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Constraining Ω with the fluctuation of the local Hubble flow

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Abstract We present an analysis of the fluctuation of the local Hubble flow using 350 galaxies in the Local Volume ($D < 5\,\mathrm{Mpc}$, hereafter LV) with accurate measurements of distances, positions and radial velocities, and compare the results with the theoretical prediction of the local Hubble flow induced by density perturbations. This allows us to set a useful constraint on the local Ω parameters: $\Omega_M \sim 0.6$ and $\Omega_\Lambda \sim 0.7$, which may serve as compelling evidence for the existence of dark energy in the local Universe.

Key words: cosmology: dark matter — cosmology: large-scale structure of universe — galaxies: peculiar — techniques: radial velocities

1 INTRODUCTION

The measurement of the Hubble constant, which started as early as the 1920s (Hubble 1929), is still a hot subject today. Despite the vast improvement in accuracy, the precise value of the Hubble constant needs further determination: the estimates range from 60 to $75\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$. Sometimes, the estimates are discrepant, even if they are determined from the same data. For example, Riess et al. (2005) found $H_0 = 73\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$, with errors of $4\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$. Sandage et al. (2006) found $H_0 = 62.3 \pm 1.3\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$. Neal (2007) argues that several astronomical effects, such as period-dependent metallicity correction, the calibration of the type Ia supernova distance scale, and differences in SNe Ia luminosities as a function of environment, will affect H_0.

Besides the nontrivial systematics brought by the astronomical effects, the Hubble constant itself can be distorted because of cosmological reasons. When the Cosmological Principle holds, the Hubble constant should be uniform and isotropic over large enough scales. So according to the Friedmann model, the Hubble constant can be defined by the Robertson-Walker metric,

$$H^2 = \frac{8\pi G\rho}{3} + \frac{\Lambda}{3} - \frac{k}{R^2}\,. \tag{1}$$

If there is a density perturbation, the Hubble constant will ripple. Such an intrinsic fluctuation can be written as:

$$(H + \Delta H)^2 = \frac{8\pi G(\rho + \Delta\rho)}{3} + \frac{\Lambda}{3} - \frac{k}{R^2}\,. \tag{2}$$

A density perturbation would produce motion of objects, which deviates from a uniform Hubble flow. Such deviated motion is known as the peculiar velocity. McClure & Dyer (2007) note that whether to conceive of the Universe expanding non-uniformly or whether to conceive of it expanding uniformly with superimposed peculiar velocities is more than just a conceptual issue.

Hudson et al. (2004) found that the peculiar velocities with respect to the Cosmic Microwave Background (CMB) are correlated such that volumes of space of order $100\,\mathrm{Mpc}$ in radius are moving with obvious bulk velocities. Moffat and Tatarski (1995) estimated the observational effects assuming we were to inhabit a local void. They found the data were better fit by a model with a local void in a homogeneous universe. Zehavi et al. (1998) also suggest that we may inhabit an underdense region. The local Hubble constant behavior in the underdense region has been discussed by Wu et al. (1995).

According to Equation (2) and the work mentioned above, the density perturbation can significantly perturb the uniform Hubble flow. However, a very puzzling fact, which was first recognized by Sandage et al. (1972) and further confirmed by others (see also Sandage 1986, 1999; Teerikorpi 1997; Ekholm et al. 1999; Giovanelli et al. 1999), is that the local Hubble flow was quite cold, where the sky distribution of the galaxies looks extremely inhomogeneous, owing to the presence of galaxy groups (Karachentsev 2005) and voids (Tikhonov & Karachentsev 2006). This fact was shown to be more convincing by the most complete and accurate data on radial velocities and distances of nearby galaxies (Karchentsev et al. 2003). Recently, some papers stated that the coldness of the local Hubble flow in the LV is attributed to the existence of dark energy (Baryshev et al 2001; Chernin et al. 2004; Teerikorpi et al. 2005; Chernin et al. 2006; Chernin et al. 2007a,b), which is supported by Macciò et al. (2005) with a set of N-body simulations. However, Hoffman et al. (2007) argue that the local Hubble flow around LG-like objects in the $\Lambda\mathrm{CDM}$ ($\Omega_\Lambda > 0$) simulation and the OCDM ($\Omega_\Lambda = 0$) simulation is indistinguishable, which means that the local Hubble flow is not affected by the Λ term in the simulations.

The real answer to this puzzle is still under discussion. Macciò (2005) suggests that it may be related to the mean density around the objects. In this paper, we try to theoretically estimate the fluctuations of the local Hubble flow. With the assumption that the geometry of the LV is described by a Friedmann-Robertson-Walker (FRW) metric, the fluctuation of the local Hubble flow can be predicted theoretically by the halo approach. The result would be the theoretical statistical limit of the fluctuation of the Hubble flow induced by density perturbations. Compared with the measured fluctuation of the local Hubble flow, we can roughly constrain the Ω parameters of the LV. Although the real metric of the LV should be more complicated, we hope that the current work will be helpful for understanding the properties of the LV.

2 FLUCTUATION OF THE HUBBLE CONSTANT IN LV

2.1 Theoretical Estimation

The matter density perturbation can be written as

$$\rho(\boldsymbol{x}) = \bar{\rho}(1 + \delta(\boldsymbol{x}))\,, \tag{3}$$

where $\bar{\rho}$ is the mean density of the Universe. The Hubble constant averaged over a large volume with a scale of $|\boldsymbol{x}|$ is given by

$$\begin{aligned} H(\boldsymbol{x})^2 &= \frac{8\pi G\bar{\rho}}{3}(1+\delta(\boldsymbol{x})) + \frac{\Lambda}{3} - \frac{c^2 k}{R^2} \\ &= \overline{H}^2(1 + \Omega_M \delta(\boldsymbol{x})), \end{aligned} \tag{4}$$

where Ω_M is the density parameter of $\frac{8\pi G\bar{\rho}}{3\overline{H}^2}$, $\Omega_\Lambda = \frac{\Lambda}{3\overline{H}^2}$, $\Omega_k = \frac{-c^2 k}{R^2\overline{H}^2}$ with $\Omega_M + \Omega_\Lambda + \Omega_k = 1$. Then, the two-point correlation function of H at scale $\boldsymbol{r}$ is related to the two-point correlation function of matter by

$$\xi_H(\boldsymbol{r}) = \frac{1}{4}\Omega_M^2 \xi_m(\boldsymbol{r})\,. \tag{5}$$

The fluctuation of the Hubble constant induced by a density perturbation is

$$\delta H = \sigma_H = \sqrt{\xi_H(r)}\,. \tag{6}$$

The two-point correlation function of matter can be obtained by the Halo model as

$$\xi_m(r) = \frac{1}{2\pi^2}\int_0^\infty P(k)\frac{\sin kr}{kr}k^2 dk\,, \tag{7}$$

where $P(k)$ is the power spectrum of the density perturbation given by

$$\begin{aligned} P(k) &= P_{1h}(k) + P_{2h}(k)\,, \\ P_{1h}(k) &= \int_{m_g}^\infty n(z,M)dM\left|\frac{\rho_h(M,k)^2}{\bar\rho}\right|\,, \\ P_{2h}(k) &= P_{lin}(z,k)\left[\int_{m_g}^\infty \bar n(z,M)b(z,M)\frac{\rho_h(M,k)}{\bar\rho}dM\right]^2, \end{aligned} \tag{8}$$

in which the power spectrum is separated into the Poisson term $P_{1h}(k)$ and the clustering term $P_{2h}(k)$; $b(z, M)$ is the bias parameter, m_g is the mass of a galaxy and $\bar\rho$ is the mean matter density.

2.2 Observational Data

Our goal is to derive the fluctuation of the local Hubble flow using galaxies in the LV. We restrict ourselves to the tracers whose distances to the Local Group center are less than $5\,\mathrm{Mpc}$. We adopt a sample of 350 galaxies in the LV from observational data summarized in the catalog of neighboring galaxies by Karachentsev et al. (2004), in which the positions, distances and radial velocities to the Local Group center are all available. The typical relative errors of the distances are $10\% \sim 15\%$.

With the assumption that the distribution of the galaxies in the LV is isotropic, we can ignore the sky distribution of the galaxies and construct a simple estimator for the fluctuation of the local Hubble flow from the samples by

$$\sigma_H = \sqrt{\frac{1}{N-1}\sum_{i=1}^{n}\frac{(v_i-\bar v)^2}{(\bar v)^2}}\,, \tag{9}$$

where v_i is the relative radial recessional velocity between each pair of galaxies.

However, the real sky distribution of the galaxies in the LV is anisotropic, and we observe only the line-of-sight component of the peculiar velocity $s_A = \boldsymbol{r}_A \cdot \boldsymbol{v}_A/r \equiv \hat r_A \cdot \boldsymbol{v}_A$, rather than the three-dimensional velocity $\boldsymbol{v}_A$; it is therefore not possible to compute v_{12} directly. Instead, following Ferreira et al. (1999) (see also Górski et al. 1989), we propose measuring the fluctuation of the local Hubble flow with the mean difference between the radial velocities of a pair of galaxies

$$\langle s_1 - s_2\rangle_\rho = v_{12}\hat{\boldsymbol{r}}\cdot(\hat{\boldsymbol{r}}_1+\hat{\boldsymbol{r}}_2)/2\,, \tag{10}$$

where $\boldsymbol{r} = \boldsymbol{r}_1 - \boldsymbol{r}_2$ and $\hat{\boldsymbol{r}}$ is the unit vector. To estimate v_{12}, we use the simplest least squares technique, which minimizes the quantity

$$\chi^2(r) = \sum[(s_A - s_B) - p_{AB}\tilde v_{12}(r)/2]^2\,, \tag{11}$$

where $p_{AB} = \hat{\boldsymbol{r}}\cdot(\hat{\boldsymbol{r}}_A + \hat{\boldsymbol{r}}_B)$, $r = |\boldsymbol{r}_A - \boldsymbol{r}_B|$. The condition $\partial\chi^2/\partial\tilde v_{12} = 0$ implies

$$\tilde v_{12}(r) = \frac{2\sum(s_A - s_B)p_{AB}}{\sum p_{AB}^2}\,. \tag{12}$$

The variance of the mean difference between the radial velocities is

$$\sigma_{(s_A - s_B)} = \sqrt{\frac{\sum[(s_A - s_B) - p_{AB}\tilde v_{12}(r)/2]^2}{N-1}}\,. \tag{13}$$

So the new estimator of the fluctuation of the Hubble flow reads as

$$\sigma_{\tilde H} = \sqrt{\frac{\sum[(s_A - s_B) - p_{AB}\tilde v_{12}(r)/2]^2}{(N-1)(\tilde v_{12}^2)}}\,. \tag{14}$$

3 RESULTS AND DISCUSSION

Before we proceed to the observational results, we present our theoretical prediction of the Hubble flow at all scales under the ΛCDM model with the cosmological parameters determined from the WMAP cosmological parameters table $(\Omega_M, \Omega_\Lambda, \sigma_8, n_s, h_0) = (0.268, 0.732, 0.776, 0.704)$ (Spergel 2007). The prediction is shown in Figure 1. Limited by linear expansion of Equation (2), only the result with sufficiently small δ is meaningful. When the scale increases, δH, i.e., the ripple of the Hubble constant, decreases. This gives the theoretical statistical limit of the fluctuation of the Hubble constant introduced by the density perturbations. At $100\,\mathrm{Mpc}$ scales, the fluctuation of the Hubble constant will only be $\sim 0.01\%$. Therefore, when the samples for determining the Hubble constant are distributed over large enough scales, the fluctuation of the Hubble constant induced by the density perturbations will be small enough to be neglected.

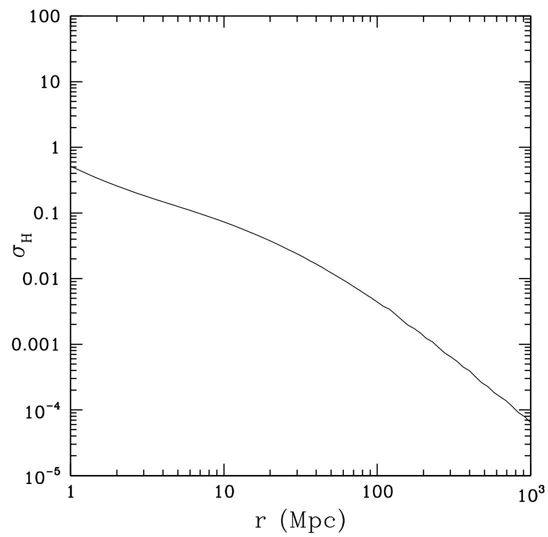

Fig. 1 Fluctuation of the Hubble flow induced by density perturbation. Here, we adopted the cosmological parameters from the WMAP with $(\Omega_M, \Omega_\Lambda, \sigma_8, n_s, h0) = (0.268, 0.732, 0.776, 0.704)$.

For the result at small scales, e.g., in the LV, the fluctuation of the Hubble constant will increase to $\sim 5\%$. From Equation (5), we can see that the fluctuation of the Hubble constant is determined by two quantities. One is the density fluctuation, and the other is the matter density parameter Ω_M. If the dark energy exists in the LV, the fluctuation of the Hubble constant will decrease with the decrease of the matter density parameter. If we have measured the fluctuation of the local Hubble flow and compared it with the theoretical estimation, we could set a rough but useful constraint on the matter density parameter Ω_M. We proceed by comparing the measured σ_H for each of the distance bins with the prediction discussed in Section 2.1. The result is demonstrated in Figure 2. The solid points with error bars represent the measured fluctuation of the Local Hubble constant σ_H from Equation (9). The open squares are the result with Equation (14), which is a more reasonable estimator. The theoretical predictions of σ_H with different Ω parameters and a constant $\Omega_\Lambda = 0.7$ are plotted as solid lines. The fluctuation predicted by the de Sitter universe ($\Omega_\Lambda = 0, \Omega_M = 1$) will be significantly larger than the measured fluctuation. The existence of dark energy can decrease the fluctuation of the local Hubble flow. The density of dark energy in the ΛCDM model does not vary with the scale. So, we only compare the theoretical predictions of σ_H with a constant $\Omega_\Lambda = 0.7$, in which the parameter $\Omega_\Lambda = 0.7$ is confident for the large scale universe and should be same of that for the LV.

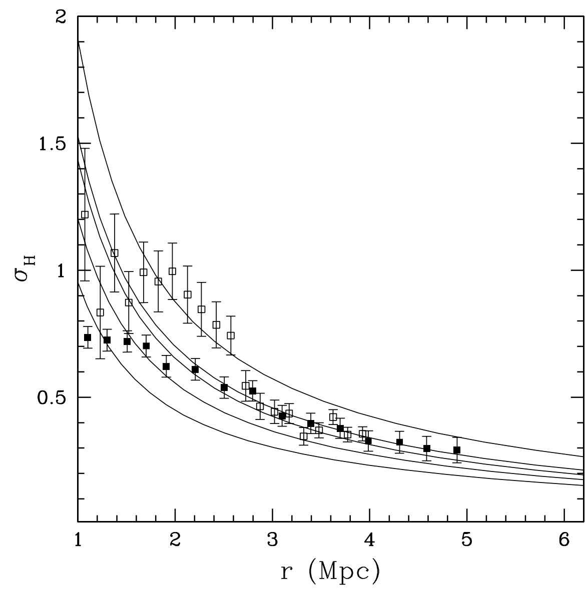

Fig. 2 Variation of δ_H with scale r. The open squares represent the pair-wise $\sigma_{\tilde{H}}$ at different scales of R measured from the nearby galaxies with the new estimator given by Eq. (14) and the solid squares represent the pair-wise σ_H directly from the radial velocity with Eq. (9). The solid lines from the bottom to top represent the theoretical predictions of the σ_H with $\Omega_M = 0.5$, 0.63, 0.75, 0.8 and 1.0.

At scale $R < 2\,\mathrm{Mpc}$, the measured points for both of the two estimators obviously deviate from the predicted lines, and this trend turns at scale $R \sim 2\,\mathrm{Mpc}$. This implies that the Hubble law emerges at scale $R = 1.5 \sim 2\,\mathrm{Mpc}$, as suggested by Sandage (1986, 1999) and Ekholm et al. (1999). The measured σ_H from Equation (14) is not only larger than the one from Equation (9) at scale $R = 2 \sim 2.5\,\mathrm{Mpc}$, but also more scattered than the one from Equation (9) at all scales, implying that the latter estimator seems more stable than the former in this work. The reason may be that the former estimator is more sensitive to the capacity of the sample than the latter, and in this work, the sample needed to investigate this is not large enough. Furthermore, a shallow selection function of the sample can induce a large fluctuation (Ferreira et al. 1999), which would also introduce systematic errors. The measured points from both estimators at scale $R = 2.5 \sim 5\,\mathrm{Mpc}$ are roughly consistent with the prediction using parameters $\Omega_M = 0.65$, $\Omega_\Lambda = 0.7$ and negative Ω_k. The local Ω_M seems much higher and more reasonable than the result estimated by Karachenstsev et al. (2004), which is only 0.1 and $2 \sim 3$ times as low as the global density of matter. Besides, the density perturbation can induce the intrinsic σ_H, astronomical effects or the measurement will also bring the systematics into the measured σ_H. Therefore, the measured σ_H could only constrain the limit of the local matter parameter Ω_M. We can still conclude that the low σ_H in the LV may be a manifestation of the dark energy in the LV.

In summary, comparing the measured fluctuation of the local Hubble flow with the theoretical prediction, we can roughly constrain the local Ω_M and Ω_Λ. This may serve as compelling evidence for the existence of dark energy in the LV.

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