LETTERS

Does a low solar cycle minimum hint at a weak upcoming cycle? *

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Abstract The maximum amplitude (R_{m}) of a solar cycle, in the term of mean sunspot numbers, is well-known to be positively correlated with the preceding minimum (R_{min}). So far as the long term trend is concerned, a low level of R_{min} tends to be followed by a weak R_{m}, and vice versa. We found that the evidence is insufficient to infer a very weak Cycle 24 from the very low R_{min} in the preceding cycle. This is concluded by analyzing the correlation in the temporal variations of parameters for two successive cycles.

Key words: Sun: activity — Sun: general — sunspots

1 INTRODUCTION

Studying the correlation between the maximum amplitude (R_{m}) of a solar cycle and the preceding minimum (R_{min}) is useful for understanding the long-term evolution of solar activity. This can provide information about the activity level of an ensuing cycle. The positive correlation between R_{m} and R_{min} is a well-known fact (Hathaway et al. 2002). As a natural consequence, the very low level of solar activity at the present time (around the onset of Cycle 24) seems to be followed by a very weak (Svalgaard et al. 2005; Schatten 2005), or even the weakest cycle (Li 2009). However, a lower R_{min} has not always been followed by a weaker cycle. For example, a small R_{min} precedes the greatest R_{m} in Cycle 19 (Wang & Sheeley 2009). Therefore, what information we can infer from the preceding minimum is worth re-analyzing carefully. We ask whether and how past cycles affect the present cycle.

A more accurate prediction of solar activity is an important task in solar physics and space weather. Knowing in advance the activity level of an upcoming cycle is helpful in the launching and operation of spacecrafts. An underestimate of the activity level for the next cycle may let down our guard. One aim of this study serves to remind the space flight mission planners that they still need to remain vigilant to avoid unexpected troubles.

In the present study, we use the 13-month running mean of Zürich relative sunspot number[1] from 1749 January to 2010 April to determine the maximum (R_{m}) and the preceding minimum (R_{min}) of the solar cycle. The correlation between R_{m} and R_{min} for different periods of time is shown in Section 2. In Section 3, we examine the varying trends of R_{m} and R_{min} using a quantity to describe

[1] *http://www.ngdc.noaa.gov/stp/SOLAR/ftpsunspotnumber.html*

whether a parameter increases or decreases. Then, in Section 4, we analyze the temporal variation in the correlation coefficient between R_{m} and $R_{\min}$ with a moving time window of five cycles. The results are briefly discussed and summarized in Section 5.

2 CORRELATION BETWEEN R_{m} AND $R_{\min}$

The parameters of R_{m} and $R_{\min}$ since cycle $n = 1$ are listed in Table 1, and shown in Figure 1(a).

Table 1 Cycle Maximum (R_{m}), Minimum ($R_{\min}$) and Minimum aa Index ($aa_{\min}$) and their Trends (V)

n	$R_{\mathrm{m}}(V)$	$R_{\min}(V)$	$aa_{\min}(V)$	n	$R_{\mathrm{m}}(V)$	$R_{\min}(V)$	$aa_{\min}(V)$
1	86.5	8.4		13	87.9 (+)	5.0 (+)	10.6 (+)
2	115.8 (+)	11.2 (+)		14	64.2 (−)	2.7 (−)	5.9 (−)
3	158.5 (+)	7.2 (−)		15	105.4 (+)	1.5 (−)	8.2 (+)
4	141.2 (−)	9.5 (+)		16	78.1 (−)	5.6 (+)	9.4 (+)
5	49.2 (−)	3.2 (−)		17	119.2 (+)	3.5 (−)	13.2 (+)
6	48.7 (−)	0.0 (−)		18	151.8 (+)	7.7 (+)	16.3 (+)
7	71.5 (+)	0.1 (+)		19	201.3 (+)	3.4 (−)	16.9 (+)
8	146.9 (+)	7.3 (+)		20	110.6 (−)	9.6 (+)	13.8 (−)
9	131.9 (−)	10.6 (+)	14.1	21	164.5 (+)	12.2 (+)	17.2 (+)
10	98.0 (−)	3.2 (−)	10.3 (−)	22	158.5 (−)	12.3 (+)	17.5 (+)
11	140.3 (+)	5.2 (+)	16.0 (+)	23	120.8 (−)	8.0 (−)	15.9 (−)
12	74.6 (−)	2.2 (−)	6.7 (−)	24	? (?)	1.7 (−)	8.4 (−)
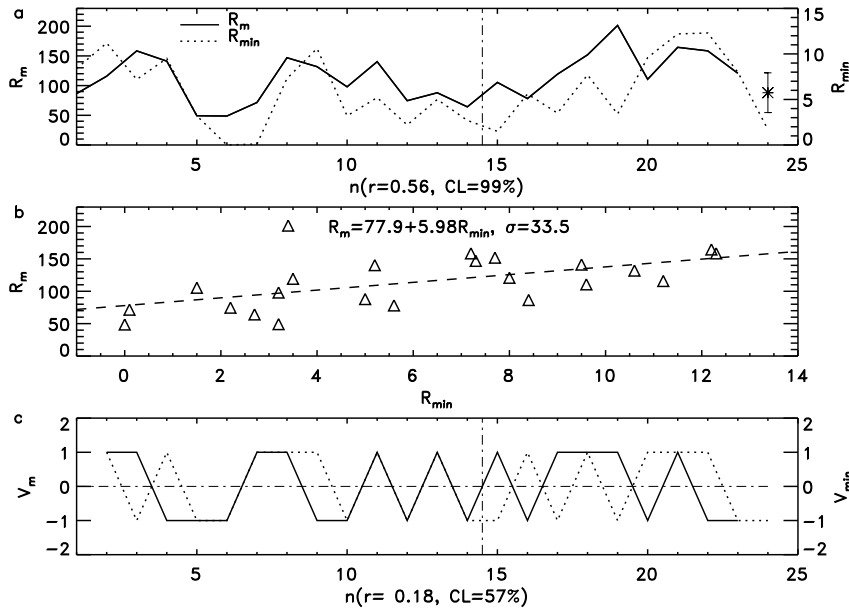
Fig. 1 (a) R_{m} (*solid line*) and $R_{\min}$ (*dotted line*) since cycle $n = 1$; (b) Scatter plot of R_{m} against $R_{\min}$ (*triangles*); (c) Varying trend: V_{m} (*solid line*) and $V_{\min}$ (*dotted line*).

It can be seen from Figure 1(a) that R_m and R_min have a similar long-term variation behavior: a lower (higher) level of R_min tends to be followed by a weaker (stronger) R_m. Their correlation coefficient is $r = 0.56$ at a confidence level (CL) of 99%. The scatter plot of R_m against R_min (triangles) is shown in Figure 1(b). Their least-squares-fit linear regression equation is

$$R_\mathrm{m} = 77.9 + 5.98 R_\mathrm{min}, \quad \sigma = 33.5, \tag{1}$$

where σ is the standard deviation of the equation. Substituting the present value of R_min (1.7) into this equation, the peak of Cycle 24 is predicted to be $R_\mathrm{m}(24) = 88.0 \pm 33.5$ (labeled by an asterisk), which is related to the 1-σ uncertainty. When using the modern era data since Cycle 10, the peak of Cycle 24 is predicted to be higher, as $R_\mathrm{m}(24) = 98.4 \pm 35.0$. When using only the most recent nine cycles since Cycle 15, an even higher value of $R_\mathrm{m}(24) = 122.3 \pm 36.5$ will be predicted for Cycle 24.

However, Li (2009) inferred a rather low level for Cycle 24, $R_\mathrm{m}(24) = 58.0 \pm 26.6$, from a relationship of $R_\mathrm{m} = 48.8 + 5.39 R_\mathrm{min} \pm 26.6$ derived by Hathaway et al. (2002). The data used to derive this relationship are those that are smoothed with the 24-month Gaussian filter, rather than the 'standard' 13-month running mean. So, the present minimum value ($R_\mathrm{min} = 1.7$) of the 13-month running mean sunspot number is inappropriate for use in inferring an $R_\mathrm{m}(24)$ value from this relationship. Besides, the R_min value in terms of the 24-month Gaussian filter is unknown within 12 months of the minimum. (Even if this value were known, the result inferred from the above relationship has a different meaning.)

We return to Figure 1(a). If using only the parameters in the earlier cycles of $n = 1$–14 (left of the vertical line in Fig. 1a), the correlation coefficient between R_m and R_min increases to $r(1$–$14) = 0.72$ at the 99% level of confidence. In contrast, for the recent cycles of $n = 15$–23, the correlation coefficient is only $r(15$–$23) = 0.23$, which is statistically insignificant (CL $< 50\%$). Therefore, the positive correlation between R_m and R_min (0.56) is mostly contributed by the earlier cycles. The recent cycles, especially for cycles 15–19, seem to behave differently from the earlier cycles. It is then necessary to analyze whether the temporal variation in the correlation affects the future R_m value.

3 TRENDS OF VARIATIONS IN R_m AND R_min

It should be noted in Figure 1(a) that, for an individual cycle, the increase or decrease of R_m does not always follow that of R_min. For example, R_min decreases while R_m increases from $n = 18$ to 19. To demonstrate the behavior of increasing or decreasing, we define the varying trends of R_m and R_min as

$$\begin{aligned} V_\mathrm{m}(n) &= \mathrm{Sgn}(R_\mathrm{m}(n) - R_\mathrm{m}(n-1)), \\ V_\mathrm{min}(n) &= \mathrm{Sgn}(R_\mathrm{min}(n) - R_\mathrm{min}(n-1)), \end{aligned} \tag{2}$$

where $y = \mathrm{Sgn}(x)$ is the sign function: $y = 1$ if $x > 0$, $y = -1$ if $x < 0$ and $y = 0$ if $x = 0$. $V_\mathrm{m}(n) = +1$ refers to an increase in $R_\mathrm{m}(n) > R_\mathrm{m}(n-1)$, and $V_\mathrm{m}(n) = -1$ refers to a decrease in $R_\mathrm{m}(n) < R_\mathrm{m}(n-1)$, and so on. $V_\mathrm{m}(n) = V_\mathrm{min}(n)$ refers to the same trend of $R_\mathrm{m}(n)$ and $R_\mathrm{min}(n)$. The values of $V_\mathrm{m}(n)$ and $V_\mathrm{min}(n)$ are listed in Table 1 and shown in Figure 1(c). For all cycles $n = 2$–23, there are 13(9) pairs of $V_\mathrm{m}(n)$ and $V_\mathrm{min}(n)$ with the same(opposite) trends. Their correlation is very weak, at $r = 0.18$, and statistically insignificant (CL $= 57\%$).

For the earlier cycles of $n = 2$–14, there are 10(3) pairs of $V_\mathrm{m}(n)$ and $V_\mathrm{min}(n)$ with the same(opposite) trends, and their correlation coefficient is $r_V(2\text{-}14) = 0.55$ at the 95% level of confidence. In contrast, for the recent cycles of $n = 15$–23, there are 3(6) pairs with the same(opposite) trends, and their correlation coefficient becomes negative, at $r_V(2 - 14) = -0.35$, at the 63% level of confidence. If we consider only the case of $V_\mathrm{min}(n) = -1$, as for $n = 3, 5, 6, 10, 12, 14, 15, 17, 19$, and 23, there are 6(4) pairs with the same(opposite) trends. For the earlier cycles of $n \le 14$,

there are 5(1) pairs for $V_{\min}(n) = -1$ with the same(opposite) trends, while for the recent cycles of $n \geq 15$, there are 1(3) pairs for $V_{\min}(n) = -1$ with the same(opposite) trends. This implies that a decrease in $R_{\min}$ is indeed followed by a decrease in R_{m} in most of the earlier cycles, while in the recent cycles, a decrease of $R_{\min}$ tends to be followed by an increase of R_{m}.

In summary, in terms of the varying trend, there is no statistically significant positive correlation between V_{m} and $V_{\min}$ for all cycles ($r = 0.18$). The positive correlation between V_{m} and $V_{\min}$ exists only in the earlier cycles ($r_V(2\text{–}14) = 0.55$) at the 95% level of confidence. Concerning the recent cycles, this correlation becomes negative ($r_V(2\text{–}14) = -0.35$). The behavior of the varying trend changed in recent cycles. Therefore, we cannot infer a decrease of R_{m} from a decrease of $R_{\min}$ for Cycle 24.

4 TEMPORAL VARIATION IN THE RUNNING CORRELATION

In the previous two sections, one has noted that the correlation between R_{m} and $R_{\min}$ behaves differently for different periods of time. Now, we analyze the temporal variation in the running correlation with a moving time window of $w = 5$ cycles. For each cycle n, we calculate the correlation coefficient between $R_{\mathrm{m}}(i)$ and $R_{\min}(i)$ for $i = n - 2, n - 1, \ldots, n + 2$ (Du et al. 2009a), denoted by $r(5, n)$. The results are shown in Figure 2(a).

It can be seen that the correlation is positive before $n = 13$ and significant at the 90% level of confidence for cycles $n = 5$–9 (asterisks). This implies that a lower (higher) level of $R_{\min}$ tends to be followed by a weaker (stronger) R_{m} for these earlier cycles. However, the correlation decreases after $n = 14$, and becomes negative after $n = 18$, implying that a lower $R_{\min}$ corresponds to a
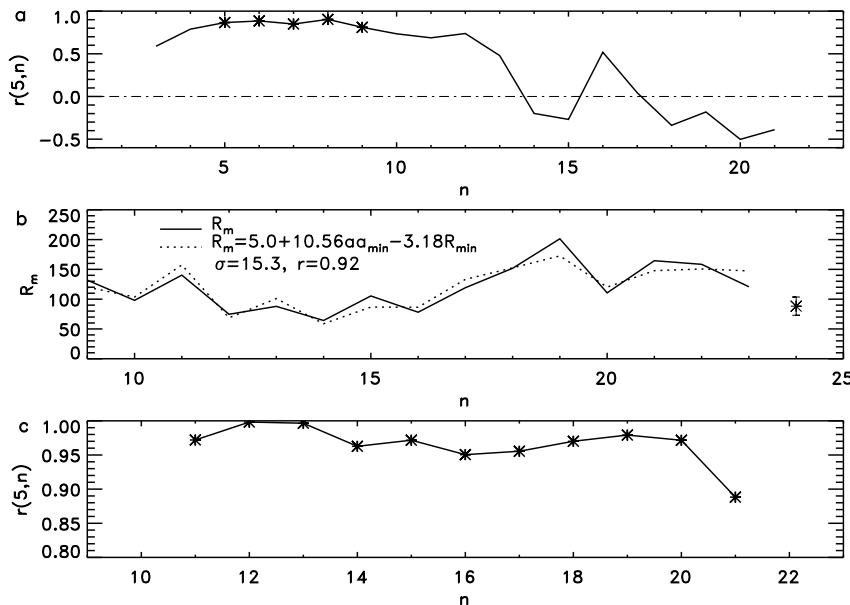
Fig. 2 (a) Running correlation coefficient between R_{m} and $R_{\min}$ for a 5-cycle moving window. Asterisks indicate that the relevant values are significant at the 90% level of confidence; (b) R_{m} (*solid line*) versus the fitted value (*dotted line*) from Eq. (3); (c) Running correlation coefficient of R_{m} with both $aa_{\min}$ and $R_{\min}$ for a 5-cycle moving window.

stronger R_m (see also $n = 15$, 17 and 19 in Fig. 1a). Therefore, a lower R_{min} has not always been followed by a weaker R_m. In other words, we cannot infer a very weak R_m of Cycle 24 from the preceding very low level of R_{min}.

5 DISCUSSION AND CONCLUSIONS

It is well known that the maximum amplitude (R_m) of a solar cycle is positively correlated with the preceding minimum (R_{min}), so that a low R_{min} tends to be followed by a weak R_m. However, this relationship is not always effective for individual cycles (Wang & Sheeley 2009), especially for the recent cycles, as shown in Figure 2(a). The correlation between R_m and R_{min} varies with time (n).

We analyzed the temporal behavior of this correlation and the varying trends (V) of R_m and R_{min}. In the recent cycles, they all show a negative correlation. Since the prediction of R_m relies more on the recent cycle rather than on the past cycles (Schatten 2005; Svalgaard et al. 2005; Du et al. 2008, 2009b), the negative correlation in the recent cycles cannot infer a very weak R_m from a very low R_{min}.

One may argue that R_m and R_{min} have a similar shape in the most recent four cycles of $n = 20$–23. Along with the developing trend of these cycles, $R_m(24)$ should be very small. However, whether this behavior holds true is questionable before and after these cycles. It should be noted in Figure 1(a) that R_m has never decreased in three successive cycles. The R_m value decreased two cycles from $n = 3$ to 5, and then leveled off to $n = 6$, and decreased two cycles from $n = 8$ to 10, and then increased to $n = 11$. Now that the R_m value decreased two cycles from $n = 21$ to 23, it seems to increase or level off according to its past behavior. On the other hand, $R_{min}(24)$ is not the lowest one ever seen. It is higher than cycles 6, 7 (Fig. 1a), and 15 (Li 2009): $R_{min}(24) > R_{min}(15) > R_{min}(7) > R_{min}(6)$. However, corresponding to these local minima, the following R_m values are not local minima: $R_m(6) \sim R_m(5)$, $R_m(7) > R_m(6)$, and $R_m(15) > R_m(14)$. From this information, we cannot yet infer that Cycle 24 is a local minimum. To say the least, it is unlikely that Cycle 24 will be the weakest cycle.

In conclusion, we have not found sufficient evidence for the low(est) level of Solar Cycle 24 inferred from the low level of the present state. The sunspot number is highly correlated with other solar activity indices, such as sunspot group number, sunspot area, solar radio flux, and so on. Therefore, the above conclusions can also be reached when using these indices.

Near the time of the solar cycle minimum, geomagnetic activity is a much better indicator of the ensuing maximum amplitude (R_m) for the sunspot cycle (Ohl 1966) than the minimum amplitude (R_{min}) is. Hathaway et al. (1999) and Hathaway (2009) tested the predictive powers of several methods for cycles 19–23, and concluded that the geomagnetic-related precursor methods outperform the others. The minimum smoothed monthly mean aa index (aa_{min}) near the time of the solar cycle minimum is shown in Table 1, in which the values of cycles 9–11 are taken from the equivalent annual values (Du et al. 2009b). One can note that the varying trend (V) of R_m follows well with that of aa_{min} — with only the two exceptions of cycles 16 and 22. The correlation coefficient between R_m and aa_{min} is usually as high as 0.9 (Du et al. 2009b). The application of aa_{min} in the prediction of R_m can be found, for example, in Hathaway (2009) and Du et al. (2009b). Wilson et al. (1998) suggested the bivariate case of both aa_{min} and R_{min} to predict R_m. Using the data for cycles 9-23 in Table 1, the bivariate-fit regression equation of R_m versus both aa_{min} and R_m is

$$R_m = 5.0 + 10.56 aa_{min} - 3.18 R_{min}, \quad \sigma = 15.3, \tag{3}$$

where σ is the standard deviation of the equation. Figure 2(b) shows the observed R_m (solid line) and the fitted R_m (dotted) from the above equation. Substituting the values of $aa_{min}(8.4)$ and R_{min} (1.7) into this equation, the peak of the next cycle is predicted to be $R_m(24) = 88.3 \pm 15.3$ (labeled by an asterisk). This prediction is close to that predicted by the single variate case of R_{min} in Equation (1). However, the correlation for the bivariate case of both aa_{min} and $R_m (r = 0.92)$ is much higher than

that of the single variate case of $R_{\min}(r = 0.56)$. If this prediction comes true, Cycle 24 will be modest rather than the lowest one.

The prediction of R_m is related to the behavior of solar activity in the past cycles. Du et al. (2009b) pointed out that Ohl's precursor method performed well only if the related correlation coefficient becomes stronger. If the correlation coefficient becomes weaker, its prediction would be questionable. Figure 2(c) shows the running correlation coefficient $r(5, n)$ of R_m with both $aa_{\min}$ and $R_{\min}$ for a five-cycle moving window. It is seen that the last value ($n = 21$, corresponding to the data for cycles 19–23) drops drastically. Therefore, other methods are needed to check the above prediction.

Predicting the future level of a solar cycle is a complex project in solar physics and space weather (Wang et al. 2009). This paper stresses that the low level of $R_{\min}$ in the present state is insufficient to infer a low(est) level for Solar Cycle 24, as suggested by Li (2009). Whether a prediction from a simple parameter succeeds is related to the behavior of solar activity in the past few cycles. When a solar cycle is well underway (two to three years after the minimum), its behavior can be predicted to a good extent with curve fitting techniques (Hathaway 2009).

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