Sodium Enrichment in Yellow Supergiants: a Perspective from the Uncertainties of Reaction Rates *

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Abstract Sodium overabundance in yellow supergiants has stumped people for more than 20 years. The purpose of this paper is to explore this problem from the perspective of nuclear physics. We investigate carefully the CNO and NeNa cycles that are responsible for sodium production. We investigate some key reactions in the appropriate network. We show whether and how the sodium output can be affected by the rate uncertainties in these reactions. In this way, we evaluate if a reaction is important enough to deserve a better determination of its rate in terrestrial laboratories.

Key words: nuclear reactions — nucleosynthesis — stars: abundances — supergiants

1 INTRODUCTION

Since its first discovery, Na enrichment in the atmosphere of F, G, K supergiants (hereafter, yellow supergiants) has been a long-standing problem of stellar astrophysics in both observational measurement and theoretical interpretation. The LTE analysis found $[Na/Fe]>0^{-1}$, in yellow supergiants and an anticorrelation of [Na/Fe] with surface gravity (Boyarchuk & Lyubimkov 1983). The results have been further confirmed by the non-LTE correction (Boyarchuk et al. 1988).

Subsequently, various models have been proposed. Denisenkov & Ivanov (1987) and Denisenkov (1988) suggested that, besides the CNO cycle, NeNa cycle also exists during main-sequence H burning, and that ${}^{22}\text{Ne}(p,\gamma){}^{23}\text{Na}$ will significantly enrich the sodium abundance. The newly built Na can be brought to the stellar surface during the first dredge-up (FDU). However, in order to explain the large values of [Na/Fe] (up to ~ 0.7) reported by Boyarchuk & Lyubimkov (1983), the initial abundance of ${}^{22}\text{Ne}$ in yellow supergiants should be at least three times as large as in the Sun (Denisenkov 1990), which is not likely to happen. Later, Prantzos et al. (1991) explored the impact of the reaction rate of ${}^{20}\text{Ne}(p,\gamma){}^{21}\text{Na}$ on sodium production. They showed an enhancement factor $f \sim 10$ could be helpful to account for the observations, but further analysis indicated that such an enhancement for this reaction was improper.

It is worthwhile to point out that measurements in the past had large observational uncertainties either because of the observational resolution or data reduction, so it was difficult to constrain any theoretical models. Fortunately, Andrievsky et al. (2002) published Na abundances in a sample of 48 F-G supergiants. These high-quality data were derived from a homogeneous spectroscopic investigation based on LTE and NLTE calculations. Based on this work, Denisenkov (2005) demonstrated that the Na enrichment in stars with $M \gtrsim 7 - 10 M_{\odot}$ is the result of the Zahn model of rotational mixing between the convective core and the radiative envelope in their main-sequence progenitors.

Nevertheless, by tracing back Denisenkov's (2005) work, we find that the nuclear reaction network used in his model, given by table 1 in his previous work (1994), was more or less incomplete. A number

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¹ $[A/B]=\log[n(A)/n(B)]_{star}-\log[n(A)/n(B)]_{\odot}$, n(A) and n(B) being the number densities of nuclides A and B.



Fig. 1 Nuclear reaction network adopted here. The grey circles mark the stable nuclei, the white circles, the unstable nuclei. The solid lines represent the nuclear reactions $(p, \gamma), (\alpha, n), (\alpha, p)$ or (p, α) , and (α, γ) , and the dashed lines represent the weak interactions. For each solid line, both forward and reverse reactions exist: the arrow indicates the direction of the reaction in the environment of stellar nucleosynthesis.

of reactions were not included in the network, so there were no reactions that connect the CNO cycle and NeNa cycle. Even though the reaction rates in the network have been updated by subsequent work, no more reactions were added, therefore the gap still existed between the two cycles. This oversight made his conclusion somewhat less reliable.

On the other hand, the problem of Na enrichment can also be solved in the framework of nuclear physics. It is known that thermal nuclear reaction rates have great uncertainties due to difficulties in detecting low-energy resonance levels in nuclear physics. These uncertainties sometimes influence the results of nucleosynthesis significantly. Recently, Izzard et al. (2006) carried out a study concerning the effect of uncertainties in the proton-capture reaction rates of the NeNa and MgAl chains on nucleosynthesis of hot bottom burning (HBB) in intermediate-mass asymptotic giant branch (AGB) stars. They showed in figure 2 of their paper that the yields of Na can be boosted by a factor of about 40 when some reaction rates are varied within evaluated limits. Thus it is worthwhile to discuss the issue from the aspect of nuclear physics. Special attention should be paid to those reactions with significant uncertainties. In the following sections we concentrate on such reactions in CNO and NeNa cycles and evaluate their impacts.

This paper is organized as follow. In Section 2, we discuss the CNO and NeNa network and figure out several reactions which may be responsible for the sodium anomaly. In Section 3, we analyze these reactions one by one to see whether they are workable or not. Our conclusions and a discussion are given in Section 4.

2 ANALYSES ON CNO AND NeNa CYCLES

Our network shown in Figure 1, is adopted from Audouze et al. (1973). This network comprises 28 isotopes from ¹²C to ²⁵Mg. It includes all the important reactions, i.e., $(p, \gamma), (\alpha, n), (\alpha, p)$ and (α, γ) , and their inverses, as well as the positron decays. The (p, n) reactions, however, are not considered because they are endothermic which require fairly high temperature to trigger.



Fig. 2 Rates of reaction ${}^{19}F(p,\gamma){}^{20}Ne$, ${}^{19}F(\alpha,p){}^{22}Ne$ and ${}^{19}F(p,\alpha){}^{16}O$ from Caughlan & Fowler (1988).

In order to identify the key reactions in such a complex network, it is appropriate to take a brief overview of the H burning, both the pp-chain and CNO cycle. The net result of the pp chain is the conversion of 4 protons into 1 helium. There are three branches to achieve this goal, i.e., ppI, ppII and ppIII. Here we just list two reactions, ${}^{7}\text{Be+e^-} \rightarrow {}^{7}\text{Li}+\nu_e$ in ppII and ${}^{7}\text{Be+p}\rightarrow {}^{8}\text{B}+\gamma$ in ppIII. It is clear that the branching of ppII or ppIII depends entirely on the behavior of ${}^{7}\text{Be}$. To be explicit, whether the rate of proton capture can exceed that of beta decay determines which pathway to take. Similar situations occur in the so called hot CNO cycle. It is well known that when the temperature is high enough, substantial deviations from the usual CNO cycle can occur. ${}^{13}\text{N}(p,\gamma){}^{14}\text{O}$, for example, can compete with ${}^{13}\text{N}(e^+\nu){}^{13}\text{C}$ to result in a quite different output of nucleosynthesis (Hoyle & Fowler 1960).

Thus we conclude that for an unstable nucleus, the competition between proton capture and beta decay can sometimes be extremely important in nuclei output and energy generation. Moreover, reactions relevant to unstable nuclei usually have greater uncertainties from theory and experiment, which leaves room for the solution of Na enrichment by changing their reaction rates. With regard to our network in this work, we consider the following reactions may be important and need more specific calculations and analyses. 1. ¹⁵O(α , γ)¹⁹Ne

This reaction is important because ${}^{15}O(\alpha, \gamma){}^{19}Ne$ is a breakout reaction from the hot CNO cycle (Fisker et al. 2006). It bridges the gap between the CNO cycle and NeNa cycle efficiently. The higher its rate is, the more nuclides are transferred from the CNO cycle to the NeNa cycle, hence producing more sodium. 2. ${}^{19}Ne(p, \gamma){}^{20}Na$

¹⁹Ne is an unstable nuclide. When the reaction rate for proton capture is not fast enough to compete with its beta decay, most of ¹⁹Ne will decay into ¹⁹F. It is clear in Figure 1 that there are three reactions involving ¹⁹F, two bring the isotope forward to ²³Na, the other one, ¹⁹Ne(p, α)¹⁶O, brings it farther away from ²³Na. Nevertheless, this reaction is far more important than the other two in the environment of stellar nucleosynthesis, as is shown in Figure 2. If the rate of proton capture exceeds that of beta decay, most of ¹⁹Ne would be converted into ²⁰Na, and then decays into ²⁰Ne. All the reactions relevant to ²⁰Ne bring it near to ²³Na. Thus one scenario for the solution of Na enrichment would require that the rate of ¹⁹Ne(p, γ)²⁰Na be higher than ¹⁹Ne($e^+\nu$)¹⁹F.

3. 22 Na $(p, \gamma)^{23}$ Mg

Since adjustment of the rate of ${}^{22}\text{Ne}(p,\gamma){}^{23}\text{Na}$ is not successful in solving the sodium puzzle, this reaction gives an alternative pathway to produce Na, i.e., via ${}^{22}\text{Na}(p,\gamma){}^{23}\text{Mg}(e^+\nu){}^{23}\text{Na}$. There is another advantage for this reaction: both ${}^{22}\text{Na}$ and ${}^{23}\text{Mg}$ lie close to ${}^{23}\text{Na}$ in the flow of nucleosynthesis, so a

tiny change in the reaction rate may influence the output of ²³Na. Note that all the five isotopes (¹⁵O, ¹⁹Ne, ²⁰Na, ²²Na and ²³Mg) in these reactions are unstable nuclei with half-lives $t_{1/2} = 122.2$ s, 17.22 s, 447.9 ms, 2.602 yr and 11.32 s, respectively. Their relatively short half-lives make it difficult to determine their thermonuclear reaction rates, leaving room for improvement.

In addition, for ${}^{22}\text{Ne}(p,\gamma){}^{23}\text{Na}$, ${}^{23}\text{Na}(p,\alpha){}^{20}\text{Ne}$ and ${}^{23}\text{Na}(p,\gamma){}^{24}\text{Mg}$, we do not intend to check the impacts of their rate uncertainties in this work. Although their rates are still uncertain, Denisenkov (2005) has already pointed out that any adjustment of the rates can not explain the large Na overabundances. In addition, the large deviation of sodium output in Izzard et al. (2006) is actually not valid here because the temperature of HBB in AGB stars is about 10^8 K and these reaction rates suffer great uncertainties at this temperature, while the rate uncertainties for the temperature we are interested in ($\sim 3 - 4 \times 10^7$ K) are very small, as is shown in figure 2 in Iliadis et al. (2001).

3 CALCULATIONS AND SIMULATIONS

In this section, we discuss the three reactions to check whether changes in their rates could help to solve the Na puzzle.

3.1 15 **O** $(\alpha, \gamma)^{19}$ **Ne**

¹⁵O(α, γ)¹⁹Ne is a breakout reaction from the hot CNO cycle. For a long time, its rate was not known experimentally and significant uncertainties were involved in the theoretical analysis. Recently, Tan et al. (2007) provided a new reaction rate, which is suitable for the high temperature environment and has been used in X-ray burst models. As a result of the new rate, the model uncertainty was reduced (Fisker et al. 2007). However, Type I X-ray bursts occur in neutron star surfaces of matter-transferring binary systems, where matter is heated to $(1-2) \times 10^8$ K (Fisker et al. 2006). As stated before, sodium in yellow supergiants is commonly thought to be synthesized during main-sequence H burning with temperature $\sim (3-4) \times 10^7$ K. The new rate actually is not helpful for us.

According to Izzard et al. (2006), HBB nucleosynthesis is associated with the production of sodium, too, while for AGB stars more massive than 4 M_{\odot} , HBB proceeds at the temperature of $\sim (6-10) \times 10^7$ K. It is possible that the new rate can improve the sodium output in HBB, which may partly solve the Na anomaly in globular cluster stars.

3.2 19 Ne $(p, \gamma)^{20}$ Na

Whether or not this reaction is important depends on its rate compared to the rate of ${}^{19}\text{Ne}(e^+\nu){}^{19}\text{F}$. Consider a reaction X(i, j)Y, the reaction rate in number of events per unit time per unit volume is (e.g., see Padmanabhan 2000)

$$r_{i\mathbf{X}} = n_i n_{\mathbf{X}} \langle \sigma v \rangle_{i\mathbf{X}},\tag{1}$$

where n_i and n_x are the number densities of particles *i* and X, respectively, and $\langle \sigma v \rangle_{ix}$ is the mean cross section. This equation can be rewritten in terms of mass density as

$$r_{i\mathrm{X}} = \rho^2 N_A \frac{X_i X_{\mathrm{X}}}{A_i A_{\mathrm{X}}} \cdot N_A \left\langle \sigma v \right\rangle_{i\mathrm{X}},\tag{2}$$

where ρ is the density, N_A is the Avogadro constant, X_i and X_x are the mass fractions. $N_A \langle \sigma v \rangle_{iX}$ are the nuclear reaction rates. If the nucleus X is unstable, the rate of its beta decay is

$$r_{b} = \rho \frac{X_{x}}{A_{x}} N_{A} \frac{\ln 2}{t_{1/2}} \,. \tag{3}$$

Dividing Equation (2) by Equation (3) we obtain

$$\frac{r_{i\mathrm{x}}}{r_{b}} = \rho \frac{X_{i}}{N_{i}} N_{\mathrm{A}} \langle \sigma v \rangle t_{1/2} / \ln 2 \,. \tag{4}$$

Now it is possible to plot the rate of ${}^{19}\text{Ne}(p,\gamma){}^{20}\text{Na}$ and the beta decay rate versus the temperature in Figure 3. As shown in Equation (4) the ratio of the two reactions is independent of the mass of ${}^{19}\text{Ne}$, our



Fig. 3 Rates of the reactions ¹⁹Ne $(p, \gamma)^{20}$ Na and ¹⁹Ne $(e^+\nu)^{19}$ F. Mass fraction of proton in this calculation is 0.7, and that of ¹⁹Ne is 10^{-10} . The core density for the main-sequence progenitor of a yellow supergiant is assumed to be $\rho = 10 \text{ g cm}^{-3}$. Although the rate of ¹⁹Ne $(p, \gamma)^{20}$ Na is uncertain, here we adopt the $N_A \langle \sigma v \rangle$ value from Caughlan & Fowler (1988) as a base of our discussion.

assumption of $X_{^{19}\text{Ne}} = 10^{-10}$ has no influence on the ratio of the two rates. The mass of proton in the core of a star does vary during stellar evolution, but we assume it to be constant here just for simplicity. Furthermore, the variation of X_p in a long period of stellar evolution is relatively small. It takes 1.62×10^7 yr for a star of $10 M_{\odot}$ to burn half of the initial H in its central region, while the time scale for the exhaustion of the central hydrogen is about 2.25×10^7 yr (Claret 2004), so our simplification is generally safe.

For a typical yellow supergiant, its main-sequence progenitor has a central temperature of $\sim (3-4) \times 10^7$ K, at which the proton capture rate of ¹⁹Ne is more than ten orders smaller than its beta decay rate. There is no hope for this reaction to exceed the beta decay rate in this astrophysical environment. Although this rate may be important under other conditions, it plays no role in solving the Na anomaly.

3.3 22 Na $(p, \gamma)^{23}$ Mg

The reason that ${}^{19}\text{Ne}(p,\gamma){}^{20}\text{Na}$ is not important lies in the fact that ${}^{19}\text{Ne}$ decays too fast. In this way, ${}^{22}\text{Na}(p,\gamma){}^{23}\text{Mg}$ may be more hopeful, because the half-life of ${}^{22}\text{Na}$ is much longer than that of ${}^{19}\text{Ne}$. For ${}^{22}\text{Na}$, we plot a similar diagram with a similar set of parameters in Figure 4. Here we do not include ${}^{22}\text{Na}(\alpha,p){}^{25}\text{Mg}$ for its rate is much smaller than these of the other two reactions due to the larger Coulomb repulsive forces between the ions.

The rates for ${}^{22}\text{Na}(p,\gamma){}^{23}\text{Mg}$ and ${}^{22}\text{Na}(e^+\nu){}^{22}\text{Ne}$ are equal at a temperature $T \simeq 7.0 \times 10^7$ K. If the rate for ${}^{22}\text{Na}(p,\gamma){}^{23}\text{Mg}$ were enhanced by a factor of 3000, then this critical temperature would decrease to $T \simeq 3.7 \times 10^7$ K, which is suitable for Na production in a main-sequence progenitor of a yellow supergiant. However, this does not mean an enhancement of 3000 times in rate is crucial to solving the problem of sodium overabundance. We can see from Figure 4 that although the rate for ${}^{22}\text{Na}(p,\gamma){}^{23}\text{Mg}$ is smaller, it is still comparable with that of ${}^{22}\text{Na}(e^+\nu){}^{22}\text{Ne}$, so a small enhancement in the reaction rate may be effective.

To examine the impact of this reaction, we ran a FORTRAN code designed for thermonuclear reaction networks ². We ran the code in a hydrostatic mode, with constant temperature and density. We include all the isotopes and reactions listed in Figure 1. The characteristic parameters of a 10 M_{\odot} star adopted here are: core temperature $T_c = 3.17 \times 10^7$ K, core density $\rho_c = 10$ g cm⁻³, and evolution time $t = 7.10 \times 10^{14}$ s. The evolution time is the time for the star to consume all its initial hydrogen in the nuclear burning location.

² This network is developed by F. X. Timmes and is available at http://cococubed.asu.edu/code_pages/burn.shtml



Fig. 4 Rates of reactions ²²Na $(p, \gamma)^{23}$ Mg and ²²Na $(e^+\nu)^{22}$ Ne. The horizontal line is the rate of beta decay, solid curve the rate of proton capture. The dotted curve represents an enhancement of proton capture rate by 30 times, the dotted-dashed curve, a 300 times enhancement, and the dashed curve, 3000 times. The rate for ²²Na $(\alpha, p)^{25}$ Mg is not plotted because its rate is much smaller than the other two. Note that the rate of ²²Na $(p, \gamma)^{23}$ Mg is uncertain. We adopt the $N_A \langle \sigma v \rangle$ value from Caughlan & Fowler (1988) as a base of our calculation.

All the parameters are adopted from Claret (2004). The nuclear reaction rates in our network are mostly adopted from Caughlan & Fowler (1988).

In our model, the initial abundances are assumed to be solar-like (i.e., Lodders 2003), so the initial abundances of all stable nuclei are given. Due to its importance, the abundance of the radioactive nuclide ²²Na is also needed prior to simulation. Among the four reactions related to ²²Na, the reactions ²²Mg($e^+\nu$)²²Na and ²²Na(α , p)²⁵Mg can be ignored because of the low abundance of ²²Mg and the small cross section of ²²Na(α , p)²⁵Mg. When the two reactions are excluded, the differential equation for ²²Na abundance can be written as

$$\frac{d\mathrm{Na}}{dt} = r_{\mathrm{pNe}} - r_{\mathrm{pNa}} - r_{b} , \qquad (5)$$

where the number density is designated by the chemical symbol. We can substitute Equations (2) and (3) into Equation (5) to obtain the explicit expression. When the abundance of ²²Na reaches equilibrium, dNa/dt = 0, we acquire its equilibrium abundance, which is about 1.76×10^{-9} . By solving Equation (5), we obtain the time for ²²Na to reach its equilibrium state to be about 2.7×10^7 s. Both Ne and H change little during this time span, so our calculation is self-consistent.

In our trial, we first use the standard rate for 22 Na $(p, \gamma)^{23}$ Mg, then we increase its value by a factor of q. When q = 40, we find the 23 Na abundance to show a small increase from a certain time on. The larger q is, the larger the increase is, and the earlier the increase can be seen. Unfortunately, although we have increased the q factor up to several hundred, the increment in the Na abundance never surpassed 10⁻⁸. In other words, the corresponding changes of Na abundance are less than 0.01%. Such a small change is meaningless for solving the problem of sodium excess.

Alpha-induced reactions suffer larger Coulomb repulsive forces than proton-induced reactions do. As a result, alpha-induced reactions are usually negligible at lower temperature. If we ignore ${}^{22}\text{Ne}(\alpha,n){}^{25}\text{Mg}$ and ${}^{22}\text{Na}(\alpha,p){}^{25}\text{Mg}$ in the network, then the flow of nuclei goes either via ${}^{21}\text{Ne}(p,\gamma){}^{22}\text{Na}(e^+\nu){}^{22}\text{Ne}(p,\gamma){}^{23}\text{Na}$ or via ${}^{21}\text{Ne}(p,\gamma){}^{22}\text{Na}(e^+\nu){}^{23}\text{Na}$. A larger q increases the chance for the nuclei to take the first path, while the chance for ${}^{23}\text{Na}$ to be synthesized via the second path is decreased. That may be the cause why sodium production changes little with q. In short, all the three reactions we proposed are incapable of solving the sodium puzzle.

4 CONCLUSIONS AND DISCUSSION

We have analyzed the network containing the CNO and NeNa cycles. It is a complex network involving 28 isotopes and dozens of reactions. In order to find out the key reactions for the Na excess, we chose some unstable nucleus because their ion capture rates are uncertain, and the competition between their proton captures and beta decays can influence the result of nucleosynthesis. We plotted the proton capture rate and the beta decay rate over temperature to see whether they are comparable. In this way, we found ${}^{22}Na(p,\gamma){}^{23}Mg$ to be of importance.

Then we ran the nuclear reaction network to check the impact of 22 Na $(p, \gamma)^{23}$ Mg. Our results show that the output of 23 Na varies with the q factor, however, the deviation is too small to produce any effect.

Since the first discovery of Na enrichment in yellow supergiants, various reactions have been proposed to evaluate their contributions. In previous works, ${}^{20}\text{Ne}(p,\gamma){}^{21}\text{Na}$, ${}^{22}\text{Ne}(p,\gamma){}^{23}\text{Na}$, ${}^{23}\text{Na}(p,\alpha){}^{20}\text{Ne}$ and ${}^{23}\text{Na}(p,\gamma){}^{24}\text{Mg}$ have been checked, while in this paper, we have investigated three reactions, namely ${}^{15}\text{O}(\alpha,\gamma){}^{19}\text{Ne}$, ${}^{19}\text{Ne}(p,\gamma){}^{19}\text{Ne}$ and ${}^{22}\text{Na}(p,\alpha){}^{23}\text{Mg}$. Among these reactions, some directly relate to the production or destruction of Na, some can efficiently bridge the gap between the CNO and NeNa cycles, and some give an alternative for Na production. In general, all the important reactions involved in the problem have been checked so far. As the puzzle is still unsolved, the solution might be hidden elsewhere.

Until now, Denisenkov's (2005) model is the most promising one, but as the adopted network is incomplete, the result seems not entirely reliable. For the model, further calculation with a more complicated nuclear network is necessary.

Meanwhile, it is still possible that the puzzle can be solved in the framework of nuclear physics. We have discussed the impacts of the three reactions individually. Actually, each reaction suffers uncertainty in its rate. When all the uncertainties take effect, the result may be quite different. In our forthcoming work on the Na overabundance we shall consider the several rate uncertainties together, by adjusting them randomly and individually within their error bars. The Monte Carlo method may be helpful here.

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