_\rho})$, and the mass of the star is small compared to the mass of the core, i.e. $(\mathcal{M}_{\rm star} \ll \mathcal{M}_{\rm core} \Rightarrow \mathcal{M}_{\rm csl} \simeq \mathcal{M}_{\rm core})$, then the MDF simplifies to

$$\mathcal{M}(r) \simeq \mathcal{M}_{\rm core}\left(\frac{r}{\mathcal{R}_{\rm core}}\right)^{3-\alpha_\rho}. \tag{37}$$

Inserting this into Equation (35) and thereafter performing some basic algebraic computations that see r as the subject of the formula, one is led to

$$r < \left[\lambda_1\left(\frac{2G\mathcal{M}_{\rm core}}{c^2\mathcal{R}_{\rm core}}\right)\mathcal{R}_{\rm core}\right]|\cos\theta|^{\frac{1}{2-\alpha_\rho}}. \tag{38}$$

Now, if we set

$$\epsilon_1^{\rm core} = \left\{\left[\lambda_1\left(\frac{2G\mathcal{M}_{\rm core}}{c^2\mathcal{R}_{\rm core}}\right)\right]^{\frac{1}{2-\alpha_\rho}}\right\}\left(\frac{\mathcal{R}_{\rm core}}{\mathcal{R}_{\rm star}}\right), \tag{39}$$

then Equation (38) reduces to

$$r < \epsilon_1^{\rm core}\mathcal{R}_{\rm star}|\cos\theta|^{\frac{1}{2-\alpha_\rho}} = l_{\max}|\cos\theta|^{\frac{1}{2-\alpha_\rho}}, \tag{40}$$

where $l_{\max} = \epsilon_1^{\rm core}\mathcal{R}_{\rm star}$. On the xy-plane as shown in Figure 2, the equation $r = l_{\max}|\cos\theta|^{\frac{1}{2-\alpha_\rho}}$, describes two lobes. For the purposes of this paper, let the volume of revolution of the lobe be called a loboid, then the loboid above the x-axis shall be called the upper loboid, and likewise the loboid below the x-axis shall be called the lower loboid.

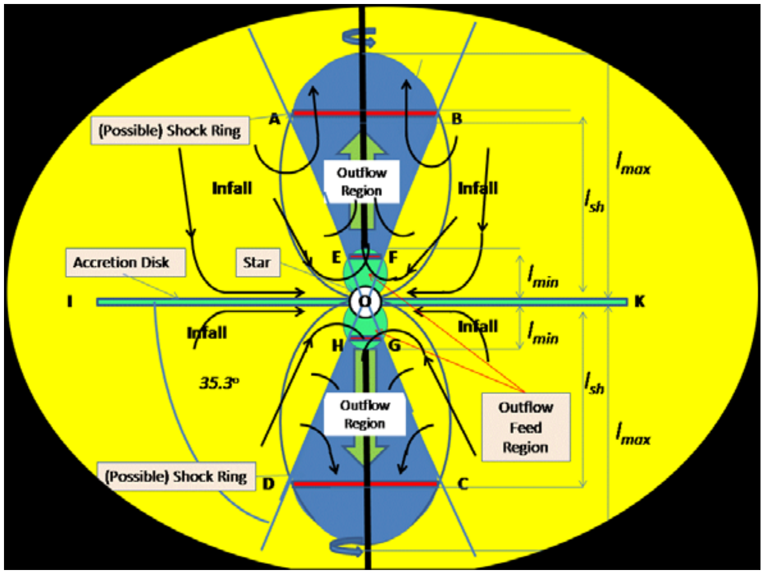

Fig. 2 Emergent picture from the azimuthally symmetric considerations of the Poisson equation. While fanning out matter in the region of repulsive gravitation, the rotating star is surrounded by an equatorial disk; once the outflow switches-on, this disk is the only channel via which the mass of the star feeds. The disk would not be affected by radiation in the sense that some of its material close to the nascent star would not be swept away by the radiation field. The force of gravity along this disk is purely radial and is directed toward the nascent star.

Now, the condition (Term II < 0) implies $[\theta < \cos^{-1}(\pm 1/\sqrt{3})]$, which means $(-54.7 < \theta < 54.7)$. Now, for the azimuthal component to be repulsive, we must have $[g_{\theta}(r,\theta) > 0]$; we will have from Equation (33) the condition

$$r > -\left(\frac{9\lambda_2}{2\lambda_1^2}\right)\left[\frac{2\lambda_1 G\mathcal{M}(r)}{c^2}\right]\cos\theta. \tag{41}$$

Now going through the same procedure as above, Equation (41) can be written as

$$r > l_{\rm min}\,|\cos\theta|^{\frac{1}{2-\alpha_\rho}}\,, \tag{42}$$

where

$$l_{\rm min} = \left(\left|\frac{9\lambda_2}{2\lambda_1^2}\right|^{\frac{1}{2-\alpha_\rho}}\right) l_{\rm max}. \tag{43}$$

Thus, combining the results, invariably one is led to conclude that the region of repulsive gravitation is

$$[l_{\rm min} < r < l_{\rm max}]\ \&\ \left[\cos^{-1}\left(-\frac{1}{\sqrt{3}}\right) < \theta < \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)\right]. \tag{44}$$

In the region described above, the gravitational field is both radially and azimuthally repulsive, that is, there is complete gravitational repulsion in this region. Pictorially, a summary of the emergent picture of the repulsive gravitational field is shown in Figure 2. This picture, in our view, fits the description of outflows; the limiting factors are the sizes of $l_{\rm max}$ and $l_{\rm min}$, and these values all depend on the one parameter λ_1, hence this parameter is the crucial parameter which determines the properties of outflows. Shortly, we will discuss this picture but before this, it will be necessary that we go through the empty space solutions as well.

4.2 Empty Space Solutions

As will be demonstrated in this section, the picture emerging from the empty space solution is not different from that of the non-empty space solution. However, there is an important difference between these two pictures and this difference needs to be stated. If our spinning gravitating body is not giving off material like the Sun, then the region of repulsive gravitation will occur inside this body. We shall consider the star to be a point mass, i.e. all of its mass is concentrated at the star's center of mass.

As stated before, from Equations (32) and (33), it is clear that regions of repulsive gravitation will exist and these will occur where $[g_r(r,\theta) < 0]$ and/or $[g_\theta(r,\theta) < 0]$. We shall as before start by treating the case $[g_r(r,\theta) < 0]$. From Equation (32), if $[g_r(r,\theta) < 0]$, then (Term I < 0) and (Term II < 0) as well. The condition (Term I < 0) implies

$$r < -\lambda_1 \left(\frac{2G\mathcal{M}}{c^2}\right) \cos\theta, \tag{45}$$

where in the present case $\mathcal{M}(r)$ must be replaced by $\mathcal{M}_{\rm star}$ and this can be written in the equivalent form

$$r < \lambda_1 \left(\frac{2G\mathcal{M}_{\rm star}}{c^2}\right) |\cos\theta| \,. \tag{46}$$

Now, if we set

$$\epsilon_1^{\rm star} = \lambda_1 \left(\frac{\mathcal{R}^s_{\rm star}}{\mathcal{R}_{\rm star}}\right), \tag{47}$$

where $\mathcal{R}^s_{\rm star} = 2G\mathcal{M}_{\rm star}/c^2$ is the Schwarzchild radius of the star, then Equation (46) reduces to

$$r < \epsilon_1^{\rm star}\mathcal{R}_{\rm star}|\cos\theta| = l_{\rm max}|\cos\theta|. \tag{48}$$

Now, the condition (Term II < 0), as before, implies $[\theta < \cos^{-1}(\pm 1/\sqrt{3})]$, which means $(-54.7 < \theta < 54.7)$. Again as before, for the azimuthal component to be repulsive, we must have $[g_\theta(r,\theta) > 0]$; we will have from Equation (33) that

$$r > \left(\frac{9\lambda_2}{2\lambda_1^2}\right)\left(\frac{2\lambda_1 G\mathcal{M}_{\rm star}}{c^2}\right) |\cos\theta| \,, \tag{49}$$

and we need not explain anymore why the above equation can be written as

$$r > l_{\rm min}|\cos\theta|, \tag{50}$$

where

$$l_{\rm min} = \left|\frac{9\lambda_2}{2\lambda_1^2}\right| l_{\rm max}. \tag{51}$$

Collecting the results, invariably, one is led to conclude that the region of repulsive gravitation is

$$[l_{\rm min} < r < l_{\rm max}] \;\&\; \left[\cos^{-1}\left(-\frac{1}{\sqrt{3}}\right) < \theta < \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)\right]. \tag{52}$$

As in the case of the non-empty space, in the region described above, the gravitational field is both radially and azimuthally repulsive, hence there is complete gravitational repulsion in this region. The emergent picture is no different from that of the case of non-empty space. The important difference is that the region of gravitational repulsion is confined in the interior of the star if $(\epsilon_1 < 1)$, so it is not visible outside. If $(\epsilon_1 < 1)$, there will exist no repulsive bipolar gravitational field that is visible beyond the surface of the spinning star. In the interior of the star, the solutions obtained for the case of non-empty space are what must apply.

5 ASGF OF A SPINNING CORE WITH AN EMBEDDED SPINNING STAR

Central to the ASTG is that the material under consideration possesses some finite spin angular momentum. In the case of a nascent star embedded inside a gravitationally bound core, we are going to have the star's spin angular frequency being different from that of the circumstellar material; because, in the early stages when the nascent star is forming, the spin angular frequency of the circumstellar material and the star will, on the average, be the same since it is expected that circumstellar material and the star will co-rotate; but, because of the increasing mass and spin angular momentum of the nascent star due to the accretion of material, at some point, the star must break-off from this co-rotational motion and spin independent of the circumstellar material. Thus in the end, the star will have a different spin angular frequency from that of the circumstellar material. The different spin angular momentum of the nascent star and the circumstellar material will have different λ-values. Assuming the circumstellar material is co-rotating with itself, it must have its own λ-value. Let us call this $\lambda_\ell^{\rm csl}$ and that for the star $\lambda_\ell^{\rm star}$.

If there is a way of calculating the ASGF of the star at point (r,θ) and that of the circumstellar material at that same point (r,θ), then one will be able to calculate the resultant ASGF at any point (r,θ) because the gravitational field is a scalar. Let $\Phi_{\rm star}$ be the Azimuthally Symmetric Gravitational Potential (ASGP) of the star and that of the circumstellar material be $\Phi_{\rm csl}$. Knowing $\Phi_{\rm star}$ and $\Phi_{\rm csl}$, clearly the resultant ASGP $\Phi_{\rm eff}$ at any point (r,θ) is $\Phi_{\rm eff}=\Phi_{\rm star}+\Phi_{\rm csl}$, hence one will be able to obtain the resultant ASGF. The ASGF of the star is not difficult to obtain. We already know that it must be given by

$$\Phi_{\rm star}=-\frac{G\mathcal{M}_{\rm star}}{r}\left[1+\frac{\lambda_1^{\rm star}G\mathcal{M}_{\rm star}\cos\theta}{rc^2}+\lambda_2^{\rm star}\left(\frac{G\mathcal{M}_{\rm star}}{rc^2}\right)^2\left(\frac{3\cos^2\theta-1}{2}\right)\right]. \qquad (53)$$

Now, we have to obtain the ASGF of a spinning core. The gravitational potential Equation (31) is the potential of a star that is co-rotating with the circumstellar material. If we remove the central star from this gravitational potential, what remains is the gravitational potential of a spinning core. Removing the central star from this potential means setting $\mathcal{M}_{\rm star}=0$, hence, the gravitational potential of a spinning core must be

$$\Phi_{\rm csl}=-\frac{G\mathcal{M}_{\rm csl}(r)}{r}\left[1+\frac{\lambda_1^{\rm csl}G\mathcal{M}_{\rm csl}(r)\cos\theta}{rc^2}+\lambda_2^{\rm csl}\left(\frac{G\mathcal{M}_{\rm csl}(r)}{rc^2}\right)^2\frac{3\cos^2\theta-1}{2}\right], \qquad (54)$$

where $\lambda_\ell^{\rm csl}$ is the λ_ℓ-value for the spinning circumstellar material and

$$\mathcal{M}_{\rm csl}(r)=\mathcal{M}_{\rm csl}\left(\frac{r^{3-\alpha_\rho}-\mathcal{R}_{\rm cav}^{3-\alpha_\rho}(t)}{\mathcal{R}_{\rm core}^{3-\alpha_\rho}(t)-\mathcal{R}_{\rm cav}^{3-\alpha_\rho}(t)}\right)\ \text{for } r\geq\mathcal{R}_{\rm cav}(t) \qquad (55)$$

is the circumstellar material enclosed in radius r. Now, as already argued $\Phi_{\rm eff}=\Phi_{\rm star}+\Phi_{\rm csl}$, thus adding these two potentials (i.e. Eqs. (53) and (54)), one obtains

$$\Phi_{\rm eff}(r,\theta)=-\sum_{\ell=0}^{\infty}c^2\left(\frac{G\left\{\lambda_\ell^{\rm star}\mathcal{M}_{\rm star}^{\ell+1}+\lambda_\ell^{\rm csl}\mathcal{M}_{\rm csl}^{\ell+1}(r)\right\}^{\frac{1}{\ell+1}}}{rc^2}\right)^{\ell+1}P_\ell(\cos\theta). \qquad (56)$$

This is the ASGP of a star that spins independently from its core. For convenience, we can write $\mathcal{M}_\ell^{\rm eff}(r)=\left\{\lambda_\ell^{\rm star}\mathcal{M}_{\rm star}^{\ell+1}+\lambda_\ell^{\rm csl}\mathcal{M}_{\rm csl}^{\ell+1}(r)\right\}^{\frac{1}{\ell+1}}$, and call this the effective gravitational mass for the $\ell^{\rm th}$ gravitational pole. By $\ell^{\rm th}$ gravitational pole, it shall be understood to mean the $\ell^{\rm th}$-term in the gravitational potential term. This means the above equation can be written in the clearer and simpler form

$$\Phi_{\rm eff}(r,\theta) = -\sum_{\ell=0}^{\infty} c^2 \left(\frac{G\mathcal{M}_{\ell}^{\rm eff}(r)}{rc^2}\right)^{\ell+1} P_{\ell}(\cos\theta). \tag{57}$$

To second order approximation, this potential is given by

$$\Phi_{\rm eff} = -\left(\frac{G\mathcal{M}_0^{\rm eff}(r)}{r}\right)\left[1+\gamma_1\lambda_1^{\rm star}\left(\frac{G\mathcal{M}_1^{\rm eff}(r)\cos\theta}{rc^2}\right)+\gamma_2\lambda_2^{\rm star}\left(\frac{G\mathcal{M}_2^{\rm eff}(r)}{rc^2}\right)^2\left(\frac{3\cos^2\theta-1}{2}\right)\right], \tag{58}$$

where $\gamma_\ell = \mathcal{M}_\ell^{\rm eff}(r)/\mathcal{M}_0^{\rm eff}(r)$. We shall assume this ASGP is for a star that spins independently from its core.

6 OUTFLOW POWER

Clearly from the ASTG model, we do have regions of repulsive gravitation whose shape is similar to that seen in outflows. If these outflows are really powered by gravity, the question is: does the gravitational field have that much energy to drive these and if so, where does this energy come from? To answer this question, one will need to know the dominant radial component of the gravitational force since outflows dominantly operate along the radial direction. Clearly, one of the new extra poles in the gravitational field must be the cause of the outflows since without them, there are no outflows. For our investigations, the correct gravitational potential to use is Equation (57) and of interest in this potential is the gravitational potential of the star. This invariably means that we are looking at Equation (53). In so doing, one sees that the first order term (involving λ_1) is an all-repulsive term as already argued while the second order term (involving λ_2) is repulsive and attractive, so it depends on the region under consideration.

Now, to ask what powers the outflows amounts to asking "What is their energy source?" If this energy source is the gravitational field, then we know that the energy stored in the gravitational field whose potential is described by $\Phi(r,\theta)$ is given by

$$\mathcal{E}_{\rm gpe}^{\rm star}(r) = \int_0^{\mathcal{M}_{\rm star}}\int_{\Phi(r,0)}^{\Phi(\infty,2\pi)} d\Phi(r,\theta)d\mathcal{M}, \tag{59}$$

and plugging into the above the ASGP, and thereafter performing the integration, one is led to

$$\mathcal{E}_{\rm gpe}^{\rm star}(r) = -\frac{G\mathcal{M}_{\rm star}^2}{2r}\left[1+\frac{\lambda_1 G\mathcal{M}_{star}}{rc^2}+\lambda_2\left(\frac{G\mathcal{M}_{\rm star}}{rc^2}\right)^2\right], \tag{60}$$

and using the fact that $\mathcal{L}_{\rm star} = \mathcal{L}_\odot\left(\mathcal{M}_{\rm star}/\mathcal{M}_\odot\right)^3$, one is further led to

$$\mathcal{E}_{\rm gpe}^{\rm star}(r) = -\frac{G\mathcal{M}_\odot^2}{2r}\left(\frac{\mathcal{L}_{\rm star}}{\mathcal{L}_\odot}\right)^{\frac{2}{3}}\left[1+\frac{\lambda_1 G\mathcal{M}_\odot}{rc^2}\left(\frac{\mathcal{L}_{\rm star}}{\mathcal{L}_\odot}\right)^{\frac{1}{3}}\right]. \tag{61}$$

If $\mathcal{M}_{\rm out}$ is the mass of the outflow at position r and $V_{\rm out}$ is the speed of this outflow at this position and $\mathcal{K}_{\rm out}$ is the kinetic energy, we know that

$$\left\langle\frac{d\mathcal{M}_{\rm out}(r)}{dt}\right\rangle = \frac{1}{V_{\rm out}^2}\frac{d\left[\mathcal{M}_{\rm out}(r)V_{\rm out}^2\right]}{dt} = \frac{2}{V_{\rm out}^2}\frac{d\mathcal{K}_{\rm out}}{dt}, \tag{62}$$

where the brackets $\langle ... \rangle$ tell us that we are looking at the average. Now if the gravitational energy $\mathcal{E}^{\rm star}_{\rm gpe}(r)$ is equal to the kinetic energy of the outflow, then from the above and coupled with what was said, one is led to

$$\left\langle \frac{d\mathcal{M}_{\rm out}(r)}{dt} \right\rangle = -\frac{\tau_G G\mathcal{M}^2_\odot V^{-2}_{\rm out}}{r}\left(\frac{\mathcal{L}_{\rm star}}{\mathcal{L}_\odot}\right)^{\frac{2}{3}}\left[1+\frac{2\lambda_1 G\mathcal{M}_\odot}{rc^2}\left(\frac{\mathcal{L}_{\rm star}}{\mathcal{L}_\odot}\right)^{\frac{1}{3}}\right], \qquad (63)$$

where $\tau_G = \dot{G}/G$ and $\dot{G}$ is the time derivative of Newton's gravitational constant. In the derivation of the above, we have considered only the first order terms and we have assumed that the gravitational constant is not a constant. Evidence that the gravitational constant may be changing exists, e.g. see Pitjeva (2005) and references therein. The ASTG model also points to a variation of the gravitational constant and the details of this are being worked out [4]; we give in the subsequent paragraphs how this comes about.

As it stands, the Poisson equation ($\boldsymbol{\nabla}^2\Phi = 4\pi G\rho$) for a time varying Φ and ρ is not in conformity with the Relativity Principle. According to our current understanding of physics and Nature, the seemingly sacrosanct Relativity Principle is a symmetry that every Law of Physics must fulfill. The Relativity Principle states that Laws of Physics must be independent of the observer's state of motion and of the coordinate system used to formulate them as well. If the Poisson equation is to be a Law of Nature, then it must successfully fulfill the Relativity Principle. This means that we must extend the Poisson equation to meet this requirement and the most natural and readily available way is

$$\boldsymbol{\nabla}^2\Phi - \frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2} = 4\pi G\rho, \qquad (64)$$

where t is the time coordinate. This equation satisfies the Relativity Principle simply because it emerges directly from Einstein's equation of the General Theory of Relativity (GTR). We know that Einstein's GTR, specifically the Law of Gravitation relating matter to the curvature of spacetime, does satisfy the Relativity Principle; hence Equation (64) also satisfies the Relativity Principle. This equation is what we are working out. We shall show that it leads to a time variable G. So, as will be shown in the near future, the time variable G in Equation (63) is not without a basis in physics.

Now from Equation (63), one sees that $\dot{\mathcal{M}}_{\rm out}(r) \propto V^{-2}_{\rm out}(r)\mathcal{L}^{2/3}_{\rm star}$. As stated in the introduction, observations show $\dot{\mathcal{M}}_{\rm out}(r) \propto V^{-1.8}_{\rm out}\mathcal{L}^{0.6}_{\rm star}$, which is close to what we have deduced here; this points to the fact that the reasoning leading to our deduction that $\dot{\mathcal{M}}_{\rm out}(r) \propto V^{-2}_{\rm out}(r)\mathcal{L}^{2/3}_{\rm star}$ may very well be on the right path of discovery. This clearly points to the need to look into these matters deeper than has been done here.

Clearly, from the above, a meticulous study of outflows should be able to measure the time variation in the gravitational constant G and this hinges on the correctness of the ASTG. This would require higher resolution observations to measure the mass outflow rate, $\dot{\mathcal{M}}_{\rm out}(r)$, at position r from the star and as well the speed of the outflow at that point and knowing the mass or luminosity of the central driving source, a graph of, e.g. $r\dot{\mathcal{M}}_{\rm out}(r)$ vs $V^{-2}_{\rm out}(r)\mathcal{L}^{2/3}_{\rm star}$, should in accordance with the ideas above, produce a straight line graph whose slope is $\tau_G G\mathcal{M}^2_\odot$. This kind of work, if it were possible, would help in independently confirming the measured time variation of Newton's constant of gravitation and it would act as further testing grounds for the falsification of the ASTG.

7 OUTFLOW ANATOMY

Briefly, we shall look into the anatomy of the outflow. We say "briefly" because each of the issues we shall look into requires a separate paper to fully address. First, before we do that, it is important

[4] We are at an advanced stage of preparation of this work and it will soon be archived on *viXra.org*: check Golden Gadzirayi Nyambuya's profile. Title of the Paper: *A Foundational Basis for Variable-G and Variable-c Theories.*

to find out when does the outflow switch-on and also when does it switch-off. Whether or not at some point in time in the evolution of a star outflows switch on and off is not a debatable issue. So, before we even look into them, it makes perfect sense to investigate this. From Figure 2, we see that the anatomy of the outflow has been identified with four regions, i.e. the *Inflow Region*, the *Outflow Feed Region*, the *Outflow Region* and the *Shock Ring*. After investigating the switching on and off of the outflow, we will look into the nature of these regions. Our analysis is qualitative rather than quantitative. We believe a quantitative analysis will require a fully-fledged numerical code. Work on this numerical code is underway.

7.1 Switching on Outflows

Let us call the loboid described by Equation (40) the outflow loboid and likewise the loboid described by Equation (42) the outflow feed loboid. From the preceding section, it is abundantly clear that we are going to have repulsive bipolar regions whose surface is described by a cone and an outflow loboid section. From this, we know that the maximum spatial extent of the repulsive gravitational field region will be given by the maximum spatial length of the lobes which occurs when: $\cos\theta = 1$, i.e. $l_{\rm max} = \epsilon_1^{\rm star}\mathcal{R}_{\rm star}$. Now, to ask the question when the outflow switches on amounts to asking when $l_{\rm max}$ is equal to the radius of the star, because the repulsive gravitational field will only be manifested beyond the surface of the star *if and only if* the maximum spatial extent of the region of repulsive gravitation is at least equal to the radius of the star, i.e. $l_{\rm max} \geq \mathcal{R}_{\rm star}$. This means $l_{\rm max} = \epsilon_1^{\rm star}\mathcal{R}_{\rm star}$; clearly, this will occur when $(\epsilon_1^{\rm star} = 1)$. Therefore, outflows will switch on when the condition $(\epsilon_1 = 1)$ is reached, otherwise when $(\epsilon_1^{\rm star} < 1)$, the repulsive gravitational field is confined inside the star.

This strongly suggests that if we are to use the ASGT to model outflows, then we must think of ϵ_1 (hence λ_1) as an evolutionary parameter of the star, i.e. this value starts from a given absolute minimum value (say $\epsilon_1^{\rm star} = 0$), and as the star evolves, this value gets larger and larger until such a time that the repulsive gravitational field is switched on when $(\epsilon_1^{\rm star} = 1)$, and thereafter it continues to grow, and as it grows, so does the spatial extent of the outflow (since this parameter controls the spatial size of the region with the repulsive gravitational field).

If the outflow switches on, as it must, the question is why it switches on at that moment when it switches on and not at any other moment. What is so special about that moment when it switches on and what triggers the outflows to switch on? As we have already argued, this special moment is when $(\epsilon_1^{\rm core} = 1)$ for a star that co-rotates with its parent core and $(\epsilon_1^{\rm star} = 1)$ for a star that rotates independently of its parent core. From Equations (39) and (47), this means we must have

$$\epsilon_1^{\rm core} = \left[\zeta\left(\frac{4\pi^2\mathcal{R}_{\rm core}^3}{G\mathcal{M}_{\rm core}\mathcal{T}_{\rm core}^2}\right)^{\zeta_0}\left(\frac{2G\mathcal{M}_{\rm core}}{c^2\mathcal{R}_{\rm core}}\right)\right]^{\frac{1}{2-\alpha_\rho}}\left(\frac{\mathcal{R}_{\rm core}}{\mathcal{R}_{\rm star}}\right) = 1, \tag{65}$$

and for a star that rotates independently of its core

$$\epsilon_1^{\rm star} = \zeta\left(\frac{4\pi^2\mathcal{R}_{\rm star}^3}{G\mathcal{M}_{\rm star}\mathcal{T}_{\rm star}^2}\right)^{\zeta_0}\left(\frac{2G\mathcal{M}_{\rm star}}{c^2\mathcal{R}_{\rm star}}\right) = 1, \tag{66}$$

where $(\mathcal{T}_{\rm core}, \mathcal{T}_{\rm star})$ are the periods of the spin of the core and the star respectively. If $\mathcal{T}_{\rm core}^{\rm on}$ is the period of the core's spin when the outflow switches on and $\mathcal{T}_{\rm star}^{\rm on}$ is the period of the spin when the outflow switches on, then, from the above equations, it follows that

$$\mathcal{T}_{\rm core}^{\rm on} = \left(\frac{\pi}{c}\right)\left[\zeta\left(\frac{2G\mathcal{M}_{\rm core}}{c^2}\right)^{1-\zeta_0}\left(\mathcal{R}_{\rm star}^{\rm on}\right)^{\alpha_\rho-1}\left(\mathcal{R}_{\rm core}^{\rm on}\right)^{3\zeta_0-\alpha_\rho+1}\right]^{\frac{1}{2\zeta_0}}, \tag{67}$$

$$\mathcal{T}_{\rm star}^{\rm on} = \left(\frac{\pi}{c}\right)\left[\zeta\left(\frac{2G\mathcal{M}_{\rm star}^{\rm on}}{c^2}\right)^{1-\zeta_0}\left(\mathcal{R}_{\rm star}^{\rm on}\right)^{3\zeta_0-1}\right]^{\frac{1}{2\zeta_0}}, \tag{68}$$

where $(\mathcal{M}_{\rm star}^{\rm on}, \mathcal{R}_{\rm star}^{\rm on}, \mathcal{R}_{\rm core}^{\rm on})$ are the mass and radius of the star and core at the time the outflow switches on respectively. From this, it follows that if the Sun were to spin on its axis once every 7.70 ± 0.40 hrs (i.e. $39.0 \pm 2.00\,\mu$Hz), the bipolar repulsive gravitational field must switch on and for the Earth, this condition would require it to spin once on its axis every 10.00 ± 2.00 min (i.e. 1.80 ± 0.50 mHz). If the above is correct, then the Earth must spin ~ 144 times its current spin in order to achieve the bipolar repulsive gravitational field while the Sun must spin ~ 5600 times its current spin rate in order to to achieve a bipolar repulsive gravitation. The spin rate of the Earth is far less than that needed to cause the bipolar repulsive gravitational to switch on, thus polar bears can smile knowing they will not fly off into space anytime soon.

Since we know that outflows are not always present, at some point in the evolution of the star, they must switch off. What could cause them to do so? Given the reality that within the outflow loboid, there is the outflow feed loboid, this too grows in size as the outflow loboid grows. At some point the outflow and the outflow that feeds the loboid will become equal, leaving the outflow with no feed point. At this point when the outflow and outflow feed loboids become equal, clearly, the outflow must switch off. This occurs when $l_{\max} = l_{\min}$ and from Equation (43) this means the condition for this to occur is $|\lambda_2| = 2\lambda_1^2/9$ and given that $\lambda_2 = -\lambda_1/96$, this means $\lambda_1^{\rm off} = 9/192$. From Equation (39), it follows that

$$\lambda_1^{\rm off} = \zeta\left(\frac{4\pi^2\mathcal{R}_{\rm star}^3}{G\mathcal{M}_{\rm star}\mathcal{T}_{\rm star}^2}\right)^{\zeta_0}\left(\frac{2G\mathcal{M}_{\rm star}}{c^2\mathcal{R}_{\rm star}}\right) = \frac{9}{192}, \tag{69}$$

which implies

$$\mathcal{T}_{\rm star}^{\rm off} = \left(\frac{192}{9}\right)^{\frac{1}{2\zeta_0}}\left(\frac{\pi}{c}\right)\left[\zeta\left(\frac{2G\mathcal{M}_{\rm star}^{\rm off}}{c^2}\right)^{1-\zeta_0}\left(\mathcal{R}_{\rm star}^{\rm off}\right)^{3\zeta_0-1}\right]^{\frac{1}{2\zeta_0}}, \tag{70}$$

where likewise $(\mathcal{M}_{\rm star}^{\rm off}, \mathcal{R}_{\rm star}^{\rm off})$ are the mass and the radius of the star at the time when the outflow switches off. We expect that $\mathcal{T}_{\rm star}^{\rm on} > \mathcal{T}_{\rm star}^{\rm off}$. If this is to hold, then

$$\left(\frac{\mathcal{M}_{\rm star}^{\rm off}}{\mathcal{M}_{\rm star}^{\rm on}}\right)^{1-\zeta_0}\left(\frac{\mathcal{R}_{\rm star}^{\rm off}}{\mathcal{R}_{\rm star}^{\rm on}}\right)^{3\zeta_0-1} < \left(\frac{9}{192}\right)^{\frac{2}{5}} = 0.30. \tag{71}$$

Hence, outflow activity will take place when $(\mathcal{T}_{\rm star}^{\rm off} \leq \mathcal{T}_{\rm star} \leq \mathcal{T}_{\rm star}^{\rm on})$. When $\mathcal{T}_{\rm star} = \mathcal{T}_{\rm star}^{\rm off}$, we have $\epsilon_1 = 9\mathcal{R}_{\rm star}^s/192\mathcal{R}_{\rm star}$. Using the approximate relation for an accreting star $\mathcal{R}_{\rm star} \sim 61\mathcal{R}_\odot\left(\mathcal{M}_{\rm star}/\mathcal{M}_\odot\right)$, one arrives at $\epsilon_1 = 3.32 \times 10^6$. This means $(\epsilon_1^{\rm on} = 1)$, and $\epsilon_1^{\rm off} = 3.32 \times 10^6$, where $\epsilon_1^{\rm on}$ and $\epsilon_1^{\rm off}$ are the values of ϵ_1 when the outflow switches on and off respectively, hence outflow activity will take place during the period which satisfies the condition

$$1 \leq \epsilon_1 < 3.32 \times 10^6. \tag{72}$$

The emerging picture is that $\mathcal{T}$ gets larger and larger as the star accretes more and more matter until a peak moment is reached (most probably when the star stops growing in mass) where the spin begins to slow down. In the process of slowing down, the inner cavity inside the lobe of the outflow is created. This inner cavity grows bigger and bigger as the star's spin slows down, until such a time when the spatial dimensions of this cavity are equal to the outflow lobe itself. Once this state is attained, the outflow switches off because the growing cavity has consumed from within all the outflow region.

Clearly, the above picture suggests that the spin of a star is what controls outflows. At some specific state, the outflow switches on; it evolves to some peak spin-value, and thereafter, its spin slows down. This means that during the outflow process after it begins to slow down, the star loses some spin angular momentum. This idea resonates with the long held suggestion discussed earlier that outflows are thought to exist as one means to tame the spin angular momentum of a star (see e.g. Larson 2003b). We will not go deeper than this in our analysis. The aim has been to show that the emergent picture of outflows from the ASTG is capable (in principle) of answering such questions. This means in a future study, these are the things to look forward to.

7.2 Outflow Feed Region

In the Outflow Feed Region, i.e. the region in Figure 2 described OEF and OGH, clearly, any material that enters this region is going to be channeled into the Outflow Region because the repulsive radial component of the gravitational field (aided by the radiation field) is going to channel this matter radially outward while the azimuthal component is going to channel this outward radially moving material toward the spin axis, hence it is expected that most of the matter will enter the Outflow Region along the spin axis of the star. It is important to state that no matter how much radiation is emitted from the star, there will be no reversal of in-falling matter outside the region of repulsive gravitation due to the radiation field of the nascent star. We shall discuss this in Section 8.

7.3 Outflow Region

The Outflow Region is comprised of a section of a cone (OAB and OCD), the outflow loboid minus the Outflow Feed Region. In this Outflow Region, the gravitational force is both radially and azimuthally repulsive, i.e. $(g_\theta > 0)$ and $(g_r > 0)$. This means once the repulsive gravitational force is switched on and it is in a fully fledged phase, all material found in this region is going to be channeled out of this region radially along with most of the matter concentrated along the spin axis. The material will be concentrated along the spin axis because the repulsive azimuthal gravitational component will channel the matter toward the spin axis. The repulsive radial component pushes the material out radially, while the repulsive azimuthal component of the gravitational force draws this material closer to the spin axis. Hence, the bulk of the outflow material must be found along the edge of the spinning axis.

Where the cone meets the outflow loboid, i.e. along AB and CD, there will be rings. Consider the ring AB. It is clear that this ring (as CD) must be a shock front since on this ring, along the radial line OA, the in-coming material will meet the outgoing material with equal but opposite radial forces. These equal and but opposite forces must create (radially) a stationery shock. This shock is going to have a ring structure; let us call this the *Shock Ring*. Like the rings AB and CD, the structures EF and GH will be rings too, but not shock things. These rings EF and GH are the mouth of the outflow and matter enters into the outflow region via this opening.

7.4 Shock Rings and Methanol Masers

Given that (1) AB and CD are shock rings, (2) that methanol masers (among other pumping mechanisms) are thought to arise in shock regions and (3) the observations of Bartkiewicz et al. (2005) were correct, where these authors discovered a ring distribution of $6.7\,$GHz methanol masers, it is logical to assume that this shock ring may well be a hub of methanol masers arising from the shock present in this ring. Recent and further work by these authors strongly suggests that a Ring of Masers is a natural occurrence in star forming regions as Bartkiewicz et al. (2009) observed.

This ring distribution of maser components, they believe, strongly suggests the existence of a central source. This is the case here; the central source must exist and it is the forming star. They found an infrared object coinciding with the center of the ring of masers within 78 mas and this source is cataloged in the 2MASS survey as 2MASS183451.56–08182114. They believe that this is an evolving protostar driving this maser via circular shocks. This is in line with the present context. Very strongly, the Bartkiewicz Ring of Masers suggests, in our opinion, that our outflow model may very well contain an element of truth, that our model contains the possible seeds of resolution of this puzzling occurrence of Ring Masers.

About this shock ring, when viewed from the projection as shown in Figure 2, the distance of the shock ring from the star will be

$$l_{\rm sh}=l_{\max}(3)^{-\frac{3-\alpha_\rho}{4-2\alpha_\rho}}, \tag{73}$$

and the radius of this shock ring will be

$$\mathcal{R}_{\rm ring}=l_{\max}(1.5)^{-\frac{3-\alpha_\rho}{4-2\alpha_\rho}}. \tag{74}$$

Clearly, for an isolated system, depending on the orientation relative to the observer, this ring can appear as a linear structure, or a circular or elliptical ring.

At present, more than 500 6.7 GHz methanol masers sources are known to exist (Malyshev & Sobolev 2003; Pestalozzi et al. 2005; Xu et al. 2003) and are associated with a very early evolutionary phase of high mass star formation. The methanol maser emitting at the 6.7 GHz frequency first discovered by Menten (1991) is the second strongest centimeter masing transition of any molecule (after the 22 GHz water transition) and is commonly found toward star formation regions. It is typically stronger than 12.2 GHz methanol masers (discovered by Batrla et al. 1987) observed toward the same region. Methanol masers have become well established tracers or sign spots of high mass star formation regions. It is thought that methanol masers occur in the very early stages of massive star formation.

While methanol masers are found in regions of massive star formation, some have been found with no associated high mass star formation activity (see e.g. Ellingsen et al. 1996, Szymczak et al. 2002). Besides this non-association, some methanol masers are and have been observed to exist in close spatial proximity to massive stars. This has lead to the classification of methanol masers into Class I and Class II. Class I masers emit at the frequencies 25.0, 44.0, 36.0 GHz, etc., while class II methanol masers emit at 6.7, 12.2, 157.0 GHz, etc. Class I methanol masers are often observed to exist apart from the continuum sources, while Class II masers are observed to exist very close, albeit, both classes often co-exist in the same star forming region inside an HII region. Clearly, $l_{\rm sh}=l_{\rm sh}(t)$ and $\mathcal{R}_{\rm ring}=\mathcal{R}_{\rm ring}(t)$ and as the star evolves, $l_{\rm sh}$ and $\mathcal{R}_{\rm ring}$ get larger. This means in the case of young stars, if this ring is a hub of methanol masers, it is expected that methanol masers will be found closer to the star for young HMS and likewise, for more evolved massive stars, methanol masers will be found further from the nascent star. If this is correct, then it may explain the aforesaid; why are Class II methanol masers mostly found close to a nascent star and why are Class I methanol masers mostly found further from the nascent star.

High resolution imaging of the 6.7 and 12.2 GHz methanol masers has found that many cases exhibit simple elongated linear or curved spatial morphologies (Norris et al. 1998; Norris et al. 1993; Minier et al. 2000) and as already stated, depending on the orientation of the observer relative to the star forming system, the ring may appear as a linear structure. These linear structures have lengths of 50 to 1300 AU. Because of this, one of the possible interpretations that has been entertained for some time is that the masers originate in the circumstellar accretion disk surrounding the newly formed star (Edris et al. 2005) and besides this they have a strong association with outflows (see e.g. Plambeck & Menten 1990; Kalenskii et al. 1992; Bachiller et al. 1995; Johnston et al. 1992). Other than originating from the circumstellar disk, it has also been entertained that a methanol maser may originate from outflows (see e.g. Pratap & Menten 1992; de Buizer et al. 2000). Clearly, the outflow origin of methanol masers resonates with the present ideas. If the ideas herein are correct, then this paper would be of value to researchers seeking an outflow origin of methanol masers.

Further, if viewed from the same view as in Figure 2, and if as argued above that masers are found on the ring, one will expect to observe a linear alignment of masers above and below the the nascent star. This would explain the observed linear alignment of methanol masers and also the observed linear alignment of masers above and below the IRAS source found in molecular cloud G69.489-0.785 (see Fish 2007). Given Fish's observations of blue and red-shifted masers in the ON1-region (Fish 2007), the suggested model of this ring of masers is interesting as it may offer an explanation of these unexplained and puzzling red and blue-shifted masers at opposite sides of the IRAS source associated with ON1.

7.5 Collimation Factor

We can calculate the collimation factor of the outflow since we know the extent ($l_{\rm max}$) and the breath of the outflow which is the size of the shock rings, i.e. the collimation factor could be $q_{\rm col} = \mathcal{R}_{\rm ring}/l_{\rm max}$, which can also be written as

$$q_{\rm col} = (1.5)^{\frac{3-\alpha_\rho}{4-2\alpha_\rho}}, \tag{75}$$

(this has been deduced from Eq. (74)). Now, it is believed that the most stable density profile is one with a density index ($\alpha_\rho = 2$). This means that molecular clouds in a state different from this density profile will tend to be attracted to the $\alpha_\rho = 2$ state. Using this assumption, we see that as $(\alpha_\rho \longmapsto 2)$ from $(\alpha_\rho = 0)$, i.e. $(\alpha_\rho\text{: } 0 \longmapsto 2)$, then we will have $(q_{\rm col} \longmapsto \infty)$. For this setting, generally $(q_{\rm col} > 1)$. We also realize that now as $(q_{\rm col} \longmapsto \infty)$, when $(\alpha_\rho\text{: } 3 \longmapsto 2)$, then $(q_{\rm col} > 1)$, and if $(\alpha_\rho\text{: } 0 \longmapsto 2)$, then generally $(q_{\rm col} \geq 1.36)$. This means that we are going to have two categories of collimation factor values, i.e. $(1 < q_{\rm col} < 1.36)$ for $(\alpha_\rho\text{: } 0 \longmapsto 2)$ and $(q_{\rm col} \geq 1.36)$ for $(\alpha_\rho\text{: } 3 \longmapsto 2)$. Because of projection effects, it is very difficult to measure the true collimation factor.

Also, because of projection effects, the collimation factor that we measure in real life is not the actual collimation factor but the projected one. If we know the actual collimation factor, we will be able to know the density index since from Equation (75) we can deduce that

$$\alpha_\rho = 2 - \left(\frac{\log q_{\rm col}^2}{\log 1.5} - 1\right)^{-1} = \frac{\log(q_{\rm col}^4/8)}{\log(1.5)}. \tag{76}$$

LMSs are known to have relatively low outflow collimation factors ($q_{\rm col} < 4$) while HMSs have significantly higher outflow collimation factors ($2 < q_{\rm col} < 10$), sometimes reaching $q_{\rm col} \sim 20$. From Equation (76) the aforesaid implies, assuming these collimation factors are a good representation of the real collimation factor, that LMSs cores have density index $\alpha_\rho = 1.56$ and HMS cores have density index $\alpha_\rho = 1.98$. This is not unreasonable, but rather very much expected. The fact that for HMS forming cores, we have $\alpha_\rho = 1.98$ and for LMS forming cores we have $\alpha_\rho = 1.56$ means that HMS cores are much denser compared to LMS forming cores.

8 RADIATION PROBLEM

While the main thrust and focus of this paper is not on the *Radiation Problem* associated with massive stars, but on the polar repulsive gravitational field and its possible association with the observed bipolar molecular outflows, we find that the ASTG affords us a window of opportunity to visit this problem. This so-called radiation problem associated with massive stars has been well articulated in Paper II. There is no need for us to go through the details of this same problem here, but we shall direct the reader to Paper II for an exposition of the radiation problem. In the subsequent paragraphs, we shall, for the sake of achieving a smooth continuous paper, present the findings of Paper II in nutshell.

In general, a massive star is defined to be one with mass greater than $\sim$ 8–10 $\mathcal{M}_\odot$ and central to the ongoing debate on how these objects (massive stars) come into being is this so-called radiation problem. For nearly forty years, it has been argued that the radiation field emanating from massive stars is high enough to cause a global reversal of direct radial in-fall of material onto the nascent star. In Paper II, it is argued that only in the case of a non-spinning isolated star does the gravitational field of the nascent star overcome the radiation field. An isolated non-spinning star is a non-spinning star without any circumstellar material around it, and the gravitational field beyond its surface is described exactly by Newton's inverse square law. The supposed fact that massive stars have a gravitational field that is much stronger than their radiation field is drawn from the analysis of a non-spinning isolated massive star. In this case, the gravitational field is (correctly) much stronger than the radiation field. This conclusion has been erroneously extended to the case of non-spinning massive stars enshrouded in gas and dust.

It is argued there, in Paper II, for the case of a non-spinning gravitating body where the circumstellar material is taken into consideration, that at $\sim 8-10\,\mathcal{M}_\odot$, the radiation field will not reverse the radial in-fall of matter, but rather a stalemate between the radiation and gravitational field will be achieved, i.e. in-fall is halted but not reversed. Any further mass growth is stymied and the star's mass stays constant at $\sim 8-10\,\mathcal{M}_\odot$. This picture is very different from the common picture that is projected and accepted in the wider literature where at $\sim 8-10\,\mathcal{M}_\odot$, all the circumstellar material, from the surface of the star right up to the edge of the molecular core, is expected to be swept away by the all-marauding and pillaging radiation field. There in Paper II, it is argued that massive stars should be able to start their normal stellar processes if the molecular core from which they form has some rotation, because a rotating core exhibits an ASGF which causes there to be an accretion disk and along this disk the radiation is not powerful enough to pillage the in-falling material. We show here that in the region (θ: $[125.3 < \theta < 54.7]$ and $[234.7 < \theta < 305.3]$) around a spinning star the gravitational field in the face of the radiation field will never be overcome by the radiation field, hence in-fall reversal does not take place in this region and this region is the region via which the massive nascent star forms once the repulsive outflow field and the star's mass have surpassed the critical $8-10\,\mathcal{M}_\odot$. Reiterating, in this region, i.e. (θ: $[125.3 < \theta < 54.7]$ and $[234.7 < \theta < 305.3]$), infall is never halted but continues unaborted and unabated.

There are three cases of an embedded spinning nascent star (1) Where the nascent star is spinning and the circumstellar material is not spinning or where the spin of the circumstellar material is so small compared to the star that the circumstellar material can be considered not to be spinning. (2) Where the nascent star is spinning independently of the circumstellar material which is itself spinning. (3) Where the nascent star is co-spinning or co-rotating with the circumstellar material. It should suffice to consider one case because the procedure to show that in the region (θ: $[305.3 < \theta < 54.7]$ and $[234.7 < \theta < 125.3]$), infall is never halted but continues unaborted and unabated, is the same. Of the three cases stated, the most likely scenario in Nature is the second case, i.e. where the nascent star is spinning independently of the circumstellar material which is itself spinning. We shall consider this case.

The ASGP for the case of a star that is spinning independently of its core has be argued to be given by Equation (58) and in the face of the radiation field, the resultant radial component of the gravitational field intensity is given by

$$g_r(r,\theta) = -\frac{G\mathcal{M}_0^{\rm eff}}{r^2}\left[1 - \frac{\kappa\mathcal{L}_{\rm star}}{4\pi G\mathcal{M}_0^{\rm eff}c} + \frac{2\lambda_1^{\rm star}\gamma_1 G\mathcal{M}_1^{\rm eff}\cos\theta}{rc^2} + 3\lambda_2^{\rm star}\gamma_2\left(\frac{G\mathcal{M}_2^{\rm eff}}{rc^2}\right)^2\left(\frac{3\cos^2\theta-1}{2}\right)\right]. \tag{77}$$

For the radiation component to be attractive, we must have $[g_r(r,\theta) < 0]$, and for this to be so, the term in the square brackets must be greater than zero, which implies

$$\left[1 - \frac{\kappa\mathcal{L}_{\rm star}}{4\pi G\mathcal{M}_0^{\rm eff}c}\right]r^2 + \left[\frac{2\lambda_1^{\rm star}\gamma_1 G\mathcal{M}_1^{\rm eff}\cos\theta}{c^2}\right]r + \left[3\lambda_2^{\rm star}\gamma_2\left(\frac{G\mathcal{M}_2^{\rm eff}}{c^2}\right)^2\left(\frac{3\cos^2\theta-1}{2}\right)\right] > 0. \tag{78}$$

This inequality is quadratic in r and can be written as $(Ar^2 + Br + C > 0)$, where A, B, and C can easily be obtained by making a comparison. Since $(Br > 0)$, for $(Ar^2 + Br + C > 0)$ to hold absolutely, [5] we must have $(Ar^2 > 0 \Rightarrow A > 0)$ and $(C > 0)$. The condition $(A > 0)$ implies

$$\mathcal{M}(r) > \frac{\kappa_{\rm eff}\mathcal{L}_{\rm star}}{4\pi Gc}. \tag{79}$$

To arrive at the above, one must remember that $\mathcal{M}_0^{\rm eff} = \mathcal{M}_{\rm star} + \mathcal{M}_{\rm csl}(r) = \mathcal{M}(r)$ (see Eq. (5) and Sect. 5), the condition (79) for $\mathcal{M}_{\rm star} > 8 - 10\,\mathcal{M}_{\odot}$, leads to the formation of a cavity inside the star forming core. In this cavity, the radiation field is powerful enough to halt infall reversal but outside, it is not.

Now, for the condition $(C > 0)$ to hold (remember $\lambda_2^{\rm star} < 0$), this means $(3\cos^2\theta - 1 < 0)$, hence (θ: $[125.3 < \theta < 54.7]$ and $[234.7 < \theta < 305.3]$). The result just obtained invariably means that inside the cavity created by the radiation field, the region (θ: $[125.3 < \theta < 54.7]$ and $[234.7 < \theta < 305.3]$) will have an attractive gravitational field, hence matter will still be able to fall onto the nascent star via this region and this in-falling of matter is completely independent of the opacity of the material of the core! Hence we expect spinning massive stars to not face any radiation problem at all. Clearly, if $(\lambda_2 > 0)$, then in the region (θ: $[125.3 < \theta < 54.7]$ and $[234.7 < \theta < 305.3]$), the gravitational field was going to cause in-fall reversal in the cavity, disallowing the star to continue its accretion. This would obviously have been at odds with experience, thus we have the strongest reason for setting $(\lambda_2 > 0)$, otherwise the ASTG would be seriously at odds with physical and natural reality as we know it. Besides, the condition $(\lambda_2 > 0)$ is supported by the solar data (see Paper I). The fact that in the regions (θ: $[125.3 < \theta < 54.7]$ and $[234.7 < \theta < 305.3]$), which are regions of attractive gravitation, it is clear that the ASGF will form a disk around the nascent star. Although no detailed study of accretion disks has been made (Brogan et al. 2007; Araya et al. 2008) and this is due to technological challenges in obtaining much higher resolution observations on the scale of these accretion disks, it has long been thought that the accretion disk is a means by which accretion of matter on the nascent stars continues soon after radiation has (significantly) sounded its presence on the star formation podium (see e.g. Chini et al. 2004; Beltr án et al. 2004). If our investigations are proved correct, as we believe they will be, then researchers have been right to think that the accretion disk serves as a platform for further accretion of mass by the nascent star.

9 DISCUSSION AND CONCLUSIONS

This paper should be taken more as a genesis that lays down the mathematical foundations that seek to lead to the resolution of the problem of outflows, vis what their origin is. Also, we should say that, if this paper is anything to go by, i.e. if it proves itself to have a real direct correspondence with the experience of physical reality, then not only have we laid down the mathematical foundations that may lead to the understanding of outflows, but we have also laid down a three fold foundation that could lead to the resolution of three problems, and these problems are

(1) The origins and nature of outflows;
(2) The radiation problem thought to exist for HMS;
(3) The origin of linear and ring structures of methanol masers.

We have arrived at all this after the consideration of the azimuthal symmetry arising from the spin of a gravitating body. This symmetry has been applied to the gravitational field and there upon we have come up with the ASTG. In Paper I, we did show that the ASTG can explain the perihelion shift of planets in the solar system and therein, the ASTG as it lays there, suffers the setback that

[5] In Paper I, we argued that r can take both negative and positive values, and further argued that the set up of the coordinate system of the ASGF is such that $[r > 0$ and $\cos\theta > 0]$ and $[r < 0$ and $\cos\theta < 0]$, hence $r\cos\theta > 0$, which implies $(Br > 0)$.

the "constants" λ_ℓ are unknown. We have gone so far in the present as to suggest a way to solve this problem but this suggestion is subject to revision pending any new data.

It should be said that, to the best of what we can remember, ever since we learned that the force of gravity is what causes an apple to fall to the ground and that the very same force causes the Moon and the planets to stay in their orbits, we have never really been convinced of gravitation as being a repulsive force, let alone that it possibly can have anything to do with the power behind outflows. Just as anyone would find these ideas in violation of their intuition, we find ourselves in the same situation. However, one thing is clear, which is the picture emerging from the associated mathematics is hard to dismiss. It calls one to take a closer look at what the Poisson equation is "saying to us."

In closing, allow me to say that as things stand in the present, while we firmly believe we have discovered something worthwhile, it is difficult to make any bold conclusions. Perhaps we should only mention that work has begun on a numerical model of outflows based on what we have discovered herein. Only then, we believe, will it be possible to make any bold conclusions.

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