Research in Astron. Astrophys. **2009** Vol. **9** No. **4**, 409–421
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Research in
Astronomy and
Astrophysics

Testing Lorentz violation using propagating UHECRs [*]

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Received 2008 May 9; accepted 2008 July 17

Abstract Lorentz invariant violation (LIV) test is important for studying modern physics. All the known astrophysical constraints either have a very small examinable parameter space or are only suitable for some special theoretical models. Here, we suggest that it is possible to directly detect the time-delay of ultra-high-energy cosmic-rays (UHECRs). We discuss some difficulties in our method, including the intergalactic magnetic fields. It seems that none of them are crucial, hence this method could give a larger examinable parameter space and a stronger constraint on LIV.

Key words: cosmic rays — gamma rays: bursts — ISM: magnetic fields — relativity

1 INTRODUCTION

Lorentz invariant violation (LIV) test (Pavlopoulos 1967) is significant for the applicability of Special Relativity, even if it has few trustworthy theoretical foundations. The theoretical approaches for LIV are mainly from modern physics, including the Standard-Model Extension, noncommutative geometry (Szabo 2003), loop quantum gravity (Rovelli 1998) and string theory (Green et al. 1987; Polchinski 1998).

The Standard-Model Extension approach [1] (Colladay & Kostelecký 1997, 1998) is the most straightforward one, which introduces LIV as an assumption. LIV may be caused by spontaneously violating the vacuum solution, if not by the theory itself. The minimum Standard-Model Extension wishes to maintain all the conventional desirable properties of the Standard-Model besides allowing for violations of Lorentz symmetry, hence it does not have many astrophysical (time-integral-type) applications. However, the Standard-Model Extension can actually induce some kinds of birefringence effects for photons (Kostelecký & Mewes 2001).

Noncommutative geometry has a lot of phenomenological applications. However, most of them are based on terrestrial experiments (Hinchliffe et al. 2002; Konopka & Major 2002). The derivation of *particle* Lorentz-violating terms from noncommutative geometry (Carroll et al. 2001) seems more natural than other approaches, but unfortunately, it does not have (at least we do not know how it can have) a beautiful and feasible way to be tested by time-integral-type experiments. The differences are as follows. Some theoretical models result in a constant space of light (e.g. by κ-Minkowski spacetime (Tamaki et al. 2002)). Researchers seem to have different opinions on whether the Lorentz-violating term $\theta^{\mu\nu}$ depends or not on position, energy or momentum [2].

The loop quantum gravity approach seems the most usable one. The propagational calculations of photons (Alfaro et al. 2002a; Gambini & Pullin 1999) and neutrinos (or other massive spin-$1/2$

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[*] Supported by the National Natural Science Foundation of China.

[1] *http://www.physics.indiana.edu/~kostelec/faq.html*

[2] The kind of experiments we are interested in only work if $\theta^{\mu\nu}$ depends on energy and momentum, but is independent or almost independent of position for us to integrate the effect.

fermions) (Alfaro et al. 2000, 2002b) are fulfilled. The propagation speeds in both cases are non-trivial, with velocity departure linearly depending on particle energy. Furthermore, photons have a first order and neutrinos have a second order birefringence effect. However, although "foam" structure (Doplicher et al. 1995; Garay 1998; Hawking 1978; Wheeler 1964) is really an intuitive way to understand the nature of quantum space-time, we have to warn ourselves time and again that loop quantum gravity theory itself has some theoretical problems (Ashtekar et al. 1992).

Another leading (and in fact, chronologically the "first") root for the LIV calculations is from the Liouville string (Ellis et al. 1992), a phenomenological model which makes the calculations of propagation equation in the framework of string theory possible (Amelino-Camelia et al. 1997; Ellis et al. 2000a). What they can calculate are the so-called "photons" which are the endpoints of open-strings attached to D-branes, and the space-time foam is described by D-brane fluctuations. The model can result in Lorentz-violating propagation equation by LIV of the string ground state, although it also has some inconsistencies. As a result, LIV is stochastic, and the degree of velocity departure is of first order. However, there is no evidence to support birefringence, which is in conflict with loop quantum gravity results.

There are also some other ways to discuss LIV from a theoretical viewpoint, although some of them are formerly due to the so-called GZK anomaly, which may in fact be some kind of experimental errors (HiRes Collaboration 2008; Pierre Auger Collaboration 2007). The methods include simply adding tiny (first order or second order or whatever we want) Lorentz-violating terms to a conventional Lagrangian (these may be considered as some kind of Standard-Model Extension) and seeing how they can affect our observations (Coleman & Glashow 1999; Myers & Pospelov 2003), calculating the geodesic in a topological fluctuated *classical* general relativity to get some very complicated results (Yu & Ford 1999), deforming the measure of integration in Feynman graphs (which is equivalent to inventing a new renormalization skill) to get an effective LIV (Alfaro 2005a,b), calculating the graviton induced corrections to Maxwell's equations (Dalvit et al. 2001); however, the resultant speed of light correction in the last method is independent of energy. A recent work by Gogberashvili et al. (2007) deduces the dispersion relation (with no birefringence effect) from the fat brane-world scenario; but the resulting constraint seems to be too strict to trust that model.

Astrophysical experimental (dis)confirmations of LIV often use far transient sources emitting high-energy particles. The common sources are gamma-ray bursts (GRBs), which are cosmological, have very short durations and can emit high-energy photons (Gupta & Zhang 2007) and neutrinos [3]. The other common sources are giant γ-ray flares of active galactic nuclei (AGNs); however, there have not been suitable models for the shapes of the time profiles until now. If energy can affect particle speed by the LIV effect (it's not the same as the effect of particle mass, which becomes unimportant if the particle is sufficiently energetic), as some theoretical works predicted, particles which are emitted simultaneously from the source but with different energies will exhibit a time-delay when observed. The possible ways include testing the time-delay of prompt emission photons from GRBs (Amelino-Camelia et al. 1998; Ellis et al. 2000b, 2003, 2006; Norris et al. 1999) and giant γ-ray flares of AGNs (Biller et al. 1999; MAGIC Collaboration 2007), the time-delay of neutrinos from GRBs (Alfaro et al. 2000, 2002b; Bertolami & Carvalho 2000; Choubey & King 2003; Jacob & Piran 2007), the polarized photons from GRBs (Fan et al. 2007; Gleiser & Kozameh 2001; G.Mitrofanov 2003) and distant galaxies (Kostelecký & Mewes 2001) which should be destroyed by birefringence (Alfaro et al. 2002a; Gambini & Pullin 1999), the synchrotron radiation from the Crab nebula (Jacobson et al. 2002, 2003) which should not be observed if photons can be both superluminal and subluminal but electrons can only be subluminal (Myers & Pospelov 2003). There are also a lot of theoretical works to explain the GZK anomaly by Lorentz-violating terms (Alfaro & Palma 2003; Aloisio et al. 2000; Amelino-Camelia & Piran 2001; Coleman & Glashow 1999), so if the GZK cutoff (Greisen 1966; Zatsepin & Kuz'min 1966) does in fact exist, the inverse proportion may also give some kind of constraints.

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[3] There are really a lot of different models for GRBs to emit ultra-high energy neutrinos, from Waxman & Bahcall (1997) until now. Nearly all of the scenarios are $p + p$ or $p + \gamma \Rightarrow \pi^+ \Rightarrow \nu$, but in different environments. See Waxman (2001a) for a review.

The purpose of this paper is to suggest a different way to (dis)confirm the LIV effect; that is, to directly test the time-delay of ultra-high-energy cosmic-rays (UHECRs) from far away sources. This method may give a larger examinable parameter space and a stronger constraint.

2 CALCULATION

2.1 Naive Time-Delays by the LIV Effect

One possible way to (dis)confirm LIV is simply to test the time-delay of UHECRs from far away sources. Because in mainstream quantum gravity models, the departure of velocities depends on energy *linearly* (Alfaro et al. 2000, 2002a,b; Amelino-Camelia et al. 1997; Ellis et al. 2000a; Gambini & Pullin 1999) in the massless approximation, the time-delay is very sensitive to ultra-high-energy particles. A naive calculation shows that the time-delays are really huge. For example, in the standard cosmological model where H_0, $\Omega_{\rm m}$ and Ω_Λ are the customary cosmological parameters, the propagation equation and time-delay for a massless particle has the form

$$v = c\left(1 \pm \frac{E}{\xi E_{\rm pl}}\right) \tag{1}$$

and

$$\Delta t_{\rm QG} = \frac{1}{H_0}\int_0^z \left(\frac{E}{\xi E_{\rm pl}}\right)\frac{(1+z')dz'}{\sqrt{\Omega_{\rm m}(1+z')^3+\Omega_\Lambda}}, \tag{2}$$

where $E \ll E_{\rm pl}$ is the energy of the particle, z is the redshift of the source, ξ is a free parameter to describe the degree of violation (which we want to restrict) with an assumed typical value of unity, c is the speed of a low-energy photon in a vacuum and $E_{\rm pl}$ is the Planck energy. To give a straightforward example, insert $E = 10^{19.8}\,{\rm eV} \simeq 6.3\times10^{19}\,{\rm eV}$ as the GZK threshold energy, $z = 0.1 \simeq 400\,{\rm Mpc}$ as a nearby source and $\xi = 1$ as a typical dimensionless free parameter. We have

$$\Delta t_{\rm QG} \simeq 7\,{\rm yr}. \tag{3}$$

We choose $z = 0.1 \simeq 400\,{\rm Mpc}$ rather than larger distances to avoid $\Delta t_{\rm QG}$ being too large to be compared with human longevity. In this case, cosmological models are in fact irrespective, so the situation differs from considering less energetic but neutral particles (like photons or neutrinos) that come from more far away sources. Closer sources are also possible (and maybe even better), because nearly all the time-delay effects (including the intergalactic magnetic fields, which we discuss in detail in Section 2.4) caused by propagation depend *linearly* on distance, and distance is irrespective when contrasting which one of the time-delay effects is most important. Remote sources are only needed when $\Delta t_{\rm QG}$ is too small compared with the internal duration of the events themselves, which is only several seconds for GRBs and some other transient sources.

When the energy of the UHECR particles exceeds the GZK threshold, it is less probable that the source is too far away, because the particles lose energy by interacting with CMB photons. The main mechanisms of energy loss during the travel time are photomeson production (Stecker 1968) and e^+e^- pair production (Blumenthal 1970), with their mean free paths already being calculated. However, for the UHECR events with energies larger than the GZK threshold which we have *already* observed, their time-delays by the LIV effect are really interesting, because they should be more energetic and more sensitive to LIV just after being emitted.

However, the calculation of energy loss rate dE/dx (where x is the propagation distance) is very difficult, although what we have to face are trivial details of standard quantum field theory and phase space integrals. Here, we simply use the existing numerical results (Aharonian & Cronin 1994; Cronin 1992) to proceed with our calculations. Time-delays depending on different propagation distances are shown in Figure 1 (bottom). We see that although the time-delays finally tend to the same level as others (because all their energies converge to the GZK threshold energy after a long distance propagation), their differences are tremendous just after being emitted. So, confirming the sources of the UHECR

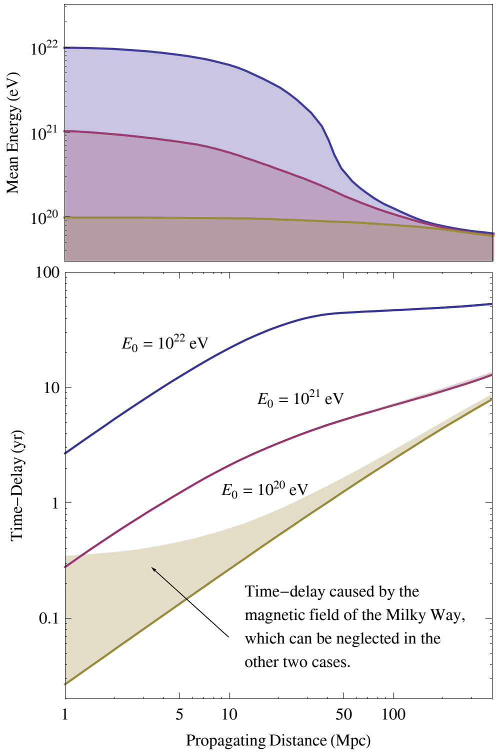

Fig. 1 Top, how particle energies change with propagation distance, if their initial energies are above the GZK threshold (Cronin 1992). The initial energies are chosen to be 10^{20}, 10^{21} and 10^{22} eV. Bottom, the total time-delays caused by the LIV effect. The shadow regions are the time-delays caused by the magnetic field of the Milky Way, which can undoubtedly be negligible in the 10^{21} and 10^{22} eV cases.

events above the GZK threshold energy may also be a way to test the LIV effect, even if the source is nearby.

We have to emphasize here that the particle energy versus propagation distance relation in Figure 1 (top) is based on some statistical results with large samples, because the photomeson interaction is stochastic (see more detailed discussion in Section 2.4.2). The mean free paths for UHECRs with energies $E \gtrsim 10^{21}$ eV are about $10\,\mathrm{Mpc}$, and larger for less energetic ones (Stecker 1968). A certain particle can only suffer from a couple of collisions before being observed, so the LIV constraint by one single UHECR event has its initial measurement errors. However, the observed time-delay can at least give an upper limit for LIV, and we can still improve our result by averaging more events or by using more advanced statistical methods.

2.2 Problem of the Applicability of the Time-delay Equation

To investigate the capacity of the method we propose, there are several problems which have to be considered carefully. The first problem is whether the naive propagation equation can be used for UHECRs. Although we are not sure what the compositions of UHECRs are, air shower data exclude photons or electrons, and the existence (HiRes Collaboration 2008; Pierre Auger Collaboration 2007) of the GZK cutoff (Greisen 1966; Zatsepin & Kuz'min 1966) suggests they are in fact protons or heavier nuclei. Although we cannot rule out other possibilities like exotic particles, we will assume in the context they are protons which are compound and have finite rest mass (discussions with the assumption that they are heavier nuclei like Fe are analogous). Mass is not a serious problem at the energy scale of the GZK threshold [4]. For example, in the case of parameters used above in Equation (3), the time-delay affected by proton mass is only 5×10^{-6} s, much shorter than the $\Delta t_{\rm QG}$ affect by the LIV effect. A more serious problem is the complexity of protons. We all know that a proton is made up of three quarks, therefore using the LIV calculations for *elementary* particles to calculate it will not be justified. A detailed study of quantum chromodynamics (QCD) with Lorentz-noninvariant terms is needed; however, it will certainly be very difficult. We still use the same propagation equation by some scaling arguments (Coleman & Glashow 1999), or by simply regarding its effect as an overall constant coefficient, just like that of turning off QCD [5]. Because the LIV confirmation is still qualitative rather than quantitative, at present, an overall constant can be neglected.

2.3 Problem of the Source

2.3.1 Time Bases

The second problem is how we can choose the time bases, and thus compute the time-delays with a suitable zero point. Because Δt should be typically very long, as shown in Equation (3), it is a serious problem to know when UHECRs should come if we change their energies (because $\Delta t_{\rm QG}$ depends on the energy of the particle) or turn off the LIV effect (as in the classical limit of $E \to 0$). Comparisons of events with photons or other low energy massless particles (which are certainly much less energetic and can be taken as in the $E \to 0$ limit; hence the LIV effect can be neglected) and other UHECRs (with energies different from each other) from the same source can scale the time-delay. The precondition is that we have confirmed the source, or that we are convinced of the fact that different signals come from the same source.

As recent powerful evidence (Pierre Auger Collaboration 2007) shows, UHECRs come from some *extragalactic* sources because they are anisotropic and correlated with the direction of the Super-galactic plane. In that case, the mainstream models for the sources are AGNs (Ginzburg & Syrovatskii 1964; Hillas 1984) and GRBs (Vietri 1995; Vietri et al. 2003; Waxman 1995, 2004; Wick et al. 2004), but other sources which are distributed within the Super-galactic plane are also possible (if they are related to, e.g. galaxy formation or stellar formation, which is always true). The main mechanism is Fermi acceleration but in different environments.

As AGNs are lasting sources, we can hardly know very well *when* the sources emitted the UHECR particles we observed. However, some recent theoretical work (Farrar & Gruzinov 2008) shows that the UHECR emissions are associated with AGN giant flares, with typical wait-times of about 10^3 to 10^4 years (Donley et al. 2002). Because the duration is much longer than the typical time-delay we gave in Equation (3), AGNs can be used in this method if the theoretical work mentioned above is true. GRBs are much better sources, because nearly all of the mainstream central engine models (including collapsars, supernovae and mergers of compact objects) tell us that they are transient and burst only one time in their whole lives. If the emission of UHECRs and the burst itself happen almost at the same time (which is the most reasonable assumption), we can scale the time-delay by the observed low energy γ-rays because they can hardly be affected by mass, electromagnetic fields and the LIV effect. Other sources are also possible, if they emit particles (photons for instance; however, they are not exclusive choices) other than UHECRs, which can be observed by our scientific equipments.

Are these kind of sources practical for our purpose? The distances of the sources mentioned above are all suitable for the constraint that $\Delta t_{\rm QG}$ given in Section 2.1 should not be too large. Short GRBs are often not too far away from us, and there are already a number of GRBs with redshifts $z \sim 0.1$, including a special one (GRB 980425) with an especially small redshift $z = 0.0085$ (Galama et al. 1998). Although the number density of AGNs decreases quickly when $z < 1$, there are already hundreds

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[4] Of course, massive and massless particles are totally different from the viewpoint of quantum field theory. However, we avoid a deep discussion about it because of the still inconsistent theoretical works.

[5] If we turn off QCD, the propagation equation can be used for every elementary particle inside a proton, so the overall effect is only a constant coefficient.

of nearby AGNs that have been observed until now (e.g. the V-C catalog (Véron-Cetty & Véron 2006) has 694 AGNs with redshifts $z \leq 0.024$). Similarly, it is reasonable to assume that other possible sources of UHECRs are not too far away, because UHECRs are not isotropically distributed on the celestial sphere.

2.3.2 *Confirmation of the UHECR Sources*

The assumption in the above paragraph is that we have confirmed the source, or we are convinced of the fact that different signals come from the same source. However, it is not always the case. Notice the fact that since UHECRs are singular events (it seldom happens that the UHECR events have clustering properties), confirming their sources by statistical correlation is very important.

Metrical bias in spatial dimensions are caused by (i) intergalactic magnetic fields and (ii) the uncertainties of detectors; the LIV effect cannot affect the orientation of UHECRs. If the collective effect of (i) and (ii) is small enough, we can confirm the sources by their locations in the celestial sphere; however, this may not be the case. If we assume that the effects of (i) and (ii) are both stochastic, confirmation of the sources is a pure statistical inferential problem. Astrophysical parameters only affect the statistical samples by (i) the UHECR energy band or (ii) the possible correlative time interval. The Pierre Auger Collaboration (2007) has already discussed the statistical correlation between the arrival directions and the positions of known AGNs. The same method can be used for our purpose; however, their arguments do not include the temporal dimension. When discussing the LIV effect, the temporal dimension is very important. Hence, we should insert by hand a possible correlative time interval when choosing the statistical samples; that is, assume that the collective time-delay caused by LIV, intergalactic magnetic fields and other reasons does not exceed this interval. Notice the fact that the observational history of UHECRs and correlative sources are at most several decades, which may be shorter than the collective time-delay; it is a good idea to ignore the temporal dimension and choose all the samples we know to perform the statistical correlation. However, if the intergalactic magnetic fields are sufficiently large, we will never know the sources of UHECRs, no matter whether LIV exists or not.

2.4 Problem of the Intergalactic Magnetic Fields

The third, but the most annoying, problem is the intergalactic magnetic fields. Because the protons are charged, their trajectories will be (Larmour) curved by magnetic fields, and the departures from straight lines will cause extra time-delays. Our method is only suitable when the time-delay $\Delta t_{\rm M}$ by the magnetic fields is less than that by the LIV effect.

Because an UHECR particle keeps constant energy inside some homogeneous magnetic field, the time-delay should be

$$\Delta t_{\rm M} \simeq \frac{1}{24}\frac{D^3}{cr_{\rm L}^2} \simeq 0.79 \left(\frac{D}{3\,{\rm kpc}}\right)^3 \left(\frac{E}{6.3\times 10^{19}\,{\rm eV}}\right)^{-2} \left(\frac{B_\perp}{1\,\mu{\rm G}}\right)^2 {\rm yr}, \tag{4}$$

where D is the linear distance of the trajectory, $B_\perp$ is the perpendicular magnitude of the magnetic field, E is the energy of the particle and $r_{\rm L} = E/(c \cdot eB_\perp)$ is the Larmour radius.

2.4.1 *Comparison with Photons*

For simplicity, we first discuss the way of comparing the UHECRs' time-delays with that of photons, because photons are not affected by the magnetic field, and their time-delays by the LIV effect can be neglected compared to that of UHECRs for their relatively lower energies.

The real trajectory can be divided into three parts: inside the host galaxy, inside our Galaxy and in the intergalactic media (IGM), that is, $\Delta t_{\rm M} = \Delta t_{\rm M,host} + \Delta t_{\rm M,Milky} + \Delta t_{\rm M,IGM}$. We have already chosen the values of D and $B_\perp$ both for a typical galaxy in Equation (4), so $\Delta t_{\rm M,Milky} \sim 0.79\,{\rm yr}$ is the typical value for the time-delay effect of the Milky Way, which can be negligible compared to $\Delta t_{\rm QG}$ we estimated in Equaton (3). Of course, $\Delta t_{\rm QG}$ decreases when the source comes nearer, but the effect of the Milky Way's magnetic field remains unaltered, so it would be troublesome when considering the use of more nearby sources to test LIV, as mentioned in Section 2.1. However, because the time-delay by magnetic fields is absolutely classical, when someday we have fine structure models for the magnetic field of our Galaxy, we can deduct this effect directly[6]. When the UHECR particles are initially more energetic than the GZK threshold, the effect from the Milky Way's magnetic field can always be negligible, as shown in Figure 1 (bottom). The time-delay by the host galaxy of the GRB will not be worse than by our Galaxy, because the energy E will be larger (if it formally exceeds the GZK threshold) or at least equal (if less than the GZK threshold) when just emitted.

However, the effect of the large scale intergalactic magnetic field is more thorny, because until now, we lacked good models for the magnitude and topological structure of the fields. A constraint from the CMB anisotropy (Barrow et al. 1997) gives

$$B_{\rm IGM} < 6.8 \times 10^{-9} (\Omega_0 h^2)^{1/2}\,{\rm G} \sim 4.9 \times 10^{-9}\,{\rm G}, \tag{5}$$

where we choose $\Omega_0 = 1$ and Hubble constant $H_0 = 72\,{\rm km\,s^{-1}\,Mpc^{-1}}$. Another constraint from the observed rotation measure (RM) of quasars (Kronberg 1994) gives

$$B_{\rm IGM} < 10^{-9} \left(\frac{\lambda}{1\,{\rm Mpc}}\right)^{-1/2} {\rm G}, \tag{6}$$

where λ denotes the correlation length (coherence length) of the magnetic fields, with a reasonable assumption that the power spectrum of magnetic fields has a large scale cut-off.

If we assume that the field is conglomerated and homogeneous inside every segment (with typical scale of correlation length λ), the UHECR particle will randomly change its direction due to Larmour's motion, but follows a nearly straight line as a whole. If $B_{\rm IGM}$ is independent of the correlation length λ, the overall time-delay should be

$$\Delta t_{\rm M,IGM} \simeq 1.18 \left(\frac{D}{400\,{\rm Mpc}}\right) \left(\frac{\lambda}{1\,{\rm Mpc}}\right)^2 \left(\frac{E}{6.3\times 10^{19}\,{\rm eV}}\right)^{-2} \left(\frac{B_\perp}{10^{-11}\,{\rm G}}\right)^2 {\rm yr}. \tag{7}$$

When B depends on the correlation length as $B = B_0 \lambda^{-1/2}$ G in Equation (6), the time-delay is

$$\Delta t_{\rm M,IGM} \simeq 1.18 \left(\frac{D}{400\,{\rm Mpc}}\right) \left(\frac{\lambda}{1\,{\rm Mpc}}\right) \left(\frac{E}{6.3\times 10^{19}\,{\rm eV}}\right)^{-2} \left(\frac{B_{0,\perp}}{10^{-11}\,{\rm G}}\right)^2 {\rm yr}. \tag{8}$$

Other possible parameters are denoted in Figure 2. We see that it is needed[7] for our purpose that $B_{\rm IGM}$ is slightly less than the upper limits given by Equations (5) and (6), unless we choose a smaller correlation length.

Things will be more interesting when considering the UHECR particles with energies exceeding the GZK threshold. Noting that $B_\perp$ ($B_{0,\perp}$) and λ in Equations (7) and (8) are independent of the source properties, we may define

$$\eta \equiv 1.18 \left(\frac{\lambda}{1\,{\rm Mpc}}\right)^2 \left(\frac{B_\perp}{10^{-11}\,{\rm G}}\right)^2 \tag{9}$$

in Equation (7) and

$$\eta \equiv 1.18 \times \left(\frac{\lambda}{1\,{\rm Mpc}}\right) \left(\frac{B_{0,\perp}}{10^{-11}\,{\rm G}}\right)^2 \tag{10}$$

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[6] Because the correlation length of the magnetic field in our galaxy should be compared with the scale of the galaxy itself. Furthermore, we know the direction where the UHECR particle is related to the local structure.

[7] It is possible in principle because $B_{\rm IGM}$ remains largely unknown by the intrinsic observational difficulties (Beck et al. 1996).

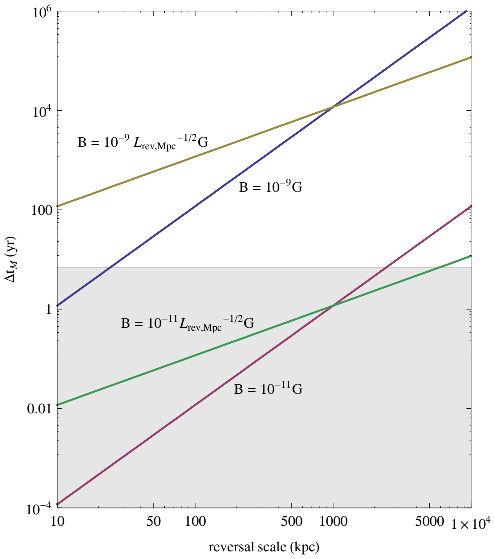

Fig. 2 Time-delay $\Delta t_{\rm M,IGM}$ caused by the intergalactic magnetic fields when $z = 0.1 \simeq 400{\rm Mpc}$ and $E = 6.3 \times 10^{19}$ eV, with different correlation lengths and strengths of the magnetic fields. The horizontal line is $\Delta t_{\rm QG} = 7\,{\rm yr}$, as the example shows in Eq. (3). The method is useful when $\Delta t_{\rm M,IGM} < \Delta t_{\rm M} < \Delta t_{\rm QG}$.

in Equation (8); then the effect of the intergalactic magnetic field has a uniform expression

$$\Delta t_{\rm M,IGM} \simeq \eta \cdot \left(\frac{D}{400\,{\rm Mpc}}\right)\left(\frac{E}{6.3\times 10^{19}\,{\rm eV}}\right)^{-2}\ {\rm yr}. \tag{11}$$

In Figure 3, we calculated $\Delta t_{\rm QG} + \Delta t_{\rm M,Milky} + \Delta t_{\rm M,IGM}$ in all, with $\eta = 1$, 70 and 5000 respectively. $\eta = 5000$ has already saturated the upper bound given by Equations (5) and (6), so $\Delta t_{\rm \ M,IGM}$ cannot be larger. Noting that when $E_0 \geq 10^{21}$ eV, the UHECR particle will absolutely not be affected by magnetic fields if it is not too far away (roughly $D \leq 10\,{\rm Mpc}$), hence the *only* thing that can make a visible time-delay is the LIV effect. HiRes and AGASA have already observed a couple of the UHECR events with energies $E > 3 \times 10^{20}$ eV (AGASA Collaboration 2003; HiRes Collaboration 2005). If their distance $D > 20\,{\rm Mpc}$, their initial energies E_0 will exceed 10^{21} eV, as shown in Figure (1, top). Hence, seeking the sources of UHECRs with energies $E > 3 \times 10^{20}$ eV will tremendously help us to (dis)confirm the LIV effect.

Things will be worse if there exists a *global* cosmic magnetic field, or the fields have structures like filaments or sheets (Ryu 1998). Those will cause larger time-delays. The first trouble can be easily seen from the expression of $\Delta t_{\rm M,IGM}$ given above, because it is equivalent to a huge λ. The second trouble is because, when a magnetic cloud collapses to $1/k$ of its diameter, the magnetic field strength will increase k^2 times its previous value. Although the particle will miss a lot of clouds it used to encounter (only for the case of filaments but not sheets, which makes things worse), we still have

$$B_\perp^2 \lambda^2 \rightarrow \frac{1}{k}(k^2 B_\perp)^2 \left(\frac{1}{k}\lambda\right)^2 = k B_\perp^2 \lambda^2\,, \tag{12}$$

and the same for the $B_\perp^2\,\lambda$ cases.

However, in fact the irrefutable observed anisotropy of UHECRs (Pierre Auger Collaboration 2007) has already given us an upper limit for $B_\perp$ and λ, and also the answer to whether the magnetic field

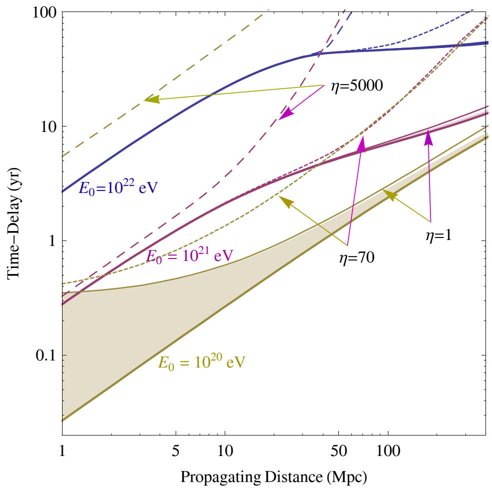

Fig. 3 The thick solid lines and shadow regions are the same as in Fig. 1. In addition, we calculated $\Delta t_{\rm QG} + \Delta t_{\rm M,Milky} + \Delta t_{\rm M,IGM}$ with $\eta = 1, 70$ and 5000 respectively. It seems that finding the sources of the UHECR events with energies $E > 3 \times 10^{20}$ eV will tremendously help us to (dis)confirm the LIV effect. See the context for detail.

has already collapsed to filaments or sheets or not. Note that 20 among 28 of the highest energy events detected by the Pierre Auger Observatory are within a 3.1° circle of nearby AGNs (with distance less than $75\,\mathrm{Mpc}$ away). No matter whether we believe that these UHECR particles originated from those AGNs or not, it is unassailable that UHECRs are unisotropically distributed, and seem to be correlated to the Super-galactic plane. So, the angular dispersion caused by the intergalactic magnetic fields should be less than a couple of degrees. The angular departure inside some homogeneous field is

$$\alpha \simeq \frac{D}{2r_{\rm L}}\,, \tag{13}$$

and the different irrelevant magnetic bulks (with typical size λ) can be considered as a random walk process. The overall angular departure is

$$\alpha \simeq \frac{\lambda}{2r_{\rm L}}\sqrt{\frac{D}{\lambda}} = \frac{\sqrt{D\cdot\lambda}}{2r_{\rm L}}\,. \tag{14}$$

Choosing $D = 100\,\mathrm{Mpc}$ as the typical scale of the Super-galactic plane, we have

$$\alpha_{\rm Milky} \simeq 1.26^\circ \left(\frac{D}{3\,\mathrm{kpc}}\right)\left(\frac{E}{6.3\times10^{19}\,\mathrm{eV}}\right)^{-1}\left(\frac{B_\perp}{10^{-6}\,\mathrm{G}}\right)\,, \tag{15}$$

and

$$\alpha_{\rm IGM} \simeq 4.21^\circ \left(\frac{D}{100\,\mathrm{Mpc}}\right)^{1/2}\left(\frac{\lambda}{1\,\mathrm{Mpc}}\right)^{1/2}\left(\frac{E}{6.3\times10^{19}\,\mathrm{eV}}\right)^{-1}\left(\frac{B_\perp}{10^{-9}\,\mathrm{G}}\right)\,. \tag{16}$$

Notice that when $\alpha_{\rm IGM}$ approaches a couple of degrees, as the above equations show, the upper limits given by Equations (5) and (6) have already been saturated. In addition, because of the fact that $\lambda^{1/2}B_\perp \to \sqrt{k}\lambda^{1/2}B_\perp$, filaments or sheets can also be suppressed.

One question is whether the Pierre Auger data tell us that $\alpha_{\rm IGM}$ should be equal to (rather than less than) several degrees. The answer is absolutely not. $\alpha_{\rm IGM}$ can also be much less (so B and λ can also be much less). Even if we have confirmed some UHECR sources, the angular dispersion can also be caused by reasons other than $\alpha_{\rm IGM}$, for example, the magnetic field of the Milky Way or simply the measurement errors.

We notice that some authors gave a larger $\Delta t_{\rm M,IGM}$ compared to ours given in Equations (5) and (6). Waxman & Miralda-Escudé (1996) gave $\Delta t_{\rm M,IGM} \sim 100\,{\rm yr}$ because their correlation length $\lambda \sim 10\,{\rm Mpc}$ is 10 times larger than ours (equivalent to $\eta = 100$ in our definition). Sigl (2001) gave $\Delta t_{\rm M,IGM} \sim 10^3\,{\rm yr}$ because he chose a really large magnetic field $B \sim 10^9$ G (equivalent to $\eta = 10^4$); however, with a smaller travelling distance D. Waxman (2001b) gave an *upper* bound of $\Delta t_{\rm M,IGM}$, even as large as $10^7\,{\rm yr}$, because his typical magnetic field $B \sim 10^8$ G is really huge. He also argued that $\Delta t_{\rm M,IGM} > 100\,{\rm yr}$ by some statistical reasons of nearby source candidates and the UHECR events above the GZK threshold. The first two estimations are consistent with our constraint from correlation of the Super-galactic plane and the UHECR events, the few discrepancies are just because we choose different typical parameters (which are all possible according to our current knowledge, because we know really little about the true value of B and λ) to write our formulas. We suggest that the anisotropy of UHECRs can give a tighter constraint on intergalactic magnetic field strength B, so we can assure that the upper bound of $\Delta t_{\rm M,IGM}$ at present should be as low as $10^4 - 10^5\,{\rm yr}$. The *lower* bound $\Delta t_{\rm M,IGM} > 100\,{\rm yr}$ can be overcome because we know really little about both possible nearby sources and the UHECRs events, and the estimation is dependent on some details of source models. In addition, all the estimations given above are only suitable for particles with energies below the GZK threshold, because the energy loss is ignored. As we show in Figure (3), the effect of the intergalactic magnetic field is much less important if the energy of the *observed* UHECR event is much larger than the GZK threshold.

2.4.2 Comparison with Other UHECR Events

We can also compare the time-delay with other UHECR events (with slightly different energies) which were emitted nearly simultaneously from the same source. Of course, because the UHECR events are really rare, it may hardly happen.

In this case, blurs in both arrival direction and time-delay have to be analyzed carefully. (i) Blurs can have two reasons. Particles with different energies follow different trajectories, thus leading to different directions and time-delays, because of the random topological distributions of the intergalactic magnetic fields. (ii) At the same time, particles above the GZK threshold energy would interact with CMB photons according to Poisson processes, introducing extra randomness. Waxman & Miralda-Escudé (1996) discussed the blurring effect with UHECRs below the GZK threshold, in which case energy loss by photomeson production can be ignored. At the end of Section 2.1, we have already discussed a little about the influence of the LIV time-delay by stochastic photomeson production.

It is easy to understand that when the particles are extremely energetic, blurs in both arrival direction and time-delay caused by the intergalactic magnetic fields become less important. However, using one of the UHECRs to scale the others may be dangerous, if their energies are large enough (e.g. larger or equal to the GZK threshold) to make us believe that they have suffered photomeson interactions one or more times. Because of the randomness of Poisson arrival photomeson interactions, particles observed with the same energy from the same source may have tremendously different interacting histories and thus possess different time-delays caused by both the intergalactic magnetic fields and the LIV effects.

However, for UHECRs less energetic than the GZK threshold, photomeson production is turned off, and comparison becomes possible. The requirement that the intergalactic magnetic fields should not be very large is the same as in the case of comparing UHECRs with photons, which we have already discussed in Section 2.4.1.

2.5 Problem of the Energy Measurements in Air Shower Detectors

Notice that the energy measurements in different mass compositions of the UHECR events and different air shower detectors have disagreements from each other which cannot be negligible, so it is necessary to discuss the influence of the LIV confirmation by energy demarcation uncertainties. In Section 2.4, we have already discussed two different methods to restrict LIV, the comparison (i) with photons and (ii) with the different UHECRs respectively from the same source.

For the reason that the investigations of the LIV confirmation are, at present, qualitative rather than quantitative, the energy metrical uncertainties are not crucial for the method (i), because it can only introduce an order one coefficient of ξ in Equation (2). When the UHECR events are not too energetic to neglect the time-delay caused by the intergalactic magnetic fields, *absolute* energy measurements are important. However, a global constraint on the collective influence of $\Delta t_{\text{ QG}} + \Delta t_{\text{M,IGM}}$, and hence the upper limits for both Δt_{QG} and $\Delta t_{\text{M,IGM}}$ respectively, are still suitable for our purpose.

For method (ii), things are a little more complicated. Uncertainties introduced by the different assumptions of mass composition are not crucial. The reason is that, when assuming that different UHECRs are the same kind of particles (protons in our context), a mistaken assumptive mass composition can only introduce an order one coefficient of ξ, just as in the case of method (i). However, it is intractable for UHECRs detected by different air shower detectors with different energy metrical techniques. A wiser way is to choose some kind of calorimetric measurements to determine UHECRs' energies by different detectors (like fluorescence light emissions (Linsley 1983; Song et al. 2000)) which are relatively model independent. As a matter of fact, the discussions of LIV are presently still superficial, we may hope that energy demarcations are finer for further investigations of LIV in the near future.

3 DISCUSSION

3.1 Two Known Events

There was an archaeological report about the association of UHECRs and GRBs (Milgrom & Usov 1995). The authors found that GRB 910503 and 921230 are associated with the two highest-energy cosmic-ray shower events, with really small error boxes and time-delays of 5.5 and 11 months respectively. If GRBs are really sources for those two UHECR events, there are very strong constraints both for LIV and the strength of intergalactic magnetic fields (as the time-delay is much shorter than the naive estimation we make in Eq. (3)), because all effects, such as rest mass, magnetic fields and quantum gravity, are addible, and to ignore some of them gives the upper constraint for the rest ones. However, we should not be too serious about drawing conclusions, because they may only be a coincidence.

3.2 Comparison with Other Models

Although there are other constraints of LIV which are much stronger than the method we suggested, the method mentioned above also has its special purpose. Birefringence (Fan et al. 2007; Gleiser & Kozameh 2001; G.Mitrofanov 2003) can only be calculated in the framework of loop quantum gravity but not in other approaches, hence it may be entirely wrong. The synchrotron radiative constraint (Jacobson et al. 2002, 2003) depends on a special theory (Myers & Pospelov 2003), which needs dimension-5 Lorentz-violating terms to induce birefringent photons and subluminal electrons (whose maximum speed cannot converge to c). The inverse proportion of the GZK anomaly may also give some stronger constraints. However, the scattering dynamical discussions are always only one-sided, which means that a scattering channel is open or suppressed only if the effect of LIV is opposite for two relative particles (and therefore their velocities as well as their effective masses are different). Although in the old days, the GZK anomaly is the most important reason for theoreticians to study LIV, its inexistence (HiRes Collaboration 2008; Pierre Auger Collaboration 2007) has not borne down the LIV subjects.

Testing the time-delay of UHECRs is a more direct way to study LIV. It can contain most kinds of theoretical works. If the intergalactic magnetic fields are sufficiently small (which is still absolutely consistent with the observations until now), it may have a larger examinable parameter space for violation scale ξ (in Equation (2)) than using photons or neutrinos. Even if its examinable parameter space is in fact much smaller, for the reasons mentioned above, the other causations are all classical and thus can someday be subtracted by models.

4 CONCLUSIONS

We have suggested to (dis)confirm LIV by simply detecting the time-delay of UHECRs. We considered some other reasons which also cause the time-delay, including the intergalactic magnetic fields. If the energies of the UHECR events we observed are *below* the GZK threshold $6.3 \times 10^{19}\,\mathrm{eV}$, a typical intergalactic magnetic field $B \lesssim 10^{-11}\,\mathrm{G}$ and correlation length $\lambda \lesssim 1\,\mathrm{Mpc}$ may be needed to give an examinable parameter space large enough to constrain LIV. However, for an UHECR event with energy larger than $3 \times 10^{20}\,\mathrm{eV}$, our method is always possible. Because of the fact that we know really little about the intergalactic magnetic field's strength, if it is much smaller than the current upper limit of $B \lesssim 10^{-9}\,\mathrm{G}$, our method may give a larger examinable parameter space and a stronger constraint on LIV than other constraints.

Acknowledgements We would like to thank Yi-Zhong Fan and Xiang-Yu Wang for helpful discussions. This work is supported by the National Natural Science Foundation of China (grants 10221001 and 10640420144) and the National Basic Research Program of China (973 program) No. 2007CB815404.

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