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Nonlinear density fluctuation field theory for large scale structure *

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Abstract We develop an effective field theory of density fluctuations for a Newtonian self-gravitating N-body system in quasi-equilibrium and apply it to a homogeneous universe with small density fluctuations. Keeping the density fluctuations up to second order, we obtain the nonlinear field equation of 2-pt correlation $\xi(r)$, which contains 3-pt correlation and formal ultra-violet divergences. By the Groth-Peebles hierarchical ansatz and mass renormalization, the equation becomes closed with two new terms beyond the Gaussian approximation, and their coefficients are taken as parameters. The analytic solution is obtained in terms of the hypergeometric functions, which is checked numerically. With one single set of two fixed parameters, the correlation $\xi(r)$ and the corresponding power spectrum $P(k)$ simultaneously match the results from all the major surveys, such as APM, SDSS, 2dfGRS, and REFLEX. The model gives a unifying understanding of several seemingly unrelated features of large scale structure from a field-theoretical perspective. The theory is worth extending to study the evolution effects in an expanding universe.

Key words: cosmology: large-scale structure — cosmology: theory — galaxies: clusters: general — gravitation —- hydrodynamics — instabilities

1 INTRODUCTION

Great progress has been made in understanding the large scale structure of the universe during the past decades. Not only have observations of major galaxy surveys such as SDSS (Tegmark et al. 2004; Zehavi et al. 2002; Zehavi et al. 2005), 2dF (Colless 2001; Hawkins et al. 2003; Madgwick et al. 2003; Percival 2005), APM (Maddox et al. 1996; Padilla & Baugh 2003), and REFLEX (Collins 2000; Schueker et al. 2001) etc., revealed cosmic structures of increasingly large sky dimension with new detailed features being found, theoretical studies have also achieved important results, through numerical simulations (White & Frenk 1991; Springel et al. 2005), perturbation methods (Peebles 1980; Davis & Peebles 1983; Fry 1984; Goroff et al. 1986; Bernardeau et al. 2002; Valageas 2004), and thermodynamics (Saslaw 2000). In view of these dynamics, the Universe, filled with galaxies and clusters, is a many-body self-gravitating system in an asymptotically relaxed state, since the cosmic time scale $1/H_0$ is longer than the local crossing time scale (Saslaw 2000). A systematic approach to statistical mechanics of many-body systems is to convert the degrees of freedom of discrete particles into a continuous field. Thereby, the fully-fledged techniques of field theory can be applied to study the systems (Zinn-Justin 1996). The Landau-Ginzburg theory is a known example in this regard. We have formulated such a density field theory of self-gravitating systems and applied it to the large scale structure of the Universe (Zhang 2007). Under the Gaussian approximation, the field equation of the 2-point correlation function and the solution has been derived explicitly. The result qualitatively interprets some

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observational features, but it suffers from insufficient clustering on small scales. In this paper, we will go beyond the Gaussian level and include nonlinear terms of density fluctuations up to second order, yielding a more satisfying description of the large scale structure.

2 NONLINEAR FIELD EQUATION OF CORRELATION FUNCTION

The universe is represented by a collection of either galaxies or clusters, including their respective dark halos, as unit cells with random velocities. Although the unit cell, galaxy or cluster, has different mass m, both cases correspond to the same mass density $\rho(\boldsymbol{r})$. We study the asymptotically relaxed state of this Newtonian self-gravitating system of N points of mass m with the Hamiltonian $H = \sum_i^N \frac{p_i^2}{2m} - \sum_{i<j}^N \frac{Gm^2}{|\boldsymbol{r}_i - \boldsymbol{r}_j|}$. Thus, evolutionary effects will not be addressed in this paper. By using the Hubbard-Stratonovich transformation (Zinn-Justin 1996), the grand partition function of this system at temperature T can be cast into the generating functional as a path integral $Z = \int D\phi \exp\left[-\beta^{-1} \int d^3\boldsymbol{r}\mathcal{L}(\phi)\right]$, where ϕ is the gravitational field, $\beta \equiv 4\pi Gm/c_s^2$ of dimension m^{-1}, $c_s \equiv (T/m)^{1/2}$ the speed of sound, the effective Lagrangian is $\mathcal{L}(\phi) = \frac{1}{2}(\bigtriangledown\phi)^2 - k_J^2 e^{\phi}$, and $k_J \equiv (4\pi G\rho_0/c_s^2)^{1/2}$ is the Jeans wavenumber. The term $-k_J^2 e^{\phi}$ has a minus sign because gravity is attractive. By Poisson's equation $\bigtriangledown^2\phi(\boldsymbol{r}) + k_J^2 e^{\phi} = 0$, the mass density ρ is related to the ϕ field by $\rho(\boldsymbol{r}) = mn(\boldsymbol{r}) = \rho_0 e^{\phi(\boldsymbol{r})}$. So, ρ_0 is the constant mass density when $\phi = 0$. We define a dimensionless re-scaled mass density field $\psi(\boldsymbol{r}) \equiv e^{\phi(\boldsymbol{r})} = \rho(\boldsymbol{r})/\rho_0$, and introduce an external source J which couples with ψ in the effective Lagrangian (Zhang 2007)

$$\mathcal{L}(\psi, J) = \frac{1}{2}\left(\frac{\bigtriangledown\psi}{\psi}\right)^2 - k_J^2\psi - J\psi\,, \tag{1}$$

J is used to handle the functional derivatives with ease. So far, c_s is the only parameter in place of temperature T, upon which β and k_J depend. The field equation of ψ in the presence of J is

$$\bigtriangledown^2\psi - \frac{1}{\psi}(\bigtriangledown\psi)^2 + k_J^2\psi^2 + J\psi^2 = 0. \tag{2}$$

The n-point connected correlation function is given by $G_c^{(n)}(\boldsymbol{r}_1, \ldots, \boldsymbol{r}_n) = \langle\delta\psi(\boldsymbol{r}_1)\ldots\delta\psi(\boldsymbol{r}_n)\rangle$, where $\delta\psi(\boldsymbol{r}) = \psi(\boldsymbol{r}) - \langle\psi(\boldsymbol{r})\rangle$ is the fluctuation of the field about the expectation value $\langle\psi(\boldsymbol{r})\rangle$, and $\langle\delta\psi\rangle = 0$. A standard way to derive the field equation of $G_c^{(2)}$ is to take the functional derivative of the expectation value of Equation (2) w.r.t. the source J, and then set $J = 0$ (Goldenfeld 1992; Zhang 2007). Assuming the large-scale homogeneity of the Universe with a constant background density $\langle\psi\rangle = \psi_0$, and keeping up to second order of small fluctuations

$$\frac{1}{\psi} = \frac{1}{\langle\psi\rangle + \delta\psi} \simeq \frac{1}{\langle\psi\rangle}\left(1 - \frac{\delta\psi}{\langle\psi\rangle} + (\frac{\delta\psi}{\langle\psi\rangle})^2\right), \tag{3}$$

we obtain the field equation of the 2-pt correlation function

$$\begin{aligned} &\nabla^2 G_c^{(2)}(\boldsymbol{r}) + 2k_J^2 G_c^{(2)}(\boldsymbol{r}) \\ +&\left[\frac{1}{\psi_0^2} G_c^{(2)}(\boldsymbol{r})\nabla^2 G_c^{(2)}(0) - \frac{1}{\psi_0}\nabla^2 G_c^{(3)}(0,\boldsymbol{r},\boldsymbol{r}) \right. \\ &\left. + \frac{2}{\psi_0}\nabla G_c^{(2)}(\boldsymbol{r})\cdot\nabla G_c^{(2)}(0)\right] = -\psi_0^2\beta\delta^{(3)}(\boldsymbol{r}), \end{aligned} \tag{4}$$

where $G_c^{(2)}(\boldsymbol{r} - \boldsymbol{r'}) = G_c^{(2)}(\boldsymbol{r}, \boldsymbol{r'}) = \beta\delta\langle\psi(\boldsymbol{r})\rangle_{J=0}/\delta J(\boldsymbol{r'})$ has been used. Equation (4) is not closed, as it involves the 3-pt correlation function $G_c^{(3)}$. If higher order terms in $\delta\psi$ were allowed in Equation (3), there would be $G_c^{(4)}$, etc., in Equation (4). Therefore, we have a hierarchy of field equations, typical for the kinetic equation of a generic many-body system. To close Equation (4), we adopt the Groth-Peebles hierarchical ansatz (Groth & Peebles 1977)

$$G_c^{(3)}(\boldsymbol{r}_1,\boldsymbol{r}_2,\boldsymbol{r}_3) = Q[G_c^{(2)}(\boldsymbol{r}_1,\boldsymbol{r}_2)G_c^{(2)}(\boldsymbol{r}_2,\boldsymbol{r}_3) + G_c^{(2)}(\boldsymbol{r}_2,\boldsymbol{r}_3)G_c^{(2)}(\boldsymbol{r}_3,\boldsymbol{r}_1) + G_c^{(2)}(\boldsymbol{r}_3,\boldsymbol{r}_1)G_c^{(2)}(\boldsymbol{r}_1,\boldsymbol{r}_2)], \tag{5}$$

where the constant $Q = 0.5 \sim 1.0$ (Fry 1984; Efstathiou & Jedrzejewski 1984; Jing & Borner 1998; Jing & Borner 2004). Then, Equation (4) becomes closed

$$\nabla^2 G_c^{(2)}(\boldsymbol{r}) + k_0^2 G_c^{(2)}(\boldsymbol{r}) + \boldsymbol{a}\cdot\nabla G_c^{(2)}(\boldsymbol{r}) - b\Big(\nabla G_c^{(2)}(\boldsymbol{r})\Big)^2 = -\psi_0^2\beta\delta^{(3)}(\boldsymbol{r}), \tag{6}$$

where $\boldsymbol{a} \equiv \big(\frac{2}{\psi_0^2} - \frac{2Q}{\psi_0}\big)\nabla G_c^{(2)}(0)$, $b \equiv Q/\psi_0 > 0$, and

$$k_0^2 \equiv 2k_J^2 + \Big(\frac{1}{\psi_0^2} - \frac{2Q}{\psi_0}\Big)\nabla^2 G_c^{(2)}(0). \tag{7}$$

Due to the higher order terms in Equation (3), the friction term $\boldsymbol{a}\cdot\nabla G$ and the nonlinear term $b(\nabla G)^2$ occur in Equation (6). As is expected for an interacting field theory, $2k_J^2$ is modified by an apparently divergent term $\nabla^2 G_c^{(2)}(0)$. We take k_0^2 as the physical wavenumber like in the standard mass renormalization (Zinn-Justin 1996). Equation (6) is a nonlinear elliptic equation with a point source $-\psi_0^2\beta\delta^{(3)}(\boldsymbol{r})$. Since $\psi_0^2\beta \propto m/c_s^2$, galaxies or clusters with greater mass have a higher correlation amplitude. This naturally explains why the correlations of clusters or galaxies increase with richness and luminosity (Zhang 2007). As mentioned earlier, galaxies and clusters are treated on equal footing as gravitating particles differing only by their masses; their correlation functions have the same functional form, differing only in the amplitude $\propto m/c_s^2$. These are the observed facts (Guzzo et al. 2000; Bahcall et al. 2003). By isotropy of the Universe, one puts $G_c^{(2)}(\boldsymbol{r}) \equiv \xi(r)$, then Equation (6) reduces to

$$\xi''(x) + \Big(\frac{2}{x} + a\Big)\xi'(x) + \xi(x) - b\xi'^{\,2}(x) = -\psi_0^2\beta k_0\frac{\delta(x)}{x^2}, \tag{8}$$

where $x \equiv k_0 r$, $\xi' \equiv \frac{d\xi}{dx}$, and $a \equiv \big(\frac{2}{\psi_0^2} - \frac{2Q}{\psi_0}\big)\xi'(0)$. Both a and b are treated as two parameters.

3 ANALYTIC AND NUMERICAL SOLUTION

The Gaussian approximation (Chavanis 2006; Zhang 2007) is recovered by setting $\boldsymbol{a} = 0 = b$ and $k_0^2 \to 2k_J^2$ in Equation (6), i.e. keeping only the term $\frac{1}{\psi} \simeq \frac{1}{\langle\psi\rangle}$ in Equation (3). The solution is $\xi(r) \propto A\frac{\cos(k_0 r)}{r}$ with $A = \frac{\psi_0^2 Gm}{c_s^2}$ and $k_0 = (\frac{8\pi G\rho_0}{c_s^2})^{\frac{1}{2}}$, and the power spectrum $P(k) = \frac{1}{2n}\frac{1}{(\frac{k}{k_0})^2-1}$, where n is the spatial number density. This result qualitatively explains several observed features, such as a stronger correlation for more massive galaxies, galaxies with a smaller n having a higher $P(k)$ (Davis & Geller 1976; Einasto 2002), the scaling of "correlation length" r_0 with the mean cluster separation d as $r_0(d) \propto d^{0.3\sim 0.5}$ (Bahcall 1996; Bahcall et al. 2003; Croft 1997; Gonzalez et al. 2002; Zandivarez et al. 2003), and damped oscillations of $\xi_c(r)$ for clusters with a wavelength $2\pi/k_0 \simeq 120\,h^{-1}$ Mpc (Broadhurst 1990; Einasto et al. 1997a,b; Einasto 2002; Tucker et al. 1997; Tago et al. 2002). Here, one sees the physical meaning of the speed of sound c_s. Using the background mass density $\rho_0 = \rho_c\Omega_m = (3/8\pi G)H_0^2 h^2\Omega_m$ leads to $c_s = \sqrt{3}H_0 h\Omega_m^{1/2}/k_0$. Taking the observed periodic length $2\pi/k_0 \simeq 100 \sim 120\,h^{-1}$ Mpc and $h\Omega_m^{1/2} \sim 0.36$ by WMAP (Spergel et al. 2003; Spergel et al. 2007), yields $c_s \simeq 1000 \sim 1200\,\mathrm{km\ s^{-1}}$, which is roughly the order of magnitude of the peculiar velocity of clusters or galaxies. Thus, viewing c_s as the random velocity of clusters or galaxies is qualitatively consistent with the observations of the periodicity in $\xi_c(r)$, of H_0, and of $h\Omega_m^{1/2}$. The shortcomings of the Gaussian solution are that $\xi(r)$ is too low at small scales, and that $P(k)$ has a sharp peak at $k = k_0$ and becomes negative for $k < k_0$ (Zhang 2007).

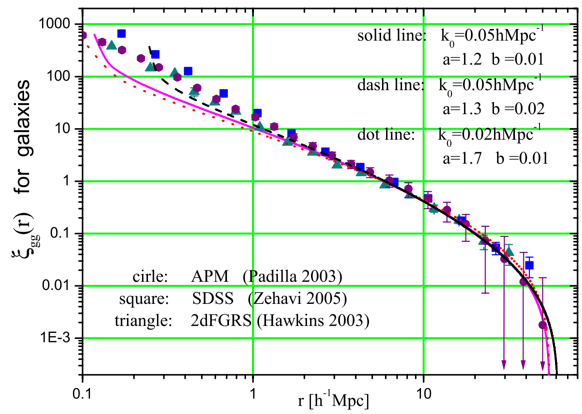

Fig. 1 $\xi_{\rm gg}(r)$ fits the galaxy correlation functions of the surveys, APM, SDSS, and 2dFGRS, simultaneously.

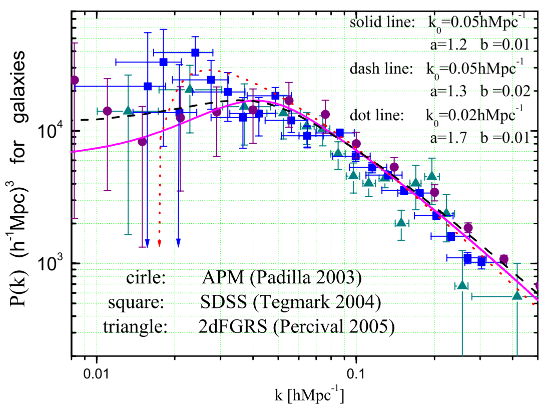

Fig. 2 $P(k)$ of Fig. 1 also fits the power spectra of APM, SDSS, and 2dFGRS. $P(k) \propto k^{-1.5}$ in $(0.05 \sim 0.5)\, h$ Mpc^{-1}.

Now, these problems are overcome by the nonlinear Equation (8), whose solution is determined by the boundary condition

$$\xi(r_c) = C, \qquad \xi'(r_c) = D, \tag{9}$$

at some r_c. Note that, in fitting with observational data, the amplitude C is higher for clusters than for galaxies, as clarified earlier, and the slope D is roughly equal for clusters and for galaxies. The range of parameters are taken to be: $a = (1.0 \sim 1.3)$, $b = (0.01 \sim 0.05)$, $k_0 = (0.03 \sim 0.06)\, h$ Mpc^{-1}. In computation, we take $r_c \simeq 0.4/k_0\, h^{-1}$ Mpc. Equation (8) is easily solved numerically. In fact, it has an analytic solution as follows. By perturbation method, since $b \ll 1$, one sets the solution as a series $\xi(x) = \sum_{i=0}^{\infty} b^i \xi_i(x)$. Equation (8) becomes $\ddot{\xi}_0 + \left(\frac{2}{x} + a\right)\dot{\xi}_0 + \xi_0 = -\psi_0^2 \beta \frac{\delta(r)}{x^2}$ and $\ddot{\xi}_i + \left(\frac{2}{x} + a\right)\dot{\xi}_i + \xi_i = g_i$, where $g_i \equiv \sum_{j=0}^{i-1} \dot{\xi}_j \dot{\xi}_{i-1-j}$ for $i > 0$. The solution for ξ_0 is a linear combination of (Gradshteyn & Ryzhik 1980)

$$y_1 \equiv e^{-\frac{1}{2}\alpha z} \Phi(\alpha, 2; z), \quad y_2 \equiv e^{-\frac{1}{2}\alpha z} \Psi(\alpha, 2; z), \tag{10}$$

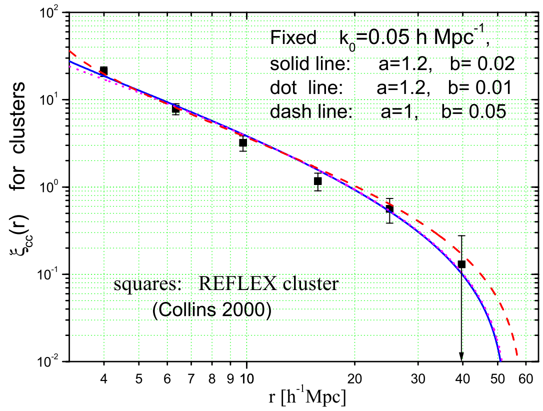

Fig. 3 $\xi_{\rm cc}(r)$ fits the correlation function of REFLEX X-ray clusters (Collins 2000).

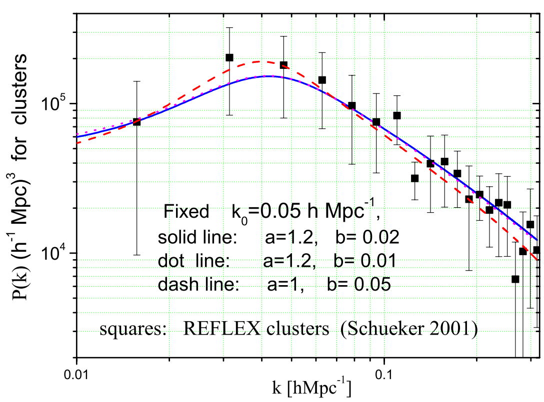

Fig. 4 $P(k)$ from Fig. 3 fits the corresponding spectrum of REFLEX X-ray clusters (Schueker et al. 2001).

where Φ and Ψ are the degenerate hypergeometric functions, $z \equiv (a^2-4)^{1/2}x$, and $\alpha \equiv 1+\frac{a}{\sqrt{a^2-4}}$. The solution of ξ_i is a linear combination of Equation (10) plus a particular solution

$$y_2(x)\int^x \frac{y_1(t)g_i(t)}{W(y_1,y_2)(t)}dt - y_1(x)\int^x \frac{y_2(t)g_i(t)}{W(y_1,y_2)(t)}dt, \tag{11}$$

where $W(y_1,y_2)$ is the Wronskian. The condition in Equation (9) is satisfied by choosing $\xi_0(r_c)=C$, $\xi_0'(r_c)=D$, and $\xi_i(r_c)=\xi_i'(r_c)=0$ for $i>0$. As is checked, up to order $i=4$, this analytic solution agrees with the numerical one. Once $\xi(r)$ is known, the power spectrum follows from Fourier transform $P(k)=4\pi\int_0^\infty \xi(r)\frac{\sin(kr)}{kr}r^2dr$. For galaxies from APM, 2dFGRAS and SDSS, the calculated $\xi_{\rm gg}(r)$ and $P(k)$ are shown in Figures 1 and 2, respectively. For REFLEX X-ray clusters, the calculated $\xi_{\rm cc}(r)$ and $P(k)$ are given in Figures 3 and 4, respectively. For SDSS clusters, $\xi_{\rm cc}(r)$ is given in Figure 5. It is seen that, for the fixed parameters $(a,b)\simeq(1.2,0.02)$, and $k_0=0.05\,h$ Mpc^{-1}, the calculated $\xi_{\rm gg}(r)$, $\xi_{\rm cc}(r)$, and their respective $P(k)$ all match very well with major surveys for both galaxies and clusters, simultaneously. Thus, our density field theory gives a decent description of the observational data.

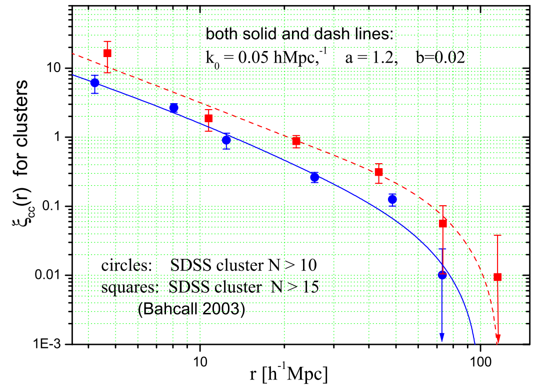

Fig. 5 $\xi_{\rm cc}(r)$ of SDSS clusters of different richness (Bahcall et al. 2003) are obtained by the same set (a, b) but different initial amplitude.

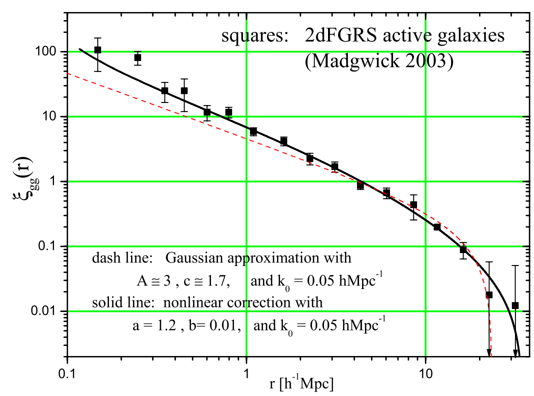

Fig. 6 Solution $\xi_{\rm gg}(r)$ of the nonlinear Eq. (8) improves the Gaussian one on small scales and matches 2dFGRAS active galaxies.

4 DISCUSSION AND CONCLUSIONS

Overall, the solution $\xi(r)$ of Equation (8) improves the Gaussian one considerably, as shown in Figure 6, in which the 2dFGRS active galaxies are taken for the purpose of demonstration. It is found that the nonlinear term $-b(\xi')^2$ has the effect of strongly enhancing $\xi(r)$ on small scales, and making $P(k)$ flatter in $k < k_0$. The friction term $a\xi'$ has the following effects: slightly increasing the height of $\xi(r)$ on small scales; moving the zeros of $\xi(r)$ to larger r; strongly damping the amplitude of oscillations of ξ on large scales (Broadhurst 1990; Einasto et al. 1997a,b; Einasto 2002; Tucker et al. 1997; Tago et al. 2002) as seen in Figure 7; smoothing out the sharp peak of $P(k)$ at k_0 and turning $P(k)$ positive for $k < k_0$.

The main conclusion of this paper is the following. The universe, containing galaxies or clusters, is viewed as a many-particle system in an asymptotically relaxed state and can be described by an effective density field, whereby the techniques of field theory apply, yielding a perspective on the large scale structure of the universe other than the conventional methods. The Jeans scale $k_0 \simeq (0.04 \sim 0.06)\, h\,{\rm Mpc}^{-1}$ appears, which is the unique scale underlying the large scale structure as a gravitational system. Up to nonlinear terms $(\delta\psi)^2$ beyond the Gaussian approximation, the nonlinear field equation of the 2-pt correlation function of density fluctuations has been derived and solved analytically. This analytic result of field theory interprets several observed features of large scale structures. With fixed

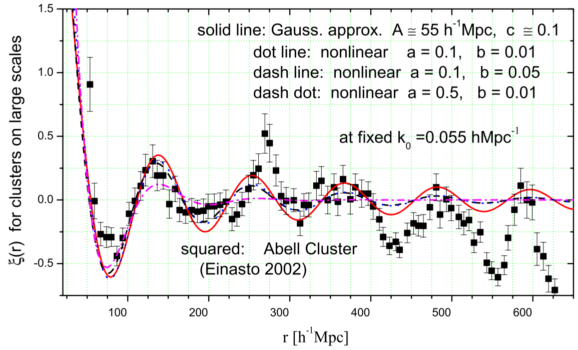

Fig. 7 Calculated $\xi_{\rm cc}(r)$ with oscillations compared with the observed $\xi_{\rm cc}(r)$ of X-ray clusters (Einasto 2002; Tago et al. 2002).

values of the parameters (a, b, k_0), the solution simultaneously matches the observed $\xi(r)$ and $P(k)$ of both galaxies and of clusters.

Although our results match the observational data on large scales very well, our model is still preliminary at the present stage. There are several problems that need to be further addressed in the future as follows.

Firstly, the calculated $\xi_{\rm gg}(r)$ for galaxies increases too fast on very small scales $r \leq 0.2\,h^{-1}$ Mpc. This indicates that the model may break down on such small scales close to the size of a galaxy. This may suggest that either higher order terms of perturbations should be taken into account, or the effects of galaxy formation and local virilization need to be included.

Next, there is a limitation in applying the Groth-Peebles ansatz in Equation (5). As is known, for descriptions of many-particle dynamic systems based on the Gibbs-Boltzmann equation, a procedure is usually taken, which decomposes the complicated equation into a set of differential equations, so that each one in the set is possibly manageable. However, a hierarchy of BBKGY type, or the like, inevitably arises. Different treatments of the hierarchy are employed for different systems, and a cut-off of the hierarchy is usually is used. For instance, in the case of the photon gas of CMB, the multipole decomposition is involved for the temperature anisotropies and polarization, and the common practice is to cut off the hierarchy of multipoles by setting higher multipole components to zero, yielding an accurate description of CMB spectra (Zhao & Zhang 2006; Zhang et al. 2007; Xia & Zhang 2008). However, for the calculation of $G_c^{(2)}(\boldsymbol{r})$ in our context, one cannot set $G_c^{(3)}(\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_3)$ to zero, since these are important and give rise to nonlinear effects, due to the long range nature of gravitational force. The Groth-Peebles ansatz has been used in our analytic treatment, because it is simple and an analytic solution can be derived. We would like to mention that the ansatz only approximately reflects the actual distribution since the forms of factor Q are in fact functions of r. In this case, Equation (6) for $G_c^{(2)}(\boldsymbol{r})$ would be more complicated and analytic solutions for the general case would be difficult to derive explicitly.

Thirdly, in fitting the observations of galaxies, we have not separated the dark matter from galaxies in our present model. Therefore, no bias is introduced and the baryon acoustic oscillations are not incorporated in our model. A comprehensive treatment of two components, dark matter and baryons, in our theory would require substantial extensions of the model discussed in the present paper.

Finally, it should be mentioned that our theoretical model deals with only the quasi-relaxed state of the large scale structure of the universe, which, as an assumption, is a qualitatively good approximation since the overall expansion rate H_0 is smaller than the particle collision rate. It would be desirable to have an extension of the present model take into account an evolutionary description. These issues will to be addressed in our further studies.

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