## LETTERS

# Predicting the start and maximum amplitude of solar cycle 24 using similar phases and a cycle grouping \*

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Abstract We find that the solar cycles 9, 11, and 20 are similar to cycle 23 in their respective descending phases. Using this similarity and the observed data of smoothed monthly mean sunspot numbers (SMSNs) available for the descending phase of cycle 23, we make a date calibration for the average time sequence made of the three descending phases of the three cycles, and predict the start of March or April 2008 for cycle 24. For the three cycles, we also find a linear correlation of the length of the descending phase of a cycle with the difference between the maximum epoch of this cycle and that of its next cycle. Using this relationship along with the known relationship between the rise-time and the maximum amplitude of a slowly rising solar cycle, we predict the maximum SMSN of cycle 24 of  $100.2\pm7.5$  to appear during the period from May to October 2012.

Key words: Sun: cycle — Sun: activity

### **1 INTRODUCTION**

The 13-point smoothed monthly mean sunspot number (SMSN or  $\overline{R}$  hereafter) is a more widely used index in various aspects, especially in determining the characteristics of a solar cycle in spite of that papers are devoted to the topic of how to determine the extrema of a solar cycle (e.g. Wilson 1995; Harvey & White 1999; Du et al. 2006 and references therein). Great progress in solar cycle prediction has been made, but how to make a reasonable prediction for the next cycle is still an open problem (Ohl 1966; Wang et al. 1975; Brown 1986; Kunches 1993; Kane 1999; Li et al. 2001; Wang et al. 2002; Lantos 2006; Du et al. 2006, 2008). Predictions for cycle 24 also show quite a large spread (Kane 2007; Pesnell 2008).

The goal of this work is to carry out a prediction for the next cycle, cycle 24, by using similar descending phases and a derived relation as well as a known relation between parameters of solar cycles. We predict that cycle 24 will begin in March or April 2008, and have a maximum SMSN of  $100.2\pm7.5$  to appear during the period from May to October 2012.

### 2 METHODOLOGY, DATA ANALYSIS, AND RESULTS

To select cycles whose descending phases are similar to that of cycle 23, the following conditions are used: (1) the selected cycles should have their maximum SMSNs around 120.8 with a deviation of

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**Fig.1**  $\overline{R}_{max}$  vs  $T_d$  for solar cycles 1~22. The number beside the point indicates the number of the cycle. Cycles in the square are considered to have descending phases similar to the one of cycle 23 in their descending phases.

**Table 1**  $\overline{R}_{max}$  and  $T_d$  of Similar Cycles and Their Correlations with Cycle 23

Cycle No.	$\overline{R}_{\max}$	$T_d$	Data Set No. 1		Data Set No. 2		Data Set No. 3		Data Set No. 4	
		(years)	$\gamma$	$\gamma_{ m C}$						
9	131.6	7.83	0.892	0.285	0.909	0.307	0.922	0.376	0.931	0.505
11	140.5	8.33	0.980	0.285	0.980	0.307	0.979	0.376	0.980	0.505
20	110.6	7.58	0.985	0.285	0.986	0.307	0.987	0.376	0.988	0.505
23	120.8	>7.5								

 $\overline{R}_{\max}$  and  $T_d$ : the maximum SMSN and descending length of the cycle;

 $\gamma$ : correlation coefficient between a similar descending phase and that of cycle 23 in the period covered by the data set;  $\gamma_C$ : critical correlation coefficient with a level of confidence of 99%;

Data Sets: see Table 2.

**Table 2** Time periods, lengths and values of  $\gamma$ ,  $\gamma_C$  and  $\chi^2$  for four data sets.

Set No.	Time period	Length (months)	$\gamma$	$\gamma_C$	$\chi^2$
1	Apr. 2000~Jun. 2006	75	0.966	0.285	1.12
2	Apr. 2000~Dec. 2006	81	0.971	0.285	1.08
3	Apr. 2000~Jun. 2007	87	0.975	0.285	1.08
4	Apr. 2000~Mar. 2008	95	0.975	0.307	1.06

 $\gamma$ : correlation coefficients for descending phases of cycle 23 with the average cycles;

 $\gamma_C$ : critical correlation coefficient

 $\pm 20$ , where 120.8 is the peak SMSN of cycle 23, and (2) the descending lengths should be larger than 7.5 years, consistent with the one of cycle 23. The relation between the maximum SMSN  $\overline{R}_{\text{max}}$  with the descending length  $T_d$  for solar cycles 1–22 is given in Figure 1 which shows that cycles 9, 11 and 20 satisfy the conditions for the selection. Parameters for the three cycles and cycle 23 are listed in the first three columns of Table 1. The correlation coefficients for each of the three cycles with cycle 23 in their descending phases for different data sets, which are given in Table 2, are respectively listed in the 4th, 6th, 8th and 10th columns of Table 1. The relevant critical correlation coefficients are also given in the same table. One can find that the correlations between cycle 23 with cycles 9, 11 and 20 in their descending phases are significant.

Using the SMSNs in the three selected descending phases and Equation (1), we make a new time sequence  $\overline{R}_i$  (3CA) whose behavior is expected to be able to describe, morphologically, the time profile of the SMSN for the descending phase of cycle 23. In case of short cycles with similar descending



**Fig. 2** Best fittings for data set 1 to set 4 respectively. The solid lines show the time profile of the observed SMSNs for the descending phase of cycle 23, and the dotted-dashed lines show the time profile of the average cycle. The dotted lines indicate the predicted beginning time for cycle 24.

phases, we consider the required SMSNs in the beginning years of the respective next cycle.

$$\overline{R}_i(3CA) = \frac{1}{3} \sum_{n=1}^{3} \overline{R}_{i,n} , \qquad (1)$$

where  $\overline{R}_i(3CA)$  is the average SMSN for month *i* of the 3 similar descending phases,  $\overline{R}_{i,n}$  is the SMSN for month *i* of cycle *n*, and n = 1, 2, and 3 indicates cycles 9, 11 and 20, respectively. Obviously, the new time sequence can be used as a model for predicting the beginning time of cycle 24 after calibrating its date with the available data for the descending phase of cycle 23.

The date calibration is done by a linear least-square fitting process. The function  $\chi^2$  is used in the process, which is defined by

$$\chi^{2} = \frac{1}{N} \sum_{i=1}^{N} \frac{[\overline{R}_{i}(23) - \overline{R}_{i}(3\text{CA})]^{2}}{\overline{R}_{i}(23)},$$
(2)

where  $\overline{R}_i(23)$  is the SMSN for month *i* in the descending phase of cycle 23 and *N* is the number of the SMSN data. The best fitting is obtained when  $\chi^2$  reaches its minimum and then the date calibration is completed. In fact, we carry out the calibrating procedure using each of the four data sets shown in Table 2, where the obtained values for  $\chi^2$  and the correlation coefficients for the four best fittings, and the relevant critical correlation coefficients are also shown. We use the 4 best fittings, as shown in Figure 2 respectively, to predict the beginning time of March ~ April 2008 for cycle 24.

In addition, we find that a linear correlation of the descending length of a cycle,  $T_{d,n}$ , with the difference between the maximum epochs of the cycle and the next cycle,  $K_{n,n+1}$ , exists for the cycle-pairs, cycles 9–10, cycles 11–12, and cycles 20–21, shown as follows:

$$K_{n,n+1} = 2.9543T_{d,n} - 11.2449, (3)$$

with an error of  $\pm 0.1$  years and  $T_{d,n}$  and  $K_{n,n+1}$  are both in units of years. The correlation coefficient for Equation (3) is 0.997 which is equal to the critical coefficient for the level of confidence of 0.95. It seems reasonable to assume that Equation (3) may be applied to the pair of cycles 23~24. Using Equation (3) and the predicted length of the descending phase of cycle 23, 7.92~8.00 years, the length of the ascending phase for cycle 24,  $T_{a,24}$ , can be derived as 4.1~4.5 years, or 4.3±0.2 years. Wang et al. (2002) pointed out that cycle 24 should be put in the slowly rising cycle group, and that the maximum amplitude of cycle 24,  $\overline{R}_{max,24}$ , can then be predicted using the formula (eq. (2) in Wang et al. 2002) for slowly rising cycles as shown by

$$\overline{R}_{\max} = 263.30 - 37.93T_{a,n}, \qquad (4)$$

where  $T_{a,n}$  is in year. This linear correlation has its correlation coefficient of 0.963 which is larger than the critical one, 0.606, at 0.99 level of confidence. So, taking  $T_{a,24} = 4.3 \pm 0.2$  years, the SMSN maximum amplitude for cycle 24 is predicted to be  $100.2\pm7.5$  by Equation (4).

### **3 DISCUSSION**

Solar cycle prediction is important for space weather forecasts. As we mentioned above, how to make a successful prediction for the next cycle, cycle 24, is still a task which we are facing, and predictions published for the cycle show quite a large spread (Kane 2007; Obridko & Shelting 2008; Pesnell 2008). In this work, we predicted the start and peak time of cycle 24 by a proposed method using similar phases for the descending phase of cycle 23, instead of similar cycles (Wang & Han 1997). Li et al. (2002, 2005) also studied the similarity of cycle 23 with other cycles and the prediction for cycle 24. The precision of a prediction by the similar phase method depends not only on the conditions for the selection of the similar phases but also on the number of the selected phases. In addition, we put cycle 24, as an even cycle, into the slowly rising cycle group which is different from the pure even cycle group (Obridko 1995; Wang et al. 2002), and then predict the maximum SMSN for cycle 24. Our prediction for the start of cycle 24 (March ~ April 2008) supports the one made by the solar cycle 24 prediction panel (*http://users. telenet. be/j.janssens/SC24.html*). Our prediction for the maximum SMSN of cycle 24 is less than 140±20 and is comparable with 90±10, both of which are also given by the panel.

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