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A logistic model for magnetic energy storage in solar active regions *

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Abstract Previous statistical analyses of a large number of SOHO/MDI full disk longitudinal magnetograms provided a result that demonstrated how responses of solar flares to photospheric magnetic properties can be fitted with sigmoid functions. A logistic model reveals that these fitted sigmoid functions might be related to the free energy storage process in solar active regions. Although this suggested model is rather simple, the free energy level of active regions can be estimated and the probability of a solar flare with importance over a threshold can be forecast within a given time window.

Key words: Sun: photosphere — Sun: magnetic field — Sun: solar activity

1 INTRODUCTION

It is well known that the energy for solar flares comes from magnetic fields in solar active regions (Gold & Hoyle 1960). Magnetic energy storage and dissipation are regarded as important physical processes in the solar corona. In order to understand these processes, the frequency distributions of solar flares are believed to be helpful clues. Aschwanden et al. (1998) listed observed solar flare frequency distributions of peak fluxes in a wide range of wavelengths during 1969–1996. Lu & Hamilton (1991) proposed solar flares to be avalanches of many small reconnection events according to the observational fact that the distribution of solar flare hard X-ray bursts is a power law in peak photon flux (Dennis 1985). Aschwanden et al. (1998) suggested a logistic avalanche model to explain the observed elementary time structures and frequency distributions of solar flares. The time profiles of resolved elementary time structures have a near-Gaussian shape and can be modeled with the logistic equation, which provides a quantitative measurement of the exponential growth time and the nonlinear saturation energy level of the underlying instability. These avalanche models describe explosive energy dissipation in the solar atmosphere, but they cannot explain the energy storage.

Since the mechanism of flare energy storage has not been clarified, solar flare forecasting models are still based on the statistical relationship between solar flares and solar magnetic field properties. In other words, solar flares are regarded as a kind of response to solar magnetic field properties. Solar magnetic fields on the photosphere have been observed for a long time (Hale 1908). Solar photospheric magnetic field properties are conventionally described with sunspot morphologic and magnetic classifications (McIntosh 1990). The response to solar magnetic field properties can be defined as solar flare

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productivity. Current solar flare forecasting models are mainly based on the statistical relationship between solar flare productivity and sunspot classifications. Conventional classifications, however, provide us with limited information on solar magnetic fields and their physical implications remain unknown.

Vector magnetic fields in solar active regions have been measured with magnetographs since the 1950s (Babcock 1953; Hagyard et al. 1982; Lites 1989; Ai 1993; Ichimoto et al. 2004). Nowadays, many parameters, such as the length of neutral lines, the horizontal gradient, the current helicity, the shear angle and the number of singular points, are used to describe solar magnetic field properties not only morphologically but also quantitatively. They have been proposed to be new measures for solar activity forecasting models (Falconer 2001; Falconer et al. 2002, 2003; Leka & Barnes 2003a,b; Jing et al. 2006; Cui et al. 2006; Georgoulis & Rust 2007; Cui et al. 2007; Li et al. 2007). Compared with conventional measures, these new ones have evident physical implications.

In order to study the solar flare productivity related to photospheric magnetic field properties, Cui et al. (2006) employed the maximum horizontal gradient, the length of neutral lines and the number of singular points to be the measures. They analyzed a large number of SOHO/MDI full disk longitudinal magnetograms, and found that the solar flare productivity can be well fitted with sigmoid functions.

Verhulst (1838) proposed a logistic model for population growth, in which the population number can be described with sigmoid functions. Nowadays, the logistic model has been wildly employed to study many natural and social phenomena, such as response of drugs or photosensors, regional economic growth and so on. It is interesting that relations of solar flare productivity with the maximum horizontal gradient, the length of neutral lines and the number of singular points can also be described with sigmoid functions, which are similar to the logistic avalanche model for solar energy dissipation. In order to understand these sigmoid functions, we structure this paper as follows: in Section 2, the statistical relationship between solar flare productivity and photospheric magnetic field properties obtained by Cui et al. (2006) will be presented. In Section 3, physical meanings of the fitted sigmoid functions will be analyzed, and finally conclusions and discussions will be given in the last section.

2 STATISTICAL ANALYSIS

Cui et al. (2006) provided their statistical results from 23990 SOHO/MDI full disk longitudinal magnetograms during the period 1996–2004. These magnetograms contain 870 observed active regions located within 30 degrees of the solar disk center, where projection effects are small enough to have little influence on solar magnetic field measurements.

In order to quantitatively describe the magnetic field properties of active regions including non-potentiality and complexity, three measures are employed: the maximum horizontal gradient $|\nabla_h B_z|_m$, the length of neutral line L, and the number of singular points η. Soft X-ray flares are regarded as the response to these magnetic field properties.

Flare productivity is defined as

$$P(X) = \frac{S_{\rm a}(X)}{S_{\rm t}(X)}, \quad (1)$$

where X is a random value of measures describing the magnetic properties, $S_{\rm a}(X)$ and $S_{\rm t}(X)$ are the number of active samples and that of the total when values of these measures are in the range [X, ∞], respectively. This definition for the flare productivity is a monotonous function of the free energy decrease rate, which is different from that for the mean flaring rate proposed by Litvinenko & Wheatland (2001).

An operational forecasting model usually pays more attention to the production of flares with importance above a threshold within a given forward looking time window. Here, the threshold for flare importance is supposed to be M1.0 equivalent. The time window is taken to be 48 h, which is long enough for the evolution of photospheric magnetic fields.

As an example, the correlation between the solar flare productivity, P, and the maximum horizontal gradient, $|\nabla_h B_z|_m$, is shown in Figure 1. The shape of data points is sigmoid and therefore we fit them

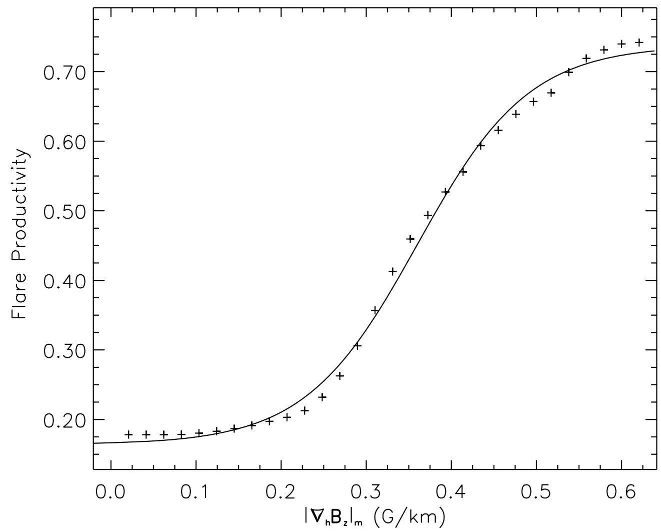

Fig. 1 Correlation between the solar flare productivity and the maximum horizontal gradient.

with a sigmoid function in a Boltzmann style,

$$P = A_2 + \frac{A_1 - A_2}{1 + \exp[(X - X_0)/W]}\,, \tag{2}$$

or equivalently

$$P = A_1 + \frac{A_2 - A_1}{1 + \exp[-(X - X_0)/W]}\,, \tag{3}$$

where A_1 and A_2 are two asymptotic values that the function approaches but never quite reaches at small and large X; W is the approximate width when the curve crosses over between A_1 and $A2$ in a region of X values, which is centered around X_0; and the slope of the curve is maximized at X_0.

Similarly, the shape of the data points obtained with the length of neutral line, L, and the number of singular points, η, can also be fitted with sigmoid functions. The four parameters of the fitted sigmoid functions are shown in Table 1.

Table 1 Four Parameters of Fitted Sigmoid Functions

	A_1	A_2	X_0	W
$\lvert\nabla_h B_z\rvert_m$	0.164	0.738	0.360	0.066
L	0.062	0.848	763.1	382.0
η	–0.196	0.730	9.343	22.66

3 PHYSICAL MEANINGS OF FITTED SIGMOID FUNCTIONS

First of all, we should explain why the shape of the fitted curves is a sigmoid. A scenario for solar magnetic energy storage is proposed as follows. Considering a closed space, Ω, in the corona with a bottom boundary surface, S, on the photosphere, we discuss the free magnetic energy variation with values of measures from photospheric magnetic fields, x, i.e., $\frac{dE}{dx}$. Let r and A be the free energy increase rate and the free energy capacity in the space, Ω, respectively. Then, we have a logistic model similar to that of population growth (Verhulst 1838). In this model, the free energy decrease rate is supposed to be

$$\kappa = r\frac{E}{A}\,, \tag{4}$$

which means that this rate is proportional to the free energy increase rate and the current amount of free energy, and is inversely proportional to the free energy capacity. This assumption is reasonable, because a large amount of energy in Ω can provide much free energy to be released, and a large energy capacity results in the system being very stable in which the free energy is hard to decrease. Now, the free energy variation has the following form:

$$\frac{dE}{dx} = \left(r - r\frac{E}{A}\right)E\,, \tag{5}$$

where r is independent of E and x, and A is a constant.

Supposing E is always smaller than A, we have $\frac{dE}{dx} > 0$, which means that there is an energy storage process in the closed space, Ω. Then, the solution of Equation (5) is obtained as follows:

$$E = \frac{A}{1+e^{-rx}}\,. \tag{6}$$

In order to simplify the monotonous relation between P and E, the solar flare productivity, P, is regarded to be a linear function of E, namely

$$P \propto (E - E_0), \tag{7}$$

where E_0 is a constant.

According to Equation (7), we set $E \propto (P - A_1)$, $A \propto (A_2 - A_1)$, and $x = X - X_0$, Equation (3) can be written in the form of Equation (6). It turns out that the parameter $1/W$ actually corresponds to the free energy increase rate, and the difference between A_2 and A_1 indicates the free energy capacity. In practice, the statistical values of $1/W$ and $A_2 - A_1$ determine the response sensitivity and dynamic response range of an individual measure. For an ideal measure, the statistical values of $1/W$ and $A_2 - A_1$ will approach the free energy increase rate and the free energy capacity.

It should be noted that the logistic model cannot be applied to all measures from photospheric magnetic fields. We only pay attention to those measures observationally verified to have sigmoidal solar flare productivity. The assumption that r is independent of E and x is a necessary condition for the logistic model. If r is a function of E or x, the flare productivity will be non-sigmoidal.

Second, the two asymptotic values, A_1 and A_2, should be explained further. For an ideal measure, A_1 and A_2 correspond to the lower free energy level and the higher, respectively. At both of the levels, the free energy variation approaches zero, which means the system has two kinds of equilibrium status. At the lower level, the free energy decrease rate approaches zero, and there is limited free energy for dissipation, while at the higher level, the free energy decrease rate approaches the increase rate, and there is a large amount of free energy for dissipation. The difference between A_2 and A_1 describes the free energy capacity. A_1 indicates that the system is unstable with respect to free energy storage, or stable for free energy dissipation, and A_2 inversely.

Third, the differences among the fitted sigmoid functions should be discussed. In order to make the comparison easy, we rescale these measures and parameters in Equation (3) according to the following algorithms,

$$t = \frac{X - X_0}{W}\,, \tag{8}$$

$$F = P - A_1\,, \tag{9}$$

$$A = A_2 - A_1\,. \tag{10}$$

Then, Equation (3) is transformed into the following form

$$F = \frac{A}{1+e^{-t}} \tag{11}$$

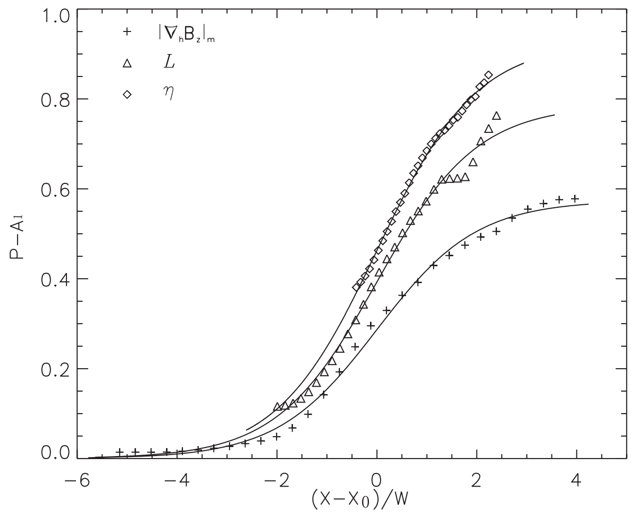

Fig. 2 Comparison of the fitted sigmoid response functions of the maximum horizontal gradient, $|\nabla_h B_z|_m$, the length of neutral line, L, and the number of singular points, η.

and three curves of the fitted sigmoid response functions according to Equation (11) are presented in Figure 2.

In Figure 2, values of $P - A_1$ correspond to the responses of the rescaled measures, $(X - X_0)/W$. Since the measures, $(X - X_0)/W$, have dimensionless values, their responses can be compared easily. It is evident that the rescaled number of singular points has the largest dynamic response range, and the rescaled maximum horizontal gradient the smallest. What is the reason for that? Let us analyze the geometrical dimension of these measures. On the boundary surface, S, the maximum horizontal gradient is a point function with zero dimension (local scale); the length of neutral line makes a measurement along the neutral line with fractal (1∼2) dimensions (line scale); and the number of singular points describes the topological complexity of photospheric magnetic fields with two dimensions (surface scale). Since the obvious discrepancy among these measures is their geometric properties, we suppose that the dynamic response ranges of these rescaled measures mainly depend on their geometric properties. A measure in the local scale indicates the free energy evaluated only in a local region of Ω, while a measure in a surface scale indicates the free energy evaluated in the whole of Ω.

Another fact which should be noted in Figure 2 is that the slope of the response curve from the rescaled number of singular points is the steepest and that of the rescaled maximum horizontal gradient the flattest. What causes that? The slope of sigmoid curves, dF/dt, indicates the response sensitivity of measures. From Equation (11), dF/dt can be derived as:

$$\frac{dF}{dt} = A\frac{e^{-t}}{(1 + e^{-t})^2}\,. \qquad (12)$$

From Equation (12), it can be seen that a measure with the largest dynamic response range, A, has the highest response sensitivity at the same value of t. When $t \to \pm\infty$, the response sensitivity of all measures approaches zero. From the values of A_1 and A_2 listed in Table 1, the values of A for three measures can be obtained as 0.574, 0.786, and 0.926, respectively. Thus, the rescaled number of singular points has the highest response sensitivity. In order to understand this fact, we give the following explanation. When there exist only two magnetic polarities in the surface, S, the number of singular points is two; when there exist two pairs of polarities, the number of singular points becomes six, i.e., the number of polarities is four and the number of saddle points is two. With the increase of the number of singular points, the topological complexity of photospheric magnetic fields increases rapidly in the surface scale. This means the dynamic response range, A, for the rescaled number of singular points is closer to the free energy capacity than the other two measures which can only evaluate the free energy in the local and line scales. It has been widely accepted that magnetic flux emergence and cancellation always lead to eruptions in the solar atmosphere (Zirin 1970; Wang & Shi 1993; Rust et al. 1994). Actually, the magnetic flux emergence and cancelation inevitably result in a change of the number of singular points.

It is a logical result of our model that the conventional magnetic classification of sunspot groups has a large dynamic response range, because this classification is also a kind of measure with surface scale, and complicated magnetic configurations mean complicated topological features. The disadvantage is this classification can hardly be quantitatively measured.

From the facts in Figure 2, we conclude that measures in a surface scale have the highest response sensitivity and largest dynamic response range. It should be pointed out that measures in a line scale are also useful, since the fractal dimensions of lines will approach two dimensions when the topology of magnetic fields becomes very complicated.

The logistic model has been put into use to setup operational models for forecasting the probability of solar flares with importance over a threshold within a given time window (Wang et al. 2008; Yu et al. 2009).

4 CONCLUSIONS AND DISCUSSION

According to the previous statistical result where responses of solar flares to photospheric magnetic properties can be fitted with sigmoid functions, which are described with four parameters, A_1, A_2, X_0, and W, we analyzed the physical meanings of these fitted sigmoid functions and obtained the following conclusions:

– Within a closed space in the corona, the free magnetic energy variation with increments of measures derived from photospheric magnetic fields can be described with a logistic model. For an ideal measure, the parameter $1/W$ in Equation (3) actually corresponds to the free energy increase rate and the difference between A_2 and A_1 indicates the free energy capacity. The two asymptotic values, A_1 and A_2, correspond to the lower free energy level and the higher, respectively. At both of the levels, the free energy variation approaches zero. At the lower level, however, the system is very stable, while at the higher level, the system is quite unstable.
– In practice, measures in a surface scale have the highest response sensitivity and largest dynamic response range. Those in a line scale are also valuable, since the fractal dimensions of lines will approach two dimensions when the topology of magnetic fields becomes very complicated.

For an individual measure, the statistical values of the parameters of the response function determine the response sensitivity and the dynamic response range, which cannot be regarded as the free energy increase rate and the free energy capacity in the solar corona. Our model provides guidance to search for an ideal measure for solar activity forecasting.

This suggested model is rather simple because the mechanism of solar magnetic energy release has not been taken into account. With values of measures, the free energy level of active regions can be estimated and the probability of a solar flare with importance over a threshold can be forecast with a given time window. However, when and where the flare will take place cannot be exactly predicted.

The projection effect is a key problem for making use of measures from photospheric magnetic fields. In order to reduce this effect, we only select observed active regions located within 30 degrees of the solar disk center. This means that our model is currently suitable for a limited number of active regions.

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