# Fitting formulae for the effects of binary interactions on lick indices and colors of stellar populations \*

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Received 2007 December 16; accepted 2008 October 16

**Abstract** More than about 50% of stars are in binaries, but most stellar population studies take single star stellar population (ssSSP) models, which do not take binary interactions into account. In fact, the integrated peculiarities of ssSSPs are different from those of stellar populations with binary interactions (bsSSPs). Therefore, it is necessary to investigate the effects of binary interactions on the Lick indices and colors of populations in detail. We show some formulae for calculating the difference between the Lick indices and colors of bsSSPs, and those of ssSSPs. Twenty-five Lick indices and 12 colors are studied in this work. The results can be conveniently used for calculating the effects of binary interactions on stellar population studies and for adding the effects of binary interactions into present ssSSP models. The electronic data and fortran procedures in the paper can be obtained on request from the authors.

Key words: galaxies: stellar content — galaxies: elliptical and lenticular, cD

#### 1 INTRODUCTION

In the golden era of studying the formation and evolution of galaxies, evolutionary stellar population synthesis has been an important technique for such works, as some stellar characteristics (e.g., stellar age and metallicity) of galaxies can be determined via this technique. Many stellar population synthesis models, e.g., Worthey (1994), Buzzoni (1995), Bressan et al. (2003), Vázquez & Leitherer (2005), Bruzual & Charlot (2003), Fioc & Rocca-Volmerange (1997), Vazdekis et al. (2003), Delgado et al. (2005), and Zhang et al. (2005), were brought forward and have been widely used for stellar population studies. However, the above models, except the one of Zhang et al. (2005), are single star stellar population (ssSSP) models that did not take the effects of binary interactions into account. According to the results of Han et al. (2001), more than 50% of stars in the Galaxy are in binaries and binaries evolve differently than single stars. The real stellar populations of galaxies and star clusters consist of not only single stars, but also binary stars. Thus ssSSPs are different from the real populations of galaxies and star clusters. In fact, binary evolution can affect the integrated peculiarities (e.g., the spectral energy distributions in UV bands) of stellar populations significantly (see, e.g., Han et al. 2007). Therefore, the effects of binary evolution should be taken into account when modeling the stellar populations of galaxies and star clusters.

A few works have tried to investigate the effects of binary evolution on stellar population synthesis. For example, Zhang et al. (2005) tried to model populations via binary stars. In addition,

<sup>\*</sup> Supported by the National Natural Science Foundation of China.

Li & Han (2008b) built an isochrone database for quickly modeling binary star stellar populations (bsSSPs) and a rapid model (hereafter RPS model) for both ssSSPs and bsSSPs. In particular, Li & Han (2008c) investigated the detailed effects of binary interactions on the results of stellar population synthesis and the results of stellar population studies. The results can help us to understand how the results obtained via ssSSPs are different from those obtained via bsSSPs, when using H $\beta$ -[MgFe] (Thomas et al. 2003) and two-color methods. According to the results of Li & Han (2008c), when we use ssSSP models to measure the stellar ages and metallicities of galaxies, we will obtain obviously younger ages or lower metallicities compared to the real values of the populations, using the H $\beta$ -[MgFe] and two-color methods, respectively. However, there is no clear relation between the real metallicities and the fitted (via ssSSPs) results of populations. One can refer to Li & Han (2008c) for more details. In this case, it is difficult to get more accurate information about the stellar metallicities of galaxies via ssSSP models, which gives insight into the chemical evolution of galaxies. Furthermore, the previous work only shows the results for the  $H\beta$ -[MgFe] method, when taking Lick indices for reference, but some other methods and indices are also used in these types of studies. Thus, it is necessary to further investigate the effects of binary interactions on the results of stellar population studies obtained via various Lick indices. The metallicity range of the above bsSSP models (Zhang et al. 2005; Li & Han 2008b) does not seem wide enough (see Li et al. 2006), as it only covers the metallicity range poorer than 0.03 ( $Z \le 0.03$ ). This is limited by the star evolution code. If we can give the relation between the effects of binary interactions and the stellar-population parameters (age and metallicity), we will be able to understand the populations of galaxies and star clusters further, and we will have more detailed investigations about galaxy formation and evolution in the future. Therefore, it is valuable to study how the effects of binary interactions on integrated peculiarities of populations change with stellar age and metallicity. We try to advance our understanding of this topic in this work. As a result, a few formulae for describing the relations between the effects of binary interactions on integrated indices (Lick indices and colors indices) and stellar-population parameters are presented.

The structure of the paper is organized as follows. In Section 2, we introduce the stellar population model used in the paper. In Section 3, we show the fitting formulae for the changes of 25 Lick indices caused by binary interactions when compared to those of ssSSPs. In Section 4, we make similar investigations of 12 colors of populations. Finally, we give our discussion and conclusions in Section 5.

#### 2 STELLAR POPULATION MODEL

The *RPS* model of Li & Han (2008b) is used in this investigation, because this is the most suitable model. The model calculated the integrated peculiarities (0.3 Å SEDs, Lick indices and colors) of both bsSSPs and ssSSPs with two widely used initial mass functions (IMFs) (Salpeter and Chabrier IMFs). Each bsSSP contains about 50% of stars that are in binaries with orbital periods less than 100 yr (the typical value in the Galaxy, see Han et al. 1995). Binary interactions, such as mass transfer, mass accretion, common-envelope evolution, collisions, supernova kicks, angular momentum loss mechanisms and tidal interactions are considered when evolving binaries via the rapid stellar evolution code of Hurley et al. (2002). Therefore, the *RPS* model is suitable for studying the effects of binary interactions on stellar population synthesis. The details about the model can be seen in the paper of Li & Han (2008b) and Li & Han (2008c). For convenience, we take stellar populations with the Salpeter IMF as our standard for investigations in this work, while the results obtained via populations with Chabrier IMF are also presented.

# 3 FITTING FORMULAE FOR THE EFFECTS OF BINARY INTERACTIONS ON 25 LICK INDICES

Lick indices are the most widely used indices in stellar population studies, because they can disentangle the well-known stellar age-metallicity degeneracy (Worthey 1994). Making use of an age-sensitive index (e.g.,  $H\beta$ ) together with a metallicity-sensitive index (e.g., [MgFe], see Thomas et al. 2003), the stellar age and metallicity of a population can be determined. Thus, investigating the effects of binary interactions on the Lick indices of stellar populations is important. The work of Li & Han (2008c)

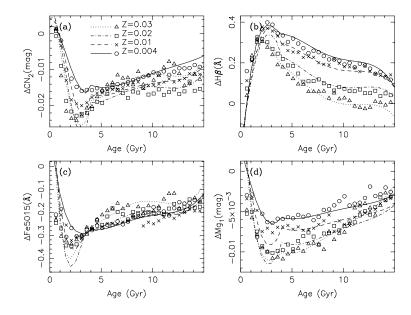


Fig. 1 Comparison of fitted and original values for the effects of binary interactions on four Lick indices. Circles, crosses, squares and triangles are for the metallicities of  $Z=0.004,\,0.01,\,0.02$  and 0.03, respectively. Solid, dashed, dash-dotted and dotted lines show the fittings for the above four metallicities, respectively. The values on the y-axis are calculated by subtracting the Lick indices of a bsSSP from that of its corresponding (with the same age and metallicity) ssSSP. Panels a), b), c) and d) are for  $CN_2$ ,  $H\beta$ , Fe5015 and  $Mg_1$ , respectively.

showed that binary interactions make the  $H\beta$  index smaller while some metal-line indices become larger as compared to those of ssSSPs. It leads to younger age estimates when we take ssSSPs for research. However, in that work, only the results obtained via the  $H\beta$ -[MgFe] method are compared to the real values of the populations. Some other Lick indices, e.g., Mg  $_2$ , H $_3$  and H $_4$  and H $_4$  are also widely used in studies (e.g., Gallazzi et al. 2005). Therefore, it is necessary to study the effects of binary interactions on more Lick indices and give the quantitative relations between binary effects and stellar-population parameters. Here, we study 25 widely used indices and fit the relations between the changes caused by binary interactions and the stellar-population parameters (age and metallicity), via a polynomial fitting method. The results can be used to calculate the differences between the 25 Lick indices of two kinds of populations with small errors (typically less than 0.03 Å or mag). All Lick indices are on the Lick system (see, e.g., Worthey 1994). When compared to ssSSPs, the effects of binary interactions on Lick indices can be calculated from stellar age and metallicity, by

$$\Delta I = \sum_{i=1}^{5} (C_{i1} + C_{i2}Z + C_{i3}Z^{2})t^{i-1},$$
(1)

where  $\Delta I$  is the change of a Lick index caused by binary interactions, and Z is stellar metallicity, while t is stellar age. The detailed coefficients for our standard investigation are shown in Tables 1 and 2.

Those for populations with Chabrier IMF are shown in the Appendix. To be clear, in Figures 1, 2 and 3, we compare the changes calculated by Equation (1) with the original values obtained in the work. Note that we only show the fittings for 12 widely used Lick indices here, because the fittings for other indices are similar. As we see, for the indices shown, the values calculated by the above equation are consistent with those obtained directly by comparing the Lick indices of bsSSPs and ssSSPs, with typical errors of 0.03 Å or mag. Therefore, the fitting formulae being presented can be used to calculate the differences of Lick indices of bsSSPs and ssSSPs, using the age and metallicity of populations.

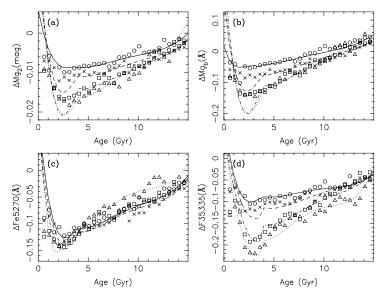
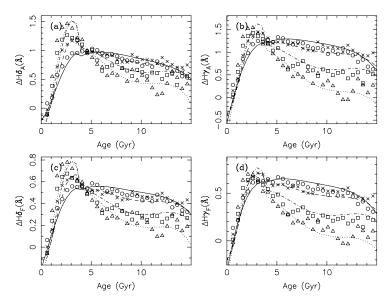


Fig. 2 Similar to Fig. 1, but for Mg<sub>2</sub>, Mg<sub>b</sub>, Fe5270 and Fe5335.



**Fig. 3** Similar to Fig. 1, but for  $H\delta_A$ ,  $H\gamma_A$ ,  $H\delta_F$  and  $H\gamma_F$ .

In addition, the results show that binary interactions make age-sensitive indices (e.g.,  $H\beta$ ,  $H\delta_A$ ,  $H\delta_F$ ,  $H\gamma_A$  and  $H\gamma_F$ ) of a bsSSP larger than those of its corresponding (with the same age and metallicity) ssSSP, while making metallicity-sensitive indices (e.g., Mg or Fe indices) of a bsSSP less than that of its corresponding ssSSP. This is similar to that shown in the paper of Li & Han (2008c). Furthermore, it is shown that the differences between Lick indices of bsSSPs and ssSSPs increase with age when stellar age is small (  $\lesssim 2.5$  Gyr), and they decrease as stellar ages become larger. This results from the star sample (i.e., the fraction of binaries and the relation between the masses of the two components of each binary) in stellar populations. As a whole, the values calculated via the fitting formulae obtained by the paper reproduce the evolution of the difference between the Lick indices of bsSSPs and ssSSPs.

 $\begin{tabular}{ll} \textbf{Table 1} & Coefficients for Eq.~(1). The coefficients are obtained via stellar populations with Salpeter IMF and can be used for populations younger than 4 Gyr (Age <math display="inline">< 4$  Gyr).

Index	j	$C_{1j}$	$C_{2j}$	$C_{3j}$	$C_{4j}$	$C_{5j}$
	1	0.0061004	-0.0124136	0.0044131	-0.0010174	0.0000862
$CN_1$	2	-0.7411409	3.6764976	-3.4540761	0.9481077	-0.0789850
	3	30.8122734	-119.0088796	98.2842545	-25.5632396	2.0732871
	1	0.0031344	-0.0060648	0.0018661	-0.0005851	0.0000607
CNI		-0.3472402	2.6820361	-2.8891002	0.8270163	-0.0703722
$CN_2$	2					
	3	20.0763558	-92.9530724	84.0072412	-22.6637506	1.8767287
	1	0.0116927	-0.0652076	0.0542984	-0.0142708	0.0011505
Ca4227	2	2.6086437	1.4449921	-5.0762513	1.6986709	-0.1529064
Ca4227					-6.3483260	
	3	-71.6010754	65.3934655	-2.6926390		0.8599647
	1	0.5779073	-1.3565672	0.6261188	-0.1240817	0.0083961
G4300	2	-50.8998137	134.5958812	-94.7989188	23.2438808	-1.8011864
	3	1740.9893518	-4249.3881941	2828.2830757	-668.7058103	50.9809128
	1	0.1013725	-0.1526486	-0.0164836	0.0124241	-0.0013244
Fe4383	2	11.0894652	-9.0004221	-12.0126956	4.9108205	-0.4569935
10.000	3	-75.5186140	-252.7173536	549.1707675	-171.1008580	14.7310943
C-1455	1	-0.0047432	0.0186972	-0.0197387	0.0040785	-0.0002709
Ca4455	2	6.5346797	-11.0259844	2.2737411	0.1733499	-0.0467838
	3	-98.3181765	98.4157661	49.1921063	-27.0189711	2.8005540
	1	0.0550772	-0.1274444	0.0222163	-0.0009561	-0.0000671
Fe4531	2	18.2809537	-32.2387726	11.1472063	-1.0492667	-0.0001559
	3	-313.2684866	374.3927838	-24.2302367	-31.1448033	4.4257825
	1	-0.0717857	0.2203558	-0.1810129	0.0435325	-0.0033410
Fe4668	2	27.2930010	-66.0581381	35.0658395	-6.9644712	0.4740158
10.000	3	-444.6620029	913.3400658	-347.7358123	51.9244201	-2.7150328
**	1	-0.4120752	0.9250669	-0.3741119	0.0622541	-0.0036591
$H_{\beta}$	2	26.4414835	-74.8894421	48.1881455	-11.2855830	0.8626001
	3	-805.0295817	2280.7328848	-1494.4764420	349.3195439	-26.6678020
	1	-0.0313311	-0.0500534	-0.0035580	-0.0037280	0.0007775
Fe5015	2	61.9785871	-107.1406403	40.6549692	-4.5784440	0.0744308
	3	-1490.9225648	2286.4644788	-734.9273433	46.6787692	3.5030950
	1	-0.0011669	0.0020035	-0.0025450	0.0006800	-0.0000553
$Mg_1$	2	1.0977834	-1.6954874	0.4725610	-0.0265407	-0.0021062
11181	3	-23.8411177	30.5518215	-4.5747311	-1.0811535	0.1935160
	1	-0.0036972	0.0016391	-0.0002221	-0.0002245	
Ma						0.0000340
$Mg_2$	2 3	2.6546194	-4.0910074 93.1999501	1.1294327 -25.0701341	-0.0429887	-0.0081361
		-63.2072524			0.5006265	0.2481080
	1	-0.1159519	0.0564072	0.0367218	-0.0207147	0.0023015
${ m Mg_b}$	2	36.6456419	-47.5077586	8.8070472	1.2388375	-0.2750762
	3	-857.5383388	1080.0808279	-207.5498638	-26.8404813	6.2521890
	1	0.1443967	-0.2658498	0.0907173	-0.0111382	0.0003331
Fe5270	2	-8.5450001	12.6759249	-7.5470939	1.5641500	-0.0991560
	3	306.8420600	-637.6523032	403.3286320	-89.3943857	6.3626512
	1					
Fe5335		0.1121199	-0.2143513	0.0776458	-0.0095392	0.0002385
	2	-6.1989089	11.7530508	-8.2144476	1.6722406	-0.0990246
	3	144.7440581	-352.4412737	218.9898664	-44.2594619	2.7694391
Fe5406	1	-0.0167597	-0.0094937	-0.0069105	0.0022746	-0.0001900
	2	18.1518764	-28.2532163	9.0539150	-0.8243859	-0.0014094
	3	-423.9701730	596.6530611	-158.4751506	4.4556600	1.3344343
	1	0.0643501	-0.1189715	0.0352640	-0.0029869	-0.0000373
Fe5709	2	-6.8927462	9.4915829	-2.5896330	0.0686243	0.0256903
	3	206.1312747	-349.9572257	151.0604011	-22.2154935	0.9194484
	3	200.1312747	-347.7314431	131.0004011	-44.4134933	0.7174404

Index	j	$C_{1j}$	$C_{2j}$	$C_{3j}$	$C_{4j}$	$C_{5j}$
	1	-0.0193549	0.0201941	-0.0227794	0.0068166	-0.0006059
Fe5782	2	5.8734339	-8.3105593	3.3800221	-0.6333537	0.0435585
	3	-145.6309054	234.0700497	-130.6085667	28.1896236	-2.0267932
	1	-0.1041283	0.0782159	-0.0131910	-0.0028878	0.0005590
$\mathrm{Na_D}$	2	35.8174765	-52.1821670	16.3021886	-1.2441912	-0.0404260
	3	-928.3347064	1387.5693179	-484.7687597	49.3839423	-0.3018862
	1	-0.0046972	-0.0031675	0.0086166	-0.0031273	0.0003031
$TiO_1$	2	2.5376502	-3.1526728	0.3308384	0.2054016	-0.0316500
	3	-70.3829758	95.4015845	-17.9727317	-3.0711691	0.6438944
	1	-0.0089825	-0.0020920	0.0096106	-0.0036755	0.0003656
$TiO_2$	2	3.9165850	-4.9045789	0.8074595	0.2009710	-0.0375203
	3	-108.7948038	146.4687943	-32.9218404	-2.5469339	0.7759394
	1	-0.3400174	0.7802154	-0.3437893	0.0824056	-0.0066930
${ m H}\delta_{ m A}$	2	36.1459627	-178.0526116	166.3449941	-45.4458738	3.7573350
	3	-1381.3144000	5283.2867289	-4334.0624242	1125.3443000	-90.9465139
	1	-0.7586478	1.6470466	-0.6800223	0.1306246	-0.0086650
${ m H}\gamma_{ m A}$	2	58.8625436	-210.0639058	174.8989489	-45.3356756	3.6089852
	3	-2043.5700337	6433.0522877	-4883.0699797	1207.4959655	-93.9809592
	1	-0.3060721	0.7125074	-0.3205038	0.0683519	-0.0050475
${ m H}\delta_{ m F}$	2	30.6677991	-121.8747783	101.4148311	-26.5930774	2.1537733
	3	-1053.7758076	3611.9972771	-2723.9526032	682.2344274	-54.0667213
·	1	-0.4440136	0.9835898	-0.4158944	0.0801404	-0.0053903
${ m H}\gamma_{ m F}$	2	35.2952498	-120.5372324	95.1523223	-24.3827679	1.9426707
	3	-1145.3060328	3586.2350776	-2658.3958905	655.6380015	-51.2487311

**Table 1** — Continued.

## 4 FITTING FORMULAE FOR THE EFFECTS OF BINARY INTERACTIONS ON 12 COLORS

Because colors can also be used for stellar population studies, we fit the formulae for calculating the color changes for the case of binary interactions when compared to ssSSPs. One can refer to, e.g., Li et al. (2007), Li & Han (2007), Li & Han (2008a), Li & Han (2008c), for the application of colors in stellar population studies. Some Johnson system colors, Sloan Digital Sky Survey system (hereafter SDSS-ugriz system) colors, and some composite colors that consist of both Johnson and an SDSS-ugriz magnitudes are studied. We only study the colors of populations with  $Z \geq 0.004$ , because it is difficult to determine the stellar age and metallicity of metal-poor (e.g., Z < 0.008) populations via colors under the typical observational uncertainties (Hi & Han 2008a). Metallicity affects the colors of metal-poor populations more strongly. Thus, one should use the results shown here for more metal-poor populations carefully. Because it is impossible to give the formulae for all colors, we give some ones for calculating the effects of binary interactions on 12 important colors, which are sensitive to stellar age or metallicity, according to the work of Li & Han (2008a). The 12 colors are (B - V), (V - K), (I - H), (R - K), (B - K), (I - K), (u - r), (r - K), (u - R), (u - R), (u - R), and  $(g - J)^1$ . Note that (B - V), (u - r), (u - R) and  $(g - L)^2$  are more sensitive to stellar age and the others to metallicity. Our work shows that the changes in the above colors caused by binary interactions can be expressed as

$$\Delta I' = \sum_{i=1}^{4} C_i t^{i-1}, \tag{2}$$

where  $\Delta I'$  is the change of colors caused by binary interactions, and t is stellar age. The coefficients of the equation are shown in Table 3. Note that the results for populations with both Salpeter IMF (standard

<sup>&</sup>lt;sup>1</sup> Colors (r-K), (u-R), (u-K), (z-K) and (g-J) are composite colors. The *UBVRIJHK* magnitudes are in the Johnson system, and the *ugriz* magnitudes in the SDSS-*ugriz* system.

**Table 2** Similar to Table 1, but for stellar populations older than  $4\,\mathrm{Gyr}$  (Age  $\geq 4\,\mathrm{Gyr}$ ).

Index	j	$C_{1j}$	$C_{2j}$	$C_{3j}$	$C_{4j}$	$C_{5j}$
	1	0.0040188	-0.0087338	0.0012365	-0.0000628	0.0000011
$CN_1$	2	0.0699149	-0.5550663	0.1605598	-0.0163788	0.0005280
	3	-4.2835098	11.3189732	-2.8137761	0.2856955	-0.0093980
	1	0.0031607	-0.0068581	0.0009171	-0.0000386	0.0000004
$CN_2$	2	0.1358253	-0.6142418	0.1616565	-0.0161493	0.0005224
	3	-5.8469187	12.5102930	-2.9058639	0.2889257	-0.0095349
	1	0.0024019	-0.0096646	0.0028396	-0.0003487	0.0000134
Ca4227	2	0.1289259	-0.3224843	-0.1484763	0.0361270	-0.0016691
	3	30.5865753	-70.1598679	19.3765757	-2.1609623	0.0788180
	1	0.1365771	-0.4119753	0.0710301	-0.0055584	0.0001697
G4300	2	-3.6369924	2.4500207	0.5456707	-0.0466573	-0.0003761
	3	28.2611775	-85.3355224	18.7904056	-2.1578069	0.1062849
	1	0.0695695	-0.1937149	0.0295201	-0.0017795	0.0000416
Fe4383	2	1.5938861	-9.3601744	2.3994053	-0.2263281	0.0068871
	3	-79.2726970	189.8780408	-38.5276774	3.5614204	-0.1108842
	1	0.0113051	-0.0260901	0.0031068	-0.0001249	0.0000015
Ca4455	2	0.1825335	-2.1257283	0.6162648	-0.0626674	0.0020383
	3	-18.3660159	52.8898164	-13.1167084	1.3350018	-0.0447127
	1	0.0241263	-0.1055130	0.0195680	-0.0014870	0.0000418
Fe4531	2	-0.8767742	-0.8527878	0.3649493	-0.0354193	0.0009578
	3	25.2479921	-36.0000062	6.6317767	-0.5259928	0.0185966
	1	0.0179749	-0.0676067	0.0083280	-0.0002240	-0.0000019
Fe4668	2	0.2262582	-4.4121200	1.5353953	-0.1944241	0.0071387
	3	-79.5135908	227.1468134	-55.0061793	6.0957633	-0.2199873
	1	-0.0625565	0.2708823	-0.0566269	0.0045862	-0.0001317
$H_{\beta}$	2	2.8700663	-5.7104589	0.9274182	-0.0705995	0.0022889
	3	-40.1033590	83.2554372	-18.0609224	1.5794001	-0.0541752
	1	0.0326274	-0.1721448	0.0331789	-0.0025354	0.0000699
Fe5015	2	-1.1476034	-1.5307235	0.5946861	-0.0674897	0.0022440
	3	7.2743924	22.9005565	-3.9738686	0.4451390	-0.0154651
	1	0.0002583	-0.0028680	0.0005454	-0.0000406	0.0000011
$Mg_1$	2	0.0303274	-0.1102308	0.0115624	-0.0003957	-0.0000009
	3	-0.0290541	0.0201172	0.2573529	-0.0302840	0.0011283
	1	0.0003494	-0.0047501	0.0010250	-0.0000857	0.0000026
$Mg_2$	2	0.0392009	-0.1979893	0.0208337	-0.0000621	-0.0000425
	3	0.4638829	-0.5123346	0.6866533	-0.0971528	0.0039890
	1	-0.0037660	-0.0157034	0.0047387	-0.0004621	0.0000160
$\mathrm{Mg_{b}}$	2	0.9698109	-4.0019894	0.5949929	-0.0254183	0.0000838
	3	-10.4139848	42.6783675	-1.0978610	-0.5308182	0.0325709
T. 5050	1	0.0135064	-0.0676152	0.0131365	-0.0009717	0.0000261
Fe5270	2	-0.3787006 2.9257542	-1.1509906 20.0839626	0.2995794	-0.0297612	0.0009629
-				-2.9644946	0.2873734	-0.0103891
E-5225	1	0.0100735	-0.0513031	0.0112253	-0.0009878	0.0000303
Fe5335	2	-0.3764122 46.0187216	-1.1131799 -72.8319926	-0.1092792 27.5059454	0.0392520 -3.2158884	-0.0018016 0.1133870
E <sub>0</sub> 5406	1	0.0061912 0.0934871	-0.0466338	0.0093712	-0.0007327 -0.0108703	0.0000206 0.0001877
Fe5406	2	1.0004186	-1.3125122 10.2495446	0.1939528 1.2408277	-0.0108703 -0.2790750	0.0001877
E <sub>0</sub> 5700	1	0.0084714	-0.0287635	0.0050787 0.0939359	-0.0003500 -0.0153071	0.0000089
Fe5709	2	-0.4407574 -0.8112771	0.0360397 19.1722212	-4.5075237	-0.0153071 0.4973342	0.0006047 -0.0184540
	3	-0.0112//1	17.1/44414	-4.5075457	0.49/3342	-0.0104340

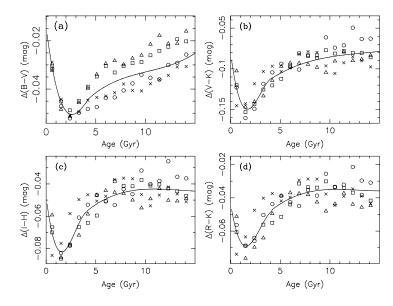
**Table 2** — Continued.

Index	j	$C_{1j}$	$C_{2j}$	$C_{3j}$	$C_{4j}$	$C_{5j}$
	1	-0.0002944	-0.0149229	0.0032162	-0.0003063	0.0000099
Fe5782	2	0.2419052	-0.2990725	-0.1171528	0.0246488	-0.0010394
	3	20.8520931	-52.8185426	16.2838692	-1.7500469	0.0595642
	1	-0.0022021	-0.0380672	0.0095990	-0.0008472	0.0000260
$Na_{\mathrm{D}}$	2	0.3464177	-0.9169326	-0.0149337	0.0195706	-0.0010391
	3	14.6152128	-38.0493377	12.0162242	-1.3802231	0.0519050
	1	-0.0004048	-0.0010965	0.0002912	-0.0000264	0.0000008
$TiO_1$	2	0.0098709	-0.0060943	-0.0015886	0.0001050	-0.0000058
	3	0.4022334	-0.0305906	-0.0854966	0.0157629	-0.0004537
	1	-0.0006406	-0.0026937	0.0006705	-0.0000589	0.0000018
$TiO_2$	2	0.0411593	0.0211796	-0.0159178	0.0015739	-0.0000518
	3	-0.4009774	-0.3431377	0.1546906	-0.0093757	0.0003560
	1	-0.1822723	0.4528323	-0.0619613	0.0036523	-0.0000907
${ m H}\delta_{ m A}$	2	-9.3242935	26.8376168	-7.6220614	0.6779245	-0.0184436
	3	273.5136304	-384.4491796	87.5549531	-6.8624029	0.1483407
	1	-0.2347224	0.6562770	-0.0985331	0.0067082	-0.0001900
$H\gamma_A$	2	-8.3702686	24.2585985	-7.8246936	0.7094328	-0.0186995
	3	392.7692657	-531.6987235	129.8789712	-11.2146737	0.2783352
	1	-0.1106862	0.2930026	-0.0443820	0.0029922	-0.0000815
${ m H}\delta_{ m F}$	2	-3.5060331	10.7215430	-3.5595302	0.3279227	-0.0089577
	3	145.3867861	-162.5243753	44.0186086	-3.7498376	0.0877458
	1	-0.1284349	0.3643152	-0.0583081	0.0041664	-0.0001196
${ m H}\gamma_{ m F}$	2	-1.3760493	7.0593899	-2.8690465	0.2678504	-0.0067945
	3	129.2354201	-186.3215690	49.9193880	-4.3279933	0.0997893

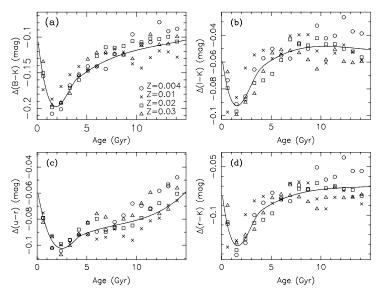
investigation) and Chabrier IMF are listed in the table. We can find that Equation (2) does not include the metallicity of populations. The reason is that colors are less sensitive to metallicity compared to Lick indices and they seem to be affected by the Monte Carlo method used to generate our star sample. The fitting of the effects of binary interactions on the 12 colors are shown in Figures 4, 5 and 6. As we see, the fitting formulae can give average color changes caused by binary interactions. However, because the results calculated using Equation (2) have typical errors of about 0.02 mag, some additional uncertainties may be brought into the results of stellar population studies.

## 5 DISCUSSION AND CONCLUSIONS

We present some formulae for conveniently computing the changes caused by binary interactions in 25 Lick indices and 12 colors, compared to the indices of single star stellar populations (ssSSPs). It is shown that the fitting formulae presented in the paper can reproduce the changes in Lick indices caused by the binary interactions with small errors and can be used to estimate similar changes in colors. It is also found that binary interactions make age-sensitive Lick indices (not only H $\beta$ , but also H $\delta$ <sub>A</sub>, H $\delta$ <sub>F</sub>,  $H\gamma_A$ ,  $H\gamma_F$ ) larger, while making metallicity-sensitive indices less compared to those of ssSSPs. This is useful for estimating the effects of binary evolution on the results of stellar population studies and for adding the effects of binary interactions into ssSSP models. Therefore, when an age-sensitive Lick index is used together with a metallicity-sensitive Lick index to determine the ages and metallicities of populations, smaller ages will be obtained, especially for metal-poor populations, see also Li & Han (2008c). Note that only binary star stellar populations (bsSSPs) and ssSSPs with four metallicities (Z = 0.004, 0.01, 0.02 and 0.03) are used in the work. This is actually limited by the metallicity coverage of the stellar population model and the lower sensitivities of colors to metallicity. Thus, the results are more suitable for studying metal-rich ( $Z \ge 0.004$ ) populations, because the differences between integrated peculiarities of populations with various metallicities seem larger for metal-poor populations. In addition, although different formulae are presented for populations with various initial mass functions

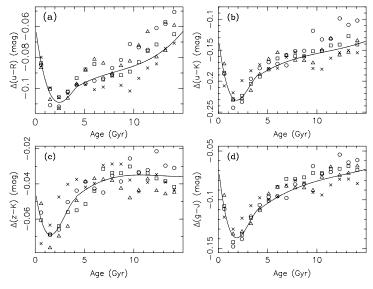


**Fig. 4** Fittings for the effects of binary interactions on four colors of populations. Circles, crosses, squares and triangles show the values obtained directly from comparing the colors of bsSSPs and ssSSPs and are for metallicities of 0.004, 0.01, 0.02 and 0.03, respectively. Solid lines show the fittings. The y-axis is obtained by subtracting the color of a bsSSP from that of an ssSSP (with the same age and metallicity as the bsSSP). The four panels are for (B-V), (V-K), (I-H) and (R-K), respectively.



**Fig. 5** Similar to Fig. 4, but for (B-K), (I-K), (u-r) and (r-K).

(IMFs), the changes calculated via two kinds of formulae (the formulae for populations with Salpeter and Chabrier IMFs) are similar. In other words, the changes calculated by the formulae obtained using populations with Salpeter IMF or Chabrier IMF can give us some picture of the effects of binary interactions. Furthermore, because the Monte Carlo technique used to generate the binary sample of stellar populations makes the evolution of integrated peculiarities of populations unsmooth, some results, espe-



**Fig. 6** Similar to Fig. 4, but for (u-R), (u-K), (z-K) and (g-J).

**Table 3** Coefficients for Eq. (2). *UBVRIJHKLMN* magnitudes are given in the Johnson system, and *ugriz* magnitudes are given in the SDSS-*ugriz* system.

IMF		Salpeter				Chabrier				
Age < 4.2 Gyr										
Color	$C_1$	$C_2$	$C_3$	$C_4$	$C_1$	$C_2$	$C_3$	$C_4$		
(B-V)	-0.014222	-0.032764	0.009111	-0.000722	-0.020059	-0.021895	0.005569	-0.000449		
(V - K)	-0.080134	-0.093961	0.038424	-0.004241	-0.086556	-0.062010	0.021278	-0.001931		
(I-H)	-0.047703	-0.049909	0.021701	-0.002461	-0.049641	-0.033034	0.011729	-0.001032		
(R-K)	-0.043288	-0.037439	0.016264	-0.001812	-0.042418	-0.025391	0.008674	-0.000701		
(B-K)	-0.094328	-0.126572	0.047424	-0.004947	-0.106087	-0.084725	0.027181	-0.002419		
(I - K)	-0.054306	-0.055956	0.024500	-0.002787	-0.055634	-0.036844	0.012924	-0.001109		
(u-r)	-0.056601	-0.042940	0.012347	-0.001002	-0.063465	-0.023075	0.005139	-0.000410		
(r-K)	-0.072748	-0.083108	0.035437	-0.004014	-0.077569	-0.054339	0.019333	-0.001782		
(u-R)	-0.059219	-0.047988	0.014264	-0.001213	-0.066913	-0.026108	0.006208	-0.000525		
(u-K)	-0.129146	-0.126222	0.047806	-0.005016	-0.140906	-0.077425	0.024514	-0.002201		
(z-K)	-0.043288	-0.037439	0.016264	-0.001812	-0.042418	-0.025391	0.008674	-0.000701		
(g-J)	-0.065198	-0.092019	0.035014	-0.003704	-0.074778	-0.061406	0.020278	-0.001863		
	•									
				$Age \ge 4.2  Gy$	r					

	$Age \ge 4.2  \mathrm{Gyr}$										
Color	$C_1$	$C_2$	$C_3$	$C_4$	$C_1$	$C_2$	$C_3$	$C_4$			
(B-V)	-0.069795	0.008858	-0.000767	0.000025	-0.062030	0.005088	-0.000384	0.000014			
(V - K)	-0.157537	0.014833	-0.001013	0.000025	-0.191178	0.028283	-0.002893	0.000103			
(I-H)	-0.087919	0.010592	-0.000813	0.000020	-0.113589	0.020964	-0.002196	0.000076			
(R-K)	-0.078362	0.010635	-0.000850	0.000022	-0.099112	0.019317	-0.002018	0.000069			
(B-K)	-0.222114	0.020910	-0.001424	0.000037	-0.245483	0.029974	-0.002870	0.000102			
(I - K)	-0.100169	0.012567	-0.000978	0.000024	-0.130611	0.024976	-0.002622	0.000090			
(u-r)	-0.119987	0.009885	-0.001017	0.000042	-0.098610	0.000563	-0.000163	0.000019			
(r-K)	-0.135641	0.013834	-0.001022	0.000026	-0.169588	0.027336	-0.002877	0.000102			
(u-R)	-0.126930	0.010203	-0.001026	0.000042	-0.106134	0.001228	-0.000236	0.000022			
(u-K)	-0.255286	0.021748	-0.001696	0.000052	-0.264095	0.024796	-0.002561	0.000101			
(z-K)	-0.078362	0.010635	-0.000850	0.000022	-0.099112	0.019317	-0.002018	0.000069			
(g-J)	-0.154783	0.014291	-0.000930	0.000024	-0.171114	0.020427	-0.001897	0.000067			

cially those for colors, may be somewhat rough. The additional uncertainties involved should be taken into account. If possible, we will give more detailed studies in the future.

**Acknowledgements** We thank the referee, Profs. Gang Zhao, Xu Zhou, Licai Deng, Xu Kong, Tinggui Wang and Li Zhang for useful suggestions or discussions. This work is supported by the Chinese National Science Foundation (Grant Nos. 10433030, 10521001 and 2007CB815406), the Youth Foundation of Knowledge Innovation Project of the Chinese Academy of Sciences (07ACX51001), and the Scientific Research Foundation of Dali University.

# Appendix A: COEFFICIENTS FOR CALCULATING THE EFFECTS OF BINARY INTERACTIONS ON 25 LICK INDICES OF POPULATIONS WITH CHABRIER IMF

**Table A.1** Coefficients for Eq. (1). The coefficients are obtained via stellar populations with Chabrier IMF and can be used for populations younger than  $3.5 \, \text{Gyr}$  (Age  $< 3.5 \, \text{Gyr}$ ).

Index	j	$C_{1j}$	$C_{2j}$	$C_{3j}$	$C_{4j}$	$C_{5j}$
	1	0.0022919	0.0011051	-0.0063997	0.0015981	-0.0001080
$CN_1$	2	0.6329579	-0.9595824	0.2427491	-0.0011151	-0.0025634
	3	-4.5940253	7.9284009	-3.9250539	0.6480545	-0.0256989
	1	0.0012774	0.0032128	-0.0069053	0.0016921	-0.0001156
$CN_2$	2	0.6334017	-1.0656913	0.3400438	-0.0309074	-0.0000476
	3	-4.4675288	9.5105511	-5.7521116	1.2233685	-0.0747440
	1	0.0795038	-0.1571070	0.0740613	-0.0128663	0.0007442
Ca4227	2	-11.9558663	20.8851588	-9.0611781	1.3421325	-0.0622930
	3	342.8235941	-529.6545151	161.4994646	-12.9148165	-0.1134096
	1	0.7660019	-1.5947618	0.6186025	-0.0971560	0.0052551
G4300	2	-87.0440009	155.4728799	-65.2201097	9.5672170	-0.4355927
	3	2358.1318663	-4145.5253913	1656.4267226	-221.2634252	8.6149062
	1	0.6089788	-0.9690479	0.3009590	-0.0329757	0.0009352
Fe4383	2	-72.6995453	111.7196061	-45.0101690	6.0333694	-0.2331824
	3	2100.1581312	-3361.5113576	1335.7159550	-174.8630448	6.5371492
	1	0.0681495	-0.1369378	0.0592712	-0.0105410	0.0006446
Ca4455	2	-7.3288591	13.2396365	-7.0194851	1.2883776	-0.0767611
	3	258.9163092	-510.1718798	260.9018384	-46.2914894	2.6884347
	1	0.2564424	-0.5328445	0.2229634	-0.0364862	0.0020371
Fe4531	2	-26.0845865	42.1057901	-17.3504899	2.4029055	-0.0987445
	3	851.0453910	-1492.6215470	616.0271336	-86.7449029	3.7433351
	1	0.1275239	-0.2791766	0.1024538	-0.0161675	0.0009372
Fe4668	2	-5.2389930	7.6979073	-2.3867283	0.1846930	-0.0012464
	3	169.0223930	-536.0764913	339.6448905	-65.7521400	4.1760430
	1	-0.4293708	0.8954661	-0.3155113	0.0432326	-0.0020233
$H_{\beta}$	2	30.1096288	-49.0583758	16.8580314	-1.8838154	0.0479103
	3	-767.8393985	1229.9316335	-430.0651511	45.5750506	-0.8795314
	1	0.2395560	-0.3782478	0.1010824	-0.0116568	0.0005056
Fe5015	2	-3.3348359	-34.6665677	26.0424965	-5.8877139	0.4132806
	3	413.3734390	45.5824955	-217.1527754	66.7394428	-5.4107732
	1	0.0057639	-0.0122970	0.0048864	-0.0007787	0.0000435
$Mg_1$	2	-0.3355850	0.6283524	-0.2850889	0.0319693	-0.0005680
	3	13.7239823	-27.6095038	10.7666288	-1.0488923	0.0060013

**Table A.1** — Continued.

Index	j	$C_{1j}$	$C_{2j}$	$C_{3j}$	$C_{4j}$	$C_{5j}$
	1	0.0123326	-0.0261803	0.0111134	-0.0018290	0.0001038
$Mg_2$	2	-0.7115473	1.0153443	-0.3612812	0.0205355	0.0016030
	3	27.2273352	-44.1981147	13.4974485	-0.5084821	-0.0847616
	1	0.1259541	-0.2256908	0.0880777	-0.0124308	0.0005828
${ m Mg_b}$	2	-8.2670312	5.7969056	0.6635323	-0.9033180	0.1008251
	3	311.7946049	-348.4131106	36.5555001	20.8760850	-2.8507846
	1	0.1526957	-0.3633561	0.1689542	-0.0304730	0.0018686
Fe5270	2	-16.8516642	33.9221551	-18.3927017	3.5096217	-0.2189850
	3	587.4969012	-1243.1886864	678.5522252	-131.0104877	8.3026556
	1	0.2076133	-0.4791791	0.2222679	-0.0377229	0.0021281
Fe5335	2	-32.4556974	65.1973802	-31.8891247	5.4057781	-0.2980594
	3	935.2591701	-1881.6098576	872.5249261	-142.2695305	7.5855211
	1	0.0953714	-0.2161430	0.0970755	-0.0171741	0.0010419
Fe5406	2	-3.9285567	6.4393293	-3.6907976	0.6355834	-0.0343946
-	3	174.8445638	-345.4890330	172.7005100	-28.2035654	1.4789489
	1	0.0665533	-0.1333817	0.0460766	-0.0056987	0.0002133
Fe5709	2	-8.4184254	11.6155577	-2.8870447	0.0168183	0.0277514
-	3	229.4380018	-346.7249858	109.9625205	-7.5674339	-0.2639790
	1	0.0724796	-0.1553226	0.0682675	-0.0111777	0.0006131
Fe5782	2	-12.3652716	23.1281644	-10.7411540	1.7511767	-0.0935373
	3	379.6596477	-665.9564600	268.9228630	-38.4448161	1.7703055
	1	0.0714688	-0.1651977	0.0664805	-0.0103341	0.0005679
$Na_{\mathrm{D}}$	2	-1.5605878	-4.9385375	5.3394247	-1.5674185	0.1304109
	3	148.3896060	-24.0899505	-130.0698652	49.4638805	-4.4865514
	1	-0.0115706	0.0112891	-0.0010864	-0.0006067	0.0000840
$TiO_1$	2	2.0705973	-2.9503555	0.8324854	-0.0401368	-0.0040348
-	3	-44.5370053	67.5680594	-20.7546877	1.4636658	0.0462995
	1	-0.0125496	0.0097243	-0.0001968	-0.0008541	0.0001063
$TiO_2$	2	2.4479466	-3.6429429	1.1610628	-0.0929928	-0.0014161
	3	-49.2619296	79.6645168	-27.3299096	2.6274999	-0.0183345
	1	-0.1701616	0.1253346	0.2496936	-0.0688490	0.0048300
${ m H}\delta_{ m A}$	2	-22.7201592	34.6318625	-16.9398410	2.8138566	-0.1414102
	3	35.0832106	-330.3012402	581.7572135	-167.8682957	12.8850332
	1	-1.5856131	2.7085039	-0.9113125	0.1227389	-0.0055250
${ m H}\gamma_{ m A}$	2	180.2526812	-282.6477080	118.3329534	-17.2376118	0.7715612
	3	-5010.4626342	7614.1515475	-2881.3416065	351.0737254	-11.0363129
	1	-0.1251870	0.1470139	0.1086295	-0.0343830	0.0025515
${ m H}\delta_{ m F}$	2	-17.3098016	30.7053467	-17.0765539	3.2519508	-0.1978084
-	3	137.3583135	-435.8552796	470.4948769	-124.2942197	9.3212486
	1	-0.6254725	1.1213832	-0.3612095	0.0484349	-0.0022475
${ m H}\gamma_{ m F}$	2	53.4441919	-86.7513473	36.9363160	-5.4971981	0.2510974
	3	-1532.4775410	2361.4227142	-884.2807841	102.2525869	-2.6516141

**Table A.2** Similar to Table A.1, but for stellar populations older than 3.5 Gyr (Age  $\geq$  3.5 Gyr).

Index	j	$C_{1j}$	$C_{2j}$	$C_{3j}$	$C_{4j}$	$C_{5j}$
	1	0.0050070	-0.0101123	0.0016866	-0.0001101	0.0000026
$CN_1$	2	-0.7387795	-0.0268996	0.0467357	-0.0067710	0.0002559
	3	1.8158227	8.1027523	-2.3010584	0.2598589	-0.0092777
	1	0.0048267	-0.0086639	0.0014493	-0.0000926	0.0000021
$CN_2$	2	-0.6713808	-0.0767864	0.0454875	-0.0063150	0.0002435
	3	2.4347829	7.4817292	-1.9820797	0.2264804	-0.0083295

Table A.2 — Continued.

Ca4227         1         0.0253654         -0.0209602         0.0048307         -0.0004936         0.0000168           Ca4227         2         2.35901788         1.5319737         -0.4799447         0.0598457         -0.0022187           3         108.6370369         -107.2440863         25.4608949         -2.5477372         0.00593647         -0.0022187           G4300         2         -3.2065131         3.8870486         -0.9523866         0.0227085         -0.0010081           Fe4383         2         -10.832039         -0.2016152         0.033234         -0.0021218         0.000535           Fe4383         2         -12.6431474         -0.2748662         1.1500142         -0.1342407         0.0045588           3         66.8341421         179.0155270         -40.031558         5.2647148         -0.180005588           4         1         0.0439274         -0.0430992         0.064975         -0.000470         0.0006266           44555         2         -6.6685854         1.4457768         -0.0185745         -0.0014817         0.0006266           54531         2         -1.65223730         5.9520743         -0.682231         0.044548         -0.00034646         -0.003472           54668	Index	j	$C_{1j}$	$C_{2j}$	$C_{3j}$	$C_{4j}$	$C_{5j}$
1 0.86370369							
G4300   2	Ca4227	2	-3.5901788	1.5319737	-0.4799447	0.0598457	-0.0022187
G4300   2		3	108.6370369	-107.2440863	25.4608949	-2.5477372	0.0859891
1		1	0.2092102	-0.4271461	0.0709524	-0.0053369	0.0001564
Fe4383   2	G4300	2	-32.0561313	13.8970486	-0.9523866	0.0227085	-0.0010081
Fe4383         2         -12.6431474         -2.2748662         1.1500142         -0.1342407         0.0045588           3         3         60.8341421         179.0155270         -49.0631558         5.2647148         -0.1806644           Ca4455         2         -6.6685854         1.4457768         -0.0535448         -0.0104817         0.0006266           1         1.00586416         -0.1084756         0.0185745         -0.0013543         0.000007249           Fe4531         2         -16.5223730         5.9250743         -0.6822331         0.0345448         -0.0007249           3         327.8776772         -131.2118033         13.2546833         -0.3619352         -0.0008992           Fe4668         2         -14.2549909         3.6173449         -0.006150         -0.00775483         0.000058           Hβ         2         1.5249999         3.6173449         -0.031692         0.0023434         -0.0006160           1         0.1188078         0.1462153         -0.0301692         0.0023434         -0.00041877           4         1         0.0327476         -0.1444852         0.0559370         -0.022244         0.040404           8         2         -2.26627396         7.4518761         -0.7875		3	388.0491119	-107.1700035	-9.9303420	2.4271319	-0.0776931
1 0.0439274			0.0852039	-0.2016152	0.0323234	-0.0021218	0.0000535
Ca4455         1         0.0439274         -0.0430992         0.0064975         -0.0004070         0.0000095           Ca4455         2         -6.6685854         1.4457768         -0.0535448         -0.0104817         0.0006266           1         0.0586416         -0.1084756         0.0185745         -0.0013543         0.0000372           Fe4531         2         -16.5223730         5.9250743         -0.6822331         0.0345448         -0.0007249           3         327.8776772         -131.2118033         13.2546833         -0.3619352         -0.0008992           6         2         -14.2549909         3.6173449         -0.0066150         -0.0775483         0.0041877           3         256.6237954         70.1814681         -29.9674944         4.6116691         -0.1954322           1         1         0.1188078         0.1462153         -0.0301692         0.002343         -0.000618           3         -6.8462900         -4.4966224         9.4442175         -1.3358200         0.045783           1         0.0327476         -0.1444852         0.0253383         -0.0018120         0.000483           Fe5015         2         -2.2667396         -7.4518761         -0.7875696         0.0244834	Fe4383		-12.6431474	-2.2748662	1.1500142	-0.1342407	0.0045588
Ca4455         2         -6.6685854         1.4457768         -0.0353448         -0.0104817         0.0006266           1         0.0586416         -0.1084756         0.0185745         -0.001343         0.0000372           Fe4531         2         -16.5223730         5.9250743         -0.6822331         0.0345448         -0.0007249           3         327.8776772         -131.2118033         13.2546833         -0.3619352         -0.0008992           1         0.0579868         -0.0883225         0.0125230         -0.0005465         0.000041877           3         256.6237954         70.1814681         -29.9674944         4.6116691         -0.1954322           1         0.1188078         0.1462153         -0.0301692         0.0023434         -0.000653           4         1         0.1188078         0.1462153         -0.0301692         0.0023434         -0.000653           4         1         0.0188090         -3.48420         0.5595370         -0.0222041         0.000457839           5         2         -22.2267396         7.4518761         -0.7875696         0.0244834         0.000054583           6         2         2-2.2267396         7.4518761         -0.7875696         0.0244834         0.0		3	60.8341421	179.0155270	-49.0631558	5.2647148	-0.1806644
1			0.0439274	-0.0430992	0.0064975	-0.0004070	
Fe4531   2	Ca4455		-6.6685854	1.4457768	-0.0535448		0.0006266
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$		3	114.6093932	-5.2478791	-4.3355245	0.8004669	-0.0341767
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$			0.0586416	-0.1084756	0.0185745	-0.0013543	0.0000372
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	Fe4531						
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		3	327.8776772	-131.2118033	13.2546833	-0.3619352	-0.0008992
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$							
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	Fe4668						
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		3	256.6237954	70.1814681		4.6116691	-0.1954322
1							-0.0000653
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$H_{\beta}$	2					
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		3	-6.8462900	-4.4966224	9.4442175	-1.3358200	0.0457839
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$			0.0327476	-0.1444852			
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	Fe5015						
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		3	369.7815953	-72.0823837	-1.2940653	1.1387862	-0.0545511
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$							
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$Mg_1$						
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$				4.5445049	-1.1536134		
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$							
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$Mg_2$						
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$							
3   158.6306124   -15.8632521   0.7547564   0.1990004   -0.0142351     1							
Fe5270         1         -0.0053182         -0.0449585         0.0074887         -0.0004761         0.0000116           Fe5270         2         -6.2888460         0.8351887         0.0980695         -0.0229461         0.0009453           3         106.4680655         11.8581491         -9.0607405         1.2154258         -0.0455243           1         -0.0299036         -0.0168634         0.0034824         -0.0003504         0.0000128           Fe5335         2         -0.8764299         -2.3631112         0.3216860         -0.0034947         -0.0004859           3         -19.1581245         20.3079448         1.5735482         -0.7362636         0.0365434           1         -0.0273466         -0.0191557         0.0032597         -0.0002321         0.0000069           Fe5406         2         -1.6859288         -1.4089294         0.3578634         -0.0309966         0.0008859           3         3.2750120         49.7275784         -12.4265243         1.1493382         -0.0351427           Fe5709         2         -3.1695351         1.3275974         -0.1282929         0.0015248         0.0001363           3         62.3172407         -5.6986762         -1.2147039         0.3086961         -0.0142722	$\mathrm{Mg}_{\mathrm{b}}$						
Fe5270         2         -6.2888460         0.8351887         0.0980695         -0.0229461         0.0009453           3         106.4680655         11.8581491         -9.0607405         1.2154258         -0.0455243           1         -0.0299036         -0.0168634         0.0034824         -0.0003504         0.0000128           Fe5335         2         -0.8764299         -2.3631112         0.3216860         -0.0034947         -0.0004859           3         -19.1581245         20.3079448         1.5735482         -0.7362636         0.0365434           1         -0.0273466         -0.0191557         0.0032597         -0.0002321         0.0000069           Fe5406         2         -1.6859288         -1.4089294         0.3578634         -0.0309966         0.0008859           3         3.2750120         49.7275784         -12.4265243         1.1493382         -0.0351427           Fe5709         2         -3.1695351         1.3275974         -0.1282929         0.0015248         0.0001363           3         62.3172407         -5.6986762         -1.2147039         0.3086961         -0.0142722           1         -0.0351759         0.0072524         -0.0009035         -0.0000295         0.0000040							
3         106.4680655         11.8581491         -9.0607405         1.2154258         -0.0455243           I         -0.0299036         -0.0168634         0.0034824         -0.0003504         0.0000128           Fe5335         2         -0.8764299         -2.3631112         0.3216860         -0.0034947         -0.0004859           3         -19.1581245         20.3079448         1.5735482         -0.7362636         0.0365434           1         -0.0273466         -0.0191557         0.0032597         -0.0002321         0.0000069           Fe5406         2         -1.6859288         -1.4089294         0.3578634         -0.0309966         0.0008859           3         3.2750120         49.7275784         -12.4265243         1.1493382         -0.0351427           Fe5709         2         -3.1695351         1.3275974         -0.1282929         0.0015248         0.0001363           3         62.3172407         -5.6986762         -1.2147039         0.3086961         -0.0142722           1         -0.0351759         0.0072524         -0.0009035         -0.0000295         0.0000040           Fe5782         2         2.9123930         -2.1632509         0.2392269         0.0009764         -0.005652	E 5070						
Fe5335         1         -0.0299036         -0.0168634         0.0034824         -0.0003504         0.0000128           Fe5335         2         -0.8764299         -2.3631112         0.3216860         -0.0034947         -0.0004859           3         -19.1581245         20.3079448         1.5735482         -0.7362636         0.0365434           1         -0.0273466         -0.0191557         0.0032597         -0.0002321         0.0000069           Fe5406         2         -1.6859288         -1.4089294         0.3578634         -0.0309966         0.0008859           3         3.2750120         49.7275784         -12.4265243         1.1493382         -0.0351427           Fe5709         2         -3.1695351         1.3275974         -0.1282929         0.0015248         0.0001363           3         62.3172407         -5.6986762         -1.2147039         0.3086961         -0.0142722           1         -0.0351759         0.0072524         -0.0009035         -0.0000295         0.0000040           Fe5782         2         2.9123930         -2.1632509         0.2392269         0.0009764         -0.005652           3         -87.2972968         23.4730212         0.8426133         -0.6095760         0.0321487 </td <td>Fe52/0</td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td>	Fe52/0						
Fe5335         2         -0.8764299         -2.3631112         0.3216860         -0.0034947         -0.0004859           3         -19.1581245         20.3079448         1.5735482         -0.7362636         0.0365434           1         -0.0273466         -0.0191557         0.0032597         -0.0002321         0.0000069           Fe5406         2         -1.6859288         -1.4089294         0.3578634         -0.0309966         0.0008859           3         3.2750120         49.7275784         -12.4265243         1.1493382         -0.0351427           Fe5709         2         -3.1695351         1.3275974         -0.1282929         0.0015248         0.0001363           3         62.3172407         -5.6986762         -1.2147039         0.3086961         -0.0142722           1         -0.0351759         0.0072524         -0.0009035         -0.000295         0.0000040           Fe5782         2         2.9123930         -2.1632509         0.2392269         0.0009764         -0.005652           3         -87.2972968         23.4730212         0.8426133         -0.6095760         0.0321487           1         -0.0349510         -0.0109978         0.0032700         -0.0003007         0.0000101		3	100.4080033	11.8381491	-9.0007403	1.2134238	-0.0433243
Fe5335         2         -0.8764299         -2.3631112         0.3216860         -0.0034947         -0.0004859           3         -19.1581245         20.3079448         1.5735482         -0.7362636         0.0365434           1         -0.0273466         -0.0191557         0.0032597         -0.0002321         0.0000069           Fe5406         2         -1.6859288         -1.4089294         0.3578634         -0.0309966         0.0008859           3         3.2750120         49.7275784         -12.4265243         1.1493382         -0.0351427           Fe5709         2         -3.1695351         1.3275974         -0.1282929         0.0015248         0.0001363           3         62.3172407         -5.6986762         -1.2147039         0.3086961         -0.0142722           1         -0.0351759         0.0072524         -0.0009035         -0.000295         0.0000040           Fe5782         2         2.9123930         -2.1632509         0.2392269         0.0009764         -0.005652           3         -87.2972968         23.4730212         0.8426133         -0.6095760         0.0321487           1         -0.0349510         -0.0109978         0.0032700         -0.0003007         0.0000101		1	-0.0299036	-0.0168634	0.0034824	-0.0003504	0.0000128
3         -19.1581245         20.3079448         1.5735482         -0.7362636         0.0365434           Fe5406         1         -0.0273466         -0.0191557         0.0032597         -0.0002321         0.0000069           Fe5406         2         -1.6859288         -1.4089294         0.3578634         -0.0309966         0.0008859           3         3.2750120         49.7275784         -12.4265243         1.1493382         -0.0351427           Fe5709         2         -3.1695351         1.3275974         -0.1282929         0.0015248         0.0001363           3         62.3172407         -5.6986762         -1.2147039         0.3086961         -0.0142722           1         -0.0351759         0.0072524         -0.0009035         -0.0000295         0.0000040           Fe5782         2         2.9123930         -2.1632509         0.2392269         0.0009764         -0.0005652           3         -87.2972968         23.4730212         0.8426133         -0.6095760         0.0321487           1         -0.0349510         -0.0109978         0.0032700         -0.0003007         0.0000101           NaD         2         -3.1483417         -0.0799733         0.0306158         0.0017984         -0.0001874	Fe5335						
Fe5406         2         -1.6859288         -1.4089294         0.3578634         -0.0309966         0.0008859           3         3.2750120         49.7275784         -12.4265243         1.1493382         -0.0351427           1         0.0058300         -0.0239914         0.0038273         -0.0002377         0.0000055           Fe5709         2         -3.1695351         1.3275974         -0.1282929         0.0015248         0.0001363           3         62.3172407         -5.6986762         -1.2147039         0.3086961         -0.0142722           1         -0.0351759         0.0072524         -0.0009035         -0.0000295         0.0000040           Fe5782         2         2.9123930         -2.1632509         0.2392269         0.0009764         -0.005652           3         -87.2972968         23.4730212         0.8426133         -0.6095760         0.0321487           1         -0.0349510         -0.0109978         0.0032700         -0.0003007         0.0000101           NaD         2         -3.1483417         -0.0799733         0.0306158         0.0017984         -0.0001874			-19.1581245	20.3079448	1.5735482	-0.7362636	0.0365434
Fe5406         2         -1.6859288         -1.4089294         0.3578634         -0.0309966         0.0008859           3         3.2750120         49.7275784         -12.4265243         1.1493382         -0.0351427           1         0.0058300         -0.0239914         0.0038273         -0.0002377         0.0000055           Fe5709         2         -3.1695351         1.3275974         -0.1282929         0.0015248         0.0001363           3         62.3172407         -5.6986762         -1.2147039         0.3086961         -0.0142722           1         -0.0351759         0.0072524         -0.0009035         -0.0000295         0.0000040           Fe5782         2         2.9123930         -2.1632509         0.2392269         0.0009764         -0.005652           3         -87.2972968         23.4730212         0.8426133         -0.6095760         0.0321487           1         -0.0349510         -0.0109978         0.0032700         -0.0003007         0.0000101           NaD         2         -3.1483417         -0.0799733         0.0306158         0.0017984         -0.0001874		1	-0.0273466	-0.0191557	0.0032597	-0.0002321	0.0000069
3         3.2750120         49.7275784         -12.4265243         1.1493382         -0.0351427           Fe5709         1         0.0058300         -0.0239914         0.0038273         -0.0002377         0.0000055           Fe5709         2         -3.1695351         1.3275974         -0.1282929         0.0015248         0.0001363           3         62.3172407         -5.6986762         -1.2147039         0.3086961         -0.0142722           1         -0.0351759         0.0072524         -0.0009035         -0.0000295         0.0000040           Fe5782         2         2.9123930         -2.1632509         0.2392269         0.0009764         -0.0005652           3         -87.2972968         23.4730212         0.8426133         -0.6095760         0.0321487           NaD         2         -3.1483417         -0.0799733         0.0306158         0.0017984         -0.0001874	Fe5406	2					
Fe5709         2         -3.1695351         1.3275974         -0.1282929         0.0015248         0.0001363           3         62.3172407         -5.6986762         -1.2147039         0.3086961         -0.0142722           1         -0.0351759         0.0072524         -0.0009035         -0.0000295         0.0000040           Fe5782         2         2.9123930         -2.1632509         0.2392269         0.0009764         -0.0005652           3         -87.2972968         23.4730212         0.8426133         -0.6095760         0.0321487           1         -0.0349510         -0.0109978         0.0032700         -0.0003007         0.0000101           NaD         2         -3.1483417         -0.0799733         0.0306158         0.0017984         -0.0001874		3	3.2750120	49.7275784	-12.4265243	1.1493382	-0.0351427
Fe5709         2         -3.1695351         1.3275974         -0.1282929         0.0015248         0.0001363           3         62.3172407         -5.6986762         -1.2147039         0.3086961         -0.0142722           1         -0.0351759         0.0072524         -0.0009035         -0.0000295         0.0000040           Fe5782         2         2.9123930         -2.1632509         0.2392269         0.0009764         -0.0005652           3         -87.2972968         23.4730212         0.8426133         -0.6095760         0.0321487           1         -0.0349510         -0.0109978         0.0032700         -0.0003007         0.0000101           NaD         2         -3.1483417         -0.0799733         0.0306158         0.0017984         -0.0001874		1	0.0058300	-0.0239914	0.0038273	-0.0002377	0.0000055
3         62.3172407         -5.6986762         -1.2147039         0.3086961         -0.0142722           1         -0.0351759         0.0072524         -0.0009035         -0.0000295         0.0000040           Fe5782         2         2.9123930         -2.1632509         0.2392269         0.0009764         -0.0005652           3         -87.2972968         23.4730212         0.8426133         -0.6095760         0.0321487           1         -0.0349510         -0.0109978         0.0032700         -0.0003007         0.0000101           NaD         2         -3.1483417         -0.0799733         0.0306158         0.0017984         -0.0001874	Fe5709	2		1.3275974			
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$			62.3172407	-5.6986762	-1.2147039	0.3086961	-0.0142722
3     -87.2972968     23.4730212     0.8426133     -0.6095760     0.0321487       1     -0.0349510     -0.0109978     0.0032700     -0.0003007     0.0000101       NaD     2     -3.1483417     -0.0799733     0.0306158     0.0017984     -0.0001874		1	-0.0351759	0.0072524	-0.0009035	-0.0000295	0.0000040
1     -0.0349510     -0.0109978     0.0032700     -0.0003007     0.0000101       Na <sub>D</sub> 2     -3.1483417     -0.0799733     0.0306158     0.0017984     -0.0001874	Fe5782			-2.1632509		0.0009764	
$Na_{\mathrm{D}}$ 2 $-3.1483417$ $-0.0799733$ $0.0306158$ $0.0017984$ $-0.0001874$		3	-87.2972968	23.4730212	0.8426133	-0.6095760	0.0321487
_		1	-0.0349510	-0.0109978	0.0032700	-0.0003007	0.0000101
3 74.8532596 -28.0695426 1.8470812 0.0045464 -0.0013129	$\mathrm{Na_{D}}$			-0.0799733			
		3	74.8532596	-28.0695426	1.8470812	0.0045464	-0.0013129

**Table A.2** — Continued.

Index	j	$C_{1j}$	$C_{2j}$	$C_{3j}$	$C_{4j}$	$C_{5j}$
	1	-0.0019580	-0.0001078	0.0001071	-0.0000133	0.0000005
$TiO_1$	2	-0.0663381	0.0341998	-0.0108762	0.0008305	-0.0000198
	3	3.2582308	-1.5575893	0.2498990	-0.0085626	-0.0000514
	1	-0.0055418	0.0002662	0.0000839	-0.0000138	0.0000006
$TiO_2$	2	0.1861284	-0.0836849	0.0091382	-0.0007762	0.0000260
- 2	3	-4.2466215	3.0328182	-0.7327127	0.0800970	-0.0027508
	1	-0.4834854	0.6747627	-0.1136504	0.0081802	-0.0002208
$H\delta_A$	2	48.2162849	-8.6259083	-0.5680772	0.1330894	-0.0044290
	3	-391.8235480	-44.3395429	41.4350353	-5.4239881	0.1828454
	1	-0.5340026	0.8499248	-0.1407715	0.0102053	-0.0002837
${ m H}\gamma_{ m A}$	2	57.5218630	-12.4432398	-1.0323349	0.2184516	-0.0069865
	3	-381.7703663	-237.3724815	109.5206537	-12.9300524	0.4258356
	1	-0.2328216	0.3790464	-0.0645870	0.0047630	-0.0001318
${ m H}\delta_{ m F}$	2	28.4618861	-8.4343118	0.2882612	0.0224536	-0.0007588
	3	-256.8427724	41.7473144	12.0094480	-2.0067178	0.0654486
	1	-0.2081433	0.4185126	-0.0709583	0.0052155	-0.0001461
${ m H}\gamma_{ m F}$	2	26.6976093	-8.4741360	0.0074237	0.0631139	-0.0020772
,-	3	-176.6172339	-82.1781969	45.6384532	-5.4871418	0.1779060

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