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A statistical study of the interdependence of solar wind parameters

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Abstract Correlation analysis of solar wind parameters, namely solar wind velocity, proton density, proton temperature and mean interplanetary magnetic field (IMF) from the ACE spacecraft data near Earth, was done. To our best knowledge, this study is a novel one since we consider here only the parameters inside the solar wind, including the mean IMF and, hence, the solar wind is a self consistent system. We have proposed a Multiple Linear Regression (MLR) model for the prediction of the response variable (solar wind velocity) using the parameters proton density, proton temperature and mean IMF measured as daily averages. About 60% of the observed value can be predicted using this model. It is shown that, in general, the correlation between solar wind parameters is significant. A deviation from the prediction at the solar maximum is interpreted. These results are verified by a graphical method.

Key words: solar wind — statistical model — correlation — solar wind parameters

1 INTRODUCTION

In this study, we have concentrated on the interdependence among the solar wind parameters, namely solar wind velocity, proton temperature, proton density and mean magnetic field in the solar wind. The interactions among solar wind parameters has not been statistically studied so far. The present study is a novel one since we consider here only various parameters inside the solar wind, including the magnetic field, and the solar wind is treated as a self consistent system. Hence, a Multiple Linear Regression (MLR) model is proposed for the prediction of the solar wind velocity (response variable) based on the explanatory variables proton density, proton temperature and mean magnetic field in the solar wind, collected from the ACE satellite data during January 1998 – May 2006.

Solar wind, usually originating from the lower corona of the Sun, travels outwards through the heliosphere. It is the supersonic outflow into the interplanetary space of plasma from the solar corona extending several solar radii into space. Beyond the Alfvén critical layer of about $10R_{\rm s}$, the solar wind flows with an approximate terminal velocity up to 1 AU. The bulk expansion of solar wind continues to accelerate until it is beyond around $8R_{\rm s}$ and the acceleration is virtually completed by $10R_{\rm s}$ (Böhm-Vitense 1989). SOHO and interplanetary scintillation results show that the fast wind reaches its terminal speed by $10R_{\rm s}$, and has already been accelerated. VLBA and EISCAT measurements show that solar wind velocity reaches a maximum value at about $10R_{\rm s}$ and it attains a terminal velocity at $10R_{\rm s}$ (Harmon et al. 2005). However, the solar wind turbulence plays a major role in the post critical journey of the solar wind. Statistical features associated with turbulence have also been identified in the density fluctuations derived from solar wind observations. Solar wind turbulence shares many statistical properties with compressible isotropic hydrodynamic turbulence (Hnat et al. 2005). As the solar wind moves outwards, velocity and temperature remain coherent, whereas density does not (Richardson 1996). Various parameters, such as solar wind velocity, proton density, proton temperature and mean magnetic field, fluctuate in this scenario. Nonlinearity in waves associated with solar wind, like Alfvén waves, can affect the process and possible wave – particle interactions, causing variations in the magnitudes of parameters. It has become increasingly apparent that Alfvén waves represent a significant contribution to the micro scale fluctuations in the solar wind at 1 AU. From the studies of the cross spectra of magnetic field and velocity fluctuations, Coleman concluded that the Alfvén waves were the principal contributors to the observed fluctuations (Hollweg 1974). The solar wind varies in density, velocity and temperature, and magnetic field properties with the solar cycle, heliographic latitude, heliocentric distance and rotational speed. It also varies in response to shocks, waves and turbulence that perturb the interplanetary flow (Richardson 1996). The magnetic field associated with the solar wind usually undergoes considerable variations in its structure, which really affects the whole set of parameters.

Observations by HELIOS, WIND, ULYSSES, IMP-8 and ACE provide the solar wind parameters between Earth and Sun. For the present study, we mainly considered the data around 1 AU from the Advanced Coronal Explorer (ACE) satellite which was launched in 1997 and supported by NOAA. ACE has four instruments (EPAM, MAG, SIS, SWEPAM) to provide data. We mainly compiled ACE SWEPAM and ACE MAG data for our studies. ACE SWEPAM provides solar wind proton density, proton temperature and proton velocity. ACE MAG provides the interplanetary magnetic field data.

Recent observational studies concentrate on various parameters of the solar wind around 1 AU (Horton et al. 2006; Boberg et al. 1999; Yermolaev & Yermolaev 2003). The three solar wind parameters B_z, n and V together with planetary magnetospheric K_p index for solar cycle 21 and 22, are downloaded from the National Space Science Data Center (NSSDC), and are used for the prediction of the K_p index (Boberg et al. 1999). Real Time data from the ACE satellite has been used for the prediction of the A_L and D_{st} indices using WINDMI model (Horton et al. 2006). A hybrid model was implemented, and the real-time prediction of the K_p index was done (Boberg et al. 1999). All of these models were used to study the interaction between solar wind parameters and magnetospheric plasma indices. The harmonic behavior of solar wind velocity, temperature and number density was investigated by analyzing data from the Pioneer XII satellite during 1980 – 1987 (Das & Ghosh 1999). Studies by Das & Ghosh were about the periodicity of the solar wind velocity, density and temperature. They concluded that the solar wind velocity and temperature have the same periodicity of about 37 days whereas the density has a smaller periodicity of about 14 days.

The equations in our model consist of the parameters, namely solar wind proton density, proton temperature, mean magnetic field and solar wind velocity. In our model, we have used the above mentioned parameters as the input, and a predicted solar wind velocity (V_{sw}) as the output. From our studies, it is observed that about 60% of the observed value of the solar wind velocity can be predicted by this model. From the graphs, it is also confirmed that solar wind velocity and temperature remains coherent while the velocity decreases nonlinearly with an increase in proton density. The velocity also varies nonlinearly with the mean magnetic field in the solar wind. It is also noted that the correlation between the parameters in the solar wind is significant from the correlation coefficients. Why? We propose that the two terms in our model expression for velocity denoted by V_0 and that due to thermal effects (ion acoustic waves), contribute more than 80% of the proposed velocity which acts as an intervening thread for the required correlation.

In the next section, the proposed model for velocity prediction is presented and the physical significance of each term in the equation is discussed. In Section 3, the data collected from the ACE satellite were used for the statistical analysis using the MLR model. The significance of the model results are analyzed. The mathematical formulation of the problem is also discussed in this section. We have obtained circles and parabolas as isocontours for the modeled equations in Section 3.1. Section 4 deals with the discussion of the results.

2 THE PROPOSED MODEL FOR THE STUDY

It is evident that the solar wind is a nonlinear complex system. As a first attempt to study the nonlinear behavior of the multiparameter low density plasma, we propose a linear model based on the MLR model. Solar wind velocity is the combined effect of the inertial uniform speed along with electron wave velocity which is manifested by Langmuir waves, thermal velocity represented by ion acoustic waves and the Alfvén velocity due to magnetic field. Hence, to study the contributions due to each of them, we use the following equation

$$V_{\rm sw} = V_0 + \sqrt{\frac{4\pi}{k^2 n}\frac{e^2}{m_e} n(t)} + \sqrt{\frac{\gamma}{M}\frac{K}{T_0} T(t)} + \frac{B(t)}{\sqrt{4\pi\rho}}\,, \tag{1}$$

where V_0 is the uniform speed due to inertia, k is the wave vector, n is the number density, B is the mean magnetic field, T is the ion temperature, K is the Boltzmann constant and M is the proton mass. From the above equation, it is evident that the solar wind velocity depends on number density, proton temperature and magnetic field.

The term multiple regression has come to describe the process by which several variables are used to predict another variable. Through multiple regression analysis, a variable can be predicted on the basis of several predictive factors. Multiple regression model is closer to reality, and it is generally used in practice. Thus, to predict the solar wind velocity by knowing proton density, proton temperature and mean magnetic field, we propose a Multiple linear regression model with solar wind velocity as the response variable, and proton density, proton temperature and mean magnetic field as the explanatory variables or regressors. The proposed MLR model is,

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3\,, \tag{2}$$

where β_1, β_2 and β_3 are the regression coefficients and β_0 is a constant (Y- intercept) which is obtained from the regression analysis and is given in Appendix A.

We have fitted the MLR model for the consolidated data as shown below,

$$V_{\rm sw}(t) = 354.907 + 0.0216 X_1(t) + 0.00117 X_2(t) - 2.3925 X_3(t)\,, \tag{3}$$

where $V_{\rm sw}(t)$ is the solar wind velocity, $X_1(t)$, $X_2(t)$ and $X_3(t)$ are the proton density, proton temperature and mean magnetic field at any time, respectively; these act as the inputs to the model. In Equation (3), the coefficients are obtained from regression analysis and are given in Appendix A. Comparing Equation (3) with Equation (1), $\sqrt{\frac{4\pi}{k^2 n}\frac{e^2}{m_e}}$ = 0.0216, $\sqrt{\frac{\gamma}{M}\frac{K}{T_0}}$ = 0.00117 and $\frac{1}{\sqrt{4\pi\rho_0}}$ =2.3925. where ρ_0 is the mean density of solar wind. The second term in Equation (3) is equivalent to the velocity of the electron plasma wave (Langmuir wave), the third term represents thermal velocity due to ion acoustic waves and the fourth term represents the Alfvén velocity. The model implies the idea that for an inhomogeneous system in time, the mean velocity can be contributed by the velocities derived from electron plasma wave velocity, thermal velocity of protons and the Alfvén velocity.

The potential from which this velocity can be derived is

$$\phi = \int m \left(\frac{d(v_e + v_t + v_A)}{dt}\right) \cdot dr\,. \tag{4}$$

Since the solar wind consists of charged particles, the driving potential can be modeled as $\Phi\,(\phi_{\,0}, \phi_1, \phi_2)$, which is expressed in the following form,

$$\Phi = \phi_0 \exp\left(\frac{-x}{\lambda_D}\right) + K\,(T_e + \gamma T_i) - \left(\frac{B^2}{8\pi}\right)\,, \tag{5}$$

where γ =3 and $\lambda_D = \sqrt{\frac{K}{8\pi}\frac{T}{ne^2}}$. As a first approximation, we took the simplest case of Equation (5). Our calculations proved that using this model, 60.53% of the variability of the observed value can be

Table 1 Year Wise Variability of the Observed Data

Year	R^2 (variability) %
1998	63.8
1999	66.16
2000	68.34
2001	27.45
2002	60.14
2003	65.45
2004	62.65
2005	68.87
2006	81.3

predicted. The density and temperature have a slight positive impact on the solar wind velocity, and they are statistically significant. $X_1(t)$, $X_2(t)$ and $X_3(t)$ are the time-varying parametric data obtained from the ACE satellite. In this model, the magnetic field affects the solar wind velocity in an inverse manner; this can be seen by analyzing the consolidated data during January 1998 – May 2006.

3 DATA FITTING AND ANALYSIS

The three solar wind parameters, namely, the mean magnetic field ($B_{\rm av}$) in nanoTesla, proton density (n) in cm^{-3} and proton temperature (T) in Kelvin, together with the solar wind velocity in km s^{-1} for January 1998 – May 2006, were downloaded from NOAA centers collected by the ACE satellite. The input parameters are daily average values to attain uniformity. The daily average data of the solar wind parameters were consolidated year-wise as well as parameter-wise, and were subjected to statistical analysis using the MLR model. The MLR model is fitted for the data of each year from January 1998 to May 2006. The significance of the fitted models was tested using ANOVA and the model results were analyzed. In multiple regression, we use R^2 (variability) as the symbol for determining coefficients. For an accepted model, R^2 should have a value greater than 0.50. Almost all the models fitted were explaining a significant part of the variability in the data. All the models except that for 2001 have $R^2 > 0.60$ as given in Table 1. For the model fitted using the consolidated data from the period of January 1998 to May 2006, $R^2 = 0.6053$.

The year-wise variability is shown in Table 1. Also, the matrix of correlation is constructed, which shows a significant positive correlation between the prediction variable and the other parameters. From the analysis, it is observed that the velocity and temperature are coherent in all cases, and the effect of temperature on velocity is also statistically significant. However, the proton density has an adverse effect on the solar wind velocity in major cases for the respective period. Density enhances the velocity for the years 1998 and 2000. From this study, it is evident that the solar wind velocity decreases nonlinearly as the proton density increases. The effect of magnetic field on the velocity is significant for the years 1998, 2004 and 2005. However, it is not statistically significant for the years 1999, 2000, 2001, 2002 and 2003. For the periods near "solar maximum," the magnetic field is not statistically significant. The physical meaning is that during solar maximum, when the Sun is more active, the magnetic field has no significant effect on the enhancement of solar wind velocity. In the year 2001, when we have obtained variability, only 27% of our model, which is meant to describe the quiet Sun, deviates much from the proposed model. During "solar maximum," CME's and SEP's originating from the solar surface play a more prominent role than the normal solar wind in the solar wind magnetospheric interactions. From January to March 2001, about 134 Plasma Transfer Events (PTE) were observed (Lundin et al. 2003). This fact is supported by the frequent geomagnetic storms during solar maximum. However, in the year 2006, which is near the solar minimum, our model has a variability of 0.81, which is in very good agreement with the observed values.

For the graphical analysis, we have studied the interdependence of two variables. Here density, temperature and the mean magnetic field associated with the solar wind are the proposed parameters, which are influencing the speed of solar winds.

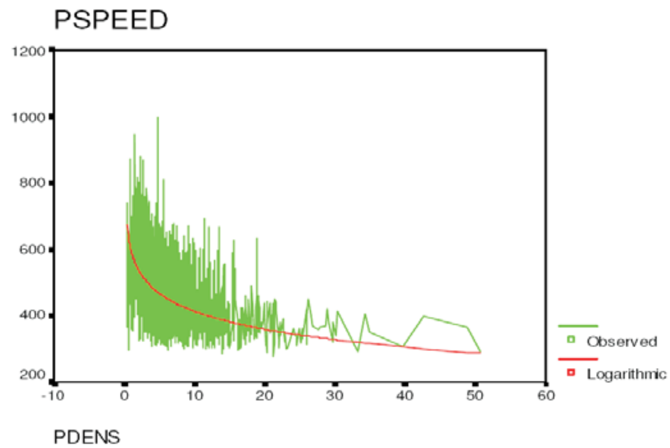

Fig. 1 Variation of solar wind proton speed (velocity) with proton density (for observed and fitted values). The abscissa is in cm^{-3} and the ordinate is in km s^{-1}.

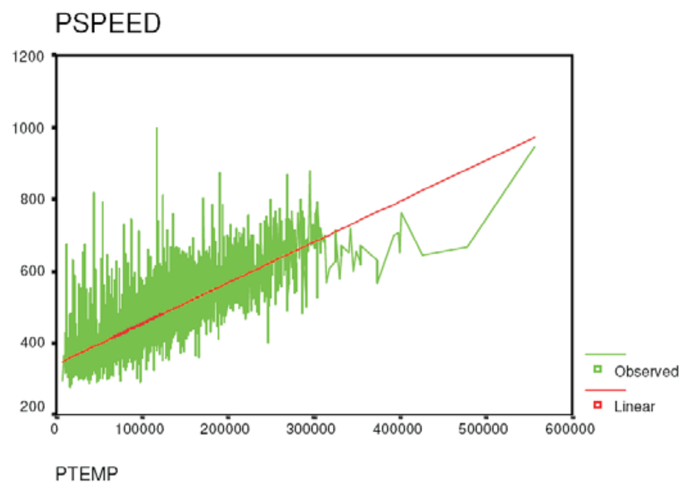

Fig. 2 Variation of solar wind proton speed (velocity) with proton temperature (for observed and fitted values). The abscissa is in Kelvin and ordinate is in km s^{-1}.

Variations of solar wind velocity with proton density are studied in Figure 1 and the relationship between them seems to be nonlinear. Figure 1 depicts the variation of solar wind speed with proton density in an inverse manner. This is due to the fact that the proton density and the magnetic field are dependent on each other due to the presence of Alfvén waves. The variation in number density is related to the variation of the magnetic field.

Variations of solar wind velocity with proton temperature are studied in Figure 2 and they are linearly correlated. Figure 2 explains that the solar wind speed also increases with temperature increase. This fact arises since the component of velocity due to the ion acoustic velocity increases with temperature.

In Figure 3, variations of solar wind velocity with mean magnetic field are studied and their relationship is also nonlinear. Solar wind speed slightly increases with an increase of average magnetic

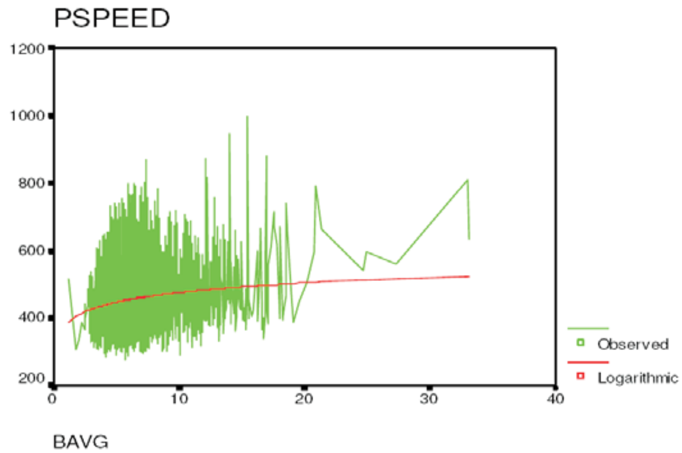

Fig. 3 Variation of solar wind proton speed (velocity) with B avg. (for observed and fitted values). The abscissa is in nano Tesla and the ordinate is in km s^{-1}.

field as shown in Figure 3, even though the contributing term due to the magnetic field in the modeling equation is negative. This term is associated with the Alfvén speed which also depends on the number density. Hence, the contribution of a changing magnetic field without a change in proton density could never be practically realized for the solar wind studies.

In multiple regression, we have used R^2 (variability) as the symbol for coefficient of determination. For an accepted model, R^2 should have a value greater than 0.50. Almost all the fitted models explain a significant part of the variability in the data. All the models except that for 2001 have $R^2 > 0.60$ as given in Table 1.

3.1 MATHEMATICAL FORMULATION OF THE PROBLEM

The pressure profile for an incompressible fluid can be expressed as a function of velocity V and magnetic field B using MHD equations (Sasidharan et al. 1995)

$$\frac{\nabla P}{\rho} = \frac{(\nabla \times B) \times B}{\rho} - (V \cdot \nabla) V - \frac{\partial V}{\partial t} . \tag{6}$$

The above model has been modified by us to include the adiabatic effects. Hence, the velocity profile for an incompressible fluid can be expressed as a function of magnetic field, pressure, number density and temperature using the following MHD equations

$$\frac{\partial V}{\partial t} = \frac{(\nabla \times B) \times B}{\rho} - (V \cdot \nabla) V + \frac{\gamma}{\gamma - 1} T - \frac{\nabla P}{\rho} , \tag{7}$$

$$\nabla \times (V \times B) - \frac{\partial B}{\partial t} = 0 , \tag{8}$$

$$\nabla \cdot V = 0$$

and

$$\frac{P}{\rho^{\gamma}} = \text{constant (say } \alpha) . \tag{9}$$

The solar wind plasma is represented by a cylindrical column of plasma of length L and radius R; here ρ is the mass density, and the force due to gravity is neglected. The above equations form a closed set of equations in variables (V, B, ρ, T). We have assumed force free and steady conditions. Hence Equation (7) can be modified as

$$\nabla\left[\frac{1}{2}V^2+\frac{1}{2}B^2+\frac{P}{\rho}+\frac{\gamma}{\gamma-1}T\right]=0\,. \tag{10}$$

Hence

$$V^2+B^2+2\alpha\rho^2+3T=\text{constant}\,, \tag{11}$$

where α is the adiabatic constant arising from Equation (9). We have obtained circles and parabolas as isocontours for the following equations as in Figure 4.

$$V^2+B^2=\text{constant (say }\varepsilon_1)\,, \tag{12}$$

$$V^2+3T=\text{constant (say }\varepsilon_2)\,, \tag{13}$$

$$B^2+3T=\text{constant(say }\varepsilon_3)\,. \tag{14}$$

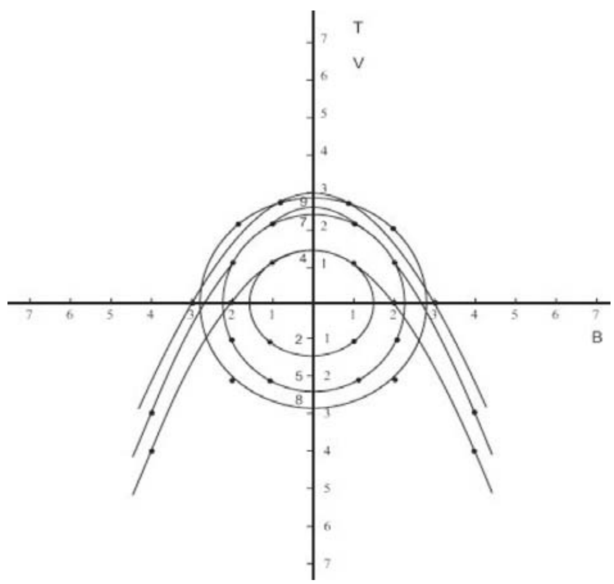

Fig. 4 Circles and the parabolas are the isocontours obtained for Equations (12) and (14). For both abscissa and ordinate, normalized arbitrary values are used for B, V and T.

In Figure 4, graphical solutions exist only for a finite number of constants. When the two fields are force free, both V and B are identical or V is proportional to B, and this can be picked up from the solutions. We conclude that the force free field becomes a finite set. Figure 4 represents a graphical method of solving Equation (11). In this figure, we assume ρ^2 = a constant plane on which V, B and $3T$ are solved. V^2+B^2 = constant is a family of circles. We have drawn Equations (13) and (14) which give a family of parabolas. These two families of curves intersect at certain points. These intersecting points are the solutions of the above Equations (12) and (14) on the ρ^2 = constant plane. The solutions are given in Table 2.

Table 2 Solutions of Equations (12) and (14) obtained from graphical method.

$\|V\|$	$\|B\|$	$\|T\|$	ε_1	ε_3
0.4	2.2	0.4	5	6.39
1	1	1	2	4
2	1	2	5	7
2.9	1	2.9	9.41	9.7
0.3	2.8	0.3	7.9	8.7
1	2.6	1	7.76	9.76

4 RESULTS AND CONCLUSIONS

The fitted regression model explains 60.53% of the variability of the consolidated data from January 1998 to May 2006. Proton temperature and solar wind velocity are coherent while the response due to density and mean magnetic field on solar wind velocity are nonlinear. We have observed that during the periods near "solar maximum," the magnetic field has no significant role to play on solar wind velocity. For all the years except 2001, the variability of the data is more than 60%. For the year 2001, due to solar maximum, the magnetic activity of the solar surface is maximum and our model, which is meant for quiet sun conditions, deviates much from the proposed model. However, for the year 2006, which is near solar minimum, our model has a very good agreement with the observed values. From the analysis of the correlation matrix, we have obtained that there is a significant correlation between solar wind velocity with the parameters proton density, proton temperature and average magnetic field in the solar wind. An inference from the results implies that the model is sensitive to the magnetic conditions of the Sun.

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Appendix A: RESULTS OF THE ANALYSIS OF THE CONSOLIDATED SOLAR WIND DATA IN ORIGIN (JANUARY 1998 TO MAY 2006)

Table A.1 Multiple Regression of consolidated Data: Independent parameters: density (B), temp (C), B avg. (D). Dependent parameter: Solar Wind Velocity (A).

Parameter	Value	Error	t-Value	Prob $> \|t\|$
Y-Intercept	354.90717	3.69122	96.14897	<0.0001
B-density	0.02162	9.28136E–4	23.2967	<0.0001
C-temp	0.00117	2.08176E–5	56.11906	<0.0001
D-B avg.	–2.39425	0.48074	–4.98033	<0.0001
R-Square (COD)	Adj. R-Square	Root-MSE (SD)		
0.6053	0.6024	70.47899		
ANOVA Table:				
Item	Degrees of Freedom	Sum of Squares	Mean Square	F Statistic
Model	3	1.8573E7	6.19101E6	1246.35541
Error	2859	1.42015E7	4967.28794	
Total	2862	3.27745E7		
Prob>F				
<0.0001				

Table A.2 Correlation Matrix

	PSPEED	PDENS	PTEMP	BAVG
PSPEED	1.0000			
PDENS	0.2680	1.0000		
PTEMP	0.6925	–0.0299	1.0000	
BAVG	0.1224	–0.0375	0.2741	1.0000

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