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A Running Average Method for Predicting the Size and Length of a Solar Cycle *

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Abstract The running correlation coefficient between the solar cycle amplitudes and the max-max cycle lengths at a given cycle lag is found to vary roughly in a cyclical wave with the cycle number, based on the smoothed monthly mean Group sunspot numbers available since 1610. A running average method is proposed to predict the size and length of a solar cycle by the use of the varying trend of the coefficients. It is found that, when a condition (that the correlation becomes stronger) is satisfied, the mean prediction error (16.1) is much smaller than when the condition is not satisfied (38.7). This result can be explained by the fact that the prediction must fall on the regression line and increase the strength of the correlation. The method itself can also indicate whether the prediction is reasonable or not. To obtain a reasonable prediction, it is more important to search for a running correlation coefficient whose varying trend satisfies the proposed condition, and the result does not depend so much on the size of the correlation coefficient. As an application, the peak sunspot number of cycle 24 is estimated as 140.4 ± 15.7, and the peak as May 2012 ± 11 months.

Key words: Sun: activity — Sun: sunspots — Sun: general

1 INTRODUCTION

Predicting the solar activity, especially in relation to the 11 - year solar cycles, is an important task in space weather. Various methods have been proposed to predict the amplitudes of solar cycles, which can be divided into two categories, statistical and precursor-based. The former depends mainly on some statistical properties of the cycles, and has tended poor results of prediction of the past few cycles (Conway 1998; Li et al. 2001), which may be caused by the intermittency and non-stationarity of the solar activity time evolution (Kremliovsky 1995; Usoskin & Mursula 2003). The latter method depends on some correlation between some preceding factor and the parameter to be predicted. Since Ohl (1966) noted a high correlation between the minimum of geomagnetic activity cycle and the amplitude of the following sunspot cycle, the precursor-based method has become important in the prediction of the cycle amplitudes. The correlation coefficients between cycle amplitudes and geomagnetically-based parameters are usually as high as $0.8 - 0.9$ (Wilson 1990; Kane 2007).

Precursor models are based on a physical relation between the poloidal solar magnetic field, estimated from geomagnetic activity in the declining phase of the preceding cycle (e.g., Hathaway et al. 1999), with the toroidal field responsible for sunspot formation. They have yielded small errors in the prediction of the size of cycles $20 - 22$ (Ohl 1976; Ohl & Ohl 1979; Kane 1978; Wilson 1990; Li et al. 2001), and so were believed to be superior to others (Kane 1978; Li et al. 2001). However, there is a large discrepancy on the precursor results for the 23rd cycle (Li et al. 2001) and for the 19th cycle (Kane 2007), although

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the correlation coefficients they were based on were very high (~ 0.9, Kane 2007). Recently, Dikpati et al. (2006) have predicted, based on model simulations of the solar dynamo, that the next sunspot cycle will be $30\% - 50\%$ higher than the current one. However, Choudhuri et al. (2007) announced, using a similar dynamo modelling, that the next cycle will be 30% lower than the current one. This indicates uncertainty in the predictive power of the new dynamo models, probably because of an important stochastic component in the solar dynamo (Charbonneau 2001; Usoskin et al. 2001).

A high correlation coefficient does not mean a successful prediction. One reason may be that solar activities are modulated by long-term periodicities (Meyer 1998; Kane 2001; Komitov & Bonev 2001; Wilson 1992; Hathaway et al. 1999; Du 2006a,b; Du et al. 2006). The size of cycle 24 has been predicted by many authors and the predictions cover a wide range (Wang et al. 2002; Li et al. 2005a; Du 2006a; Du & Du 2006; Kane 2007; Xu et al. 2008), which may again be caused by the long-term modulations and the complex correlations therein. Most of the predictions used one high correlation coefficient between the two parameters involved, which may not show the varying trend, and can hardly indicate by itself whether the prediction is reasonable or not.

In Section 2, we use the smoothed monthly mean Group sunspot numbers (Hoyt & Schatten 1998a,b) to determine the amplitudes ($R_{\rm m}$) and max-max cycle lengths ($P_{\rm m}$). The varying behavior of the running correlation coefficients between $R_{\rm m}$ and $P_{\rm m}$ at different cycle lags is investigated in Section 3. A running average method is proposed in Section 4 to improve the prediction of $R_{\rm m}$ using the varying trend found. Finally, a discussion and conclusions are given in Section 5.

2 DATA

The Wolf sunspot number has been the most important index of solar activity because of its long span of records. However, the data before 1850 are based on inaccurate or interrupted observations, and hence are less accurate than those after 1850 (Hoyt & Schatten 1998a,b). The present work uses the more homogeneous time series of Group Sunspot Number introduced by Hoyt & Schatten (1998a,b). It has a longer time series than the Wolf sunspot number, being available from 1610 to 1995. The smoothed monthly mean values *(http://www.ngdc.noaa.gov/stp/SOLAR/getdata.html)* are used to determine the $R_{\rm m}$ and $P_{\rm m}$ values listed in Table 1. The max-max cycle length was used, because it is directly correlated with $R_{\rm m}$ and is not dependent on the definition of minima. The data in the last row (cycle 23) were taken from the Wolf sunspot number, and were only used in the prediction of cycle 24, because the time series of Group Sunspot Number is close to that of Wolf sunspot number over the last few decades (Hoyt & Schatten 1998a,b).

Table 1 Values of $P_{\rm m}$ and $R_{\rm m}$ from Cycles -11 to 22

n	$P_{\rm m}$ (mon.)	$R_{\rm m}$	date	n	$P_{\rm m}$ (mon.)	$R_{\rm m}$	date
-11	131	42.8	1625 Nov	6	182	31.5	1816 Sep
-10	193	95.8	1641 Dec	7	159	64.4	1829 Dec
-9	125	6.71	1652 May	8	87	116.8	1837 Mar
-8	99	2.00	1660 Aug	9	140	93.2	1848 Nov
-7	195	2.13	1676 Nov	10	143	85.8	1860 Oct
-6	90	1.42	1684 May	11	121	99.9	1870 Nov
-5	132	0.13	1695 May	12	160	68.2	1884 Mar
-4	120	5.47	1705 May	13	118	96.0	1894 Jan
-3	174	34.2	1719 Nov	14	145	64.6	1906 Feb
-2	123	64.7	1730 Feb	15	138	111.3	1917 Aug
-1	136	57.3	1741 Jun	16	131	81.6	1928 Jul
0	105	71.7	1750 Mar	17	105	125.1	1937 Apr
1	134	71.0	1761 May	18	123	145.2	1947 Jul
2	100	106.5	1769 Sep	19	128	186.1	1958 Mar
3	113	78.2	1779 Feb	20	147	109.3	1970 Jun
4	104	90.1	1787 Oct	21	109	154.2	1979 Jul
5	165	51.1	1801 Jul	22	139	153.5	1991 Feb
				23[a]	114	120.8	2000 May

[a] Based on the Wolf numbers, only used in the prediction on cycle 24.

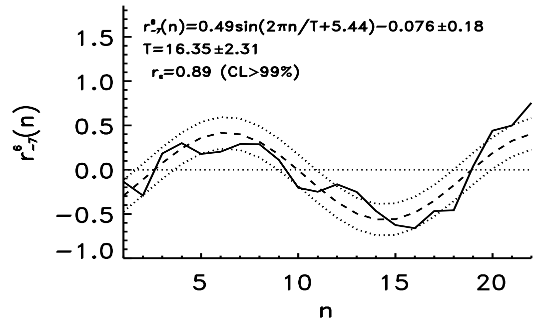

Fig. 1 Plot of $r^6_{-7}(n)$ versus n is roughly a wave with periodicity $T = 16.35 \pm 2.31$ (cycle). The solid line is the observed curve, and the dashed line is the fitting curve, with error bars on either sides (dotted lines).

3 RUNNING CORRELATION COEFFICIENTS BETWEEN R_m AND P_m AT DIFFERENT CYCLE LAGS

In order to understand the long-term behavior of solar cycles, we study the running correlation coefficient between R_m and P_m for a given cycle lag (l) at cycle n,

$$r_l^w(n) = \frac{\sum\limits_{i=n-w+1}^{n}[R_\mathrm{m}(i) - \overline{R}_\mathrm{m}][P_\mathrm{m}(i+l) - \overline{P}_\mathrm{m}]}{\sqrt{\sum\limits_{i=n-w+1}^{n}[R_\mathrm{m}(i) - \overline{R}_\mathrm{m}]^2 \sum\limits_{i=n-w+1}^{n}[P_\mathrm{m}(i+l) - \overline{P}_\mathrm{m}]^2}}\,, \tag{1}$$

where $R_\mathrm{m}(i)$ and $P_\mathrm{m}(i+l)$ are the values of R_m in cycle i and P_m in cycle $i+l$ ($-11 \le i, i+l \le 22$), respectively, w is the 'window' width for the cycles used, and

$$\overline{R}_\mathrm{m} = \frac{1}{w}\sum_{i=n-w+1}^{n} R_\mathrm{m}(i), \qquad \overline{P}_\mathrm{m} = \frac{1}{w}\sum_{i=n-w+1}^{n} P_\mathrm{m}(i+l), \tag{2}$$

are the average R_m and P_m, respectively. As an example, we show $r^6_{-7}(n)$ in Figure 1.

It should be noted in Figure 1 that $r^6_{-7}(n)$ varies in a wave with an obvious periodicity of about 16 cycles. It is positive around $n = 7$, negative around $n = 15$, and positive again around $n = 22$, indicating that the correlation varies in time. As a quantity to describe the long-term temporal evolution of sunspot numbers, $r^6_{-7}(n)$ can be fitted with a sinusoidal function,

$$r^6_{-7}(n) = 0.49\sin(2\pi n/T + 5.44) - 0.076 \pm 0.18, \tag{3}$$

$$T = 16.35 \pm 2.31, \tag{4}$$

$$r_c = 0.89(\mathrm{CL} > 99\%), \tag{5}$$

where '$\pm$' means the standard deviation, $T = 16.35 \pm 2.31$ (in units of 11 - yr cycles) is the regression periodicity, and $r_c = 0.89$ is the correlation coefficient between the fitting values (dashed line) and observed ones (solid line). The confidence level (CL) is greater than 99%. It means that $r^6_{-7}(n)$ varies with n in a sinusoidal wave with an obvious periodicity of 16.35 - cycle ($\sim$ 180 - yr).

Two more examples, $r^{10}_{-8}(n)$ and $r^9_0(n)$, are given in Figure 2, showing obvious sinusoidal variations with periodicities $T = 15.54 \pm 1.27$ (171 - yr) and 48.27 ± 8.43 (530 - yr), respectively. Although only some of the values are significant at 95% confidence level (*circled dots*), it is clearly shown that they vary regularly in sinusoidal waves. The correlation coefficients between the fitting and observed values are very high, being $r = 0.96$ in both cases. Figure 2b shows that R_m was inversely correlated with P_m of the same

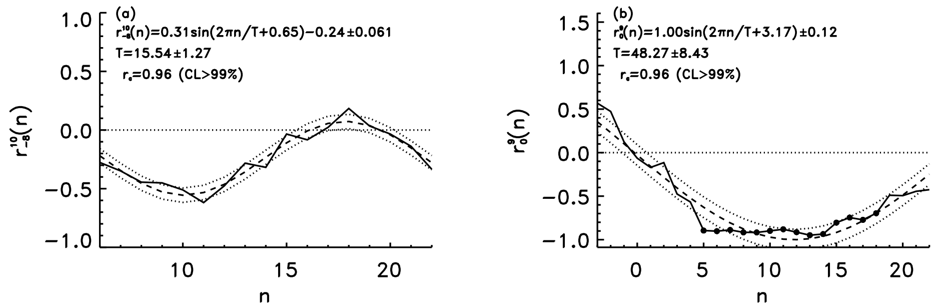

Fig. 2 (a) Plot of $r^{10}_{-8}(n)$ in periodicity of $T = 15.54 \pm 1.27$. (b) Plot of $r^{9}_{0}(n)$ in periodicity of $T = 48.27 \pm 8.43$. The circle dots denote the values significant at 95% confidence level.

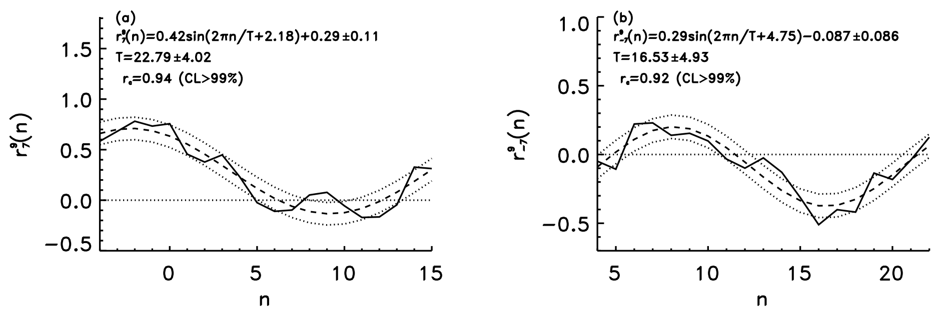

Fig. 3 (a) Plot of $r^{9}_{7}(n)$ in periodicity of $T = 22.79 \pm 4.02$. (b) Plot of $r^{9}_{-7}(n)$ in periodicity of $T = 16.53 \pm 4.93$.

cycle at 95% confidence level from $n = 5$ to 18, meaning that large amplitudes had short lengths in these cycles, but this correlation was becoming weaker from $n = 19$ onward.

The cyclical behavior of $r^{w}_{l}(n)$ is not confined to $l \leq 0$. Figure 3 shows that $r^{9}_{7}(n)$ varies cyclically in a periodicity of $T = 22.79 \pm 4.02$ (251 - yr) and that $r^{9}_{-7}(n)$ varies sinusoidally in a periodicity of $T = 16.53 \pm 4.93$ (182 - yr). The behaviors of $r^{9}_{l}(n)$ for $l = \pm 7$ are different from each other, implying that the correlation is not symmetric for $\pm l$. Figure 3a shows that $R_{\rm m}$ was positively correlated with the $P_{\rm m}$ seven cycles later at 95% confidence level from $n = -3$ to 0, while this correlation became weaker since then, and positive again now around $n = 15$.

Far from a constant, $r^{w}_{l}(n)$ is variable and even does not keep a stable sign. It varies in a cyclical wave rather than in a stochastic way. The cyclical behavior of $r^{w}_{l}(n)$ exists for nearly all l and w (in fact, all cases for $l = 0, \pm 1, \cdots, \pm 17$, $w = 3, 4, \cdots, 17$ have been examined), but shows different periodicities for different l and w (Figs. 1–3). For different w, it shows a similar behavior, larger w having a little more smooth behavior (see Figs. 1 and 3b). This cyclical behavior reflects the variations in cycle lengths and amplitudes, and may be modulated by some long-term cycles such as 80 - yr (Gleissberg 1971), 179 - yr (Jose 1965; Landscheidt 1999) and so on (notice that not all these periodicities were found in solar activities). It provides a quantity to study the long-term variations in sunspot numbers, especially when applied to the prediction on solar cycles.

4 APPLICATIONS OF $r_l^w(n)$ IN THE PREDICTION ON SOLAR CYCLES

Most workers pursue a high correlation coefficient between two or more parameters to study solar activities and/or to predict solar cycle amplitudes. Besides, they tend to make use of the time series as long as possible to seek a single correlation coefficient between the parameters involved. However, it is now known from Figures 1–3 that the coefficients vary in cyclical waves, and that the cyclical behaviors are different for different l. One coefficient can not be expected to provide enough information to show its varying trend and can not indicate whether the prediction is reasonable or not. In the present section, we show how to employ this cyclical behavior to improve the prediction on solar cycles.

4.1 A Running Average Method for Predicting the Amplitude and Length of Solar Cycles

We know from Appendix A that a prediction (extrapolated value) lies on the original regression line and increases the strength of the correlation — whether positive or negative, the correlation coefficient after including the additional pair of (prediction) data on the regression line must be stronger than the original one. However, the coefficient does not always behave in this way. If a prediction does not satisfy the varying trend of the coefficient, it may likely fail, which may be one of the reasons for some unsuccessful predictions in the past and the divergences of predictions from different authors.

Therefore, a prediction is reasonable only when its correlation coefficient and the corresponding observed one have the same varying trend, which can be expressed by the following condition,

$$\Delta r(n) = r_l^w(n-1)[r_l^w(n) - r_l^w(n-1)] > 0. \tag{6}$$

The value of $r_l^w(n)$ may not be known, whose trend can be estimated from its varying behavior.

A running average method to predict R_{m} is illustrated as follows. If we use certain $l(< 0)$ and w to predict R_{m} in cycle n, we may (i) take w pairs of $R_{\mathrm{m}}(i)$ and $P_{\mathrm{m}}(i+l)$, $i = n-w, n-w+1, \cdots, n-1$, to calculate the regression equation and the correlation coefficient $r_l^w(n-1)$; (ii) substitute the P_{m} value of cycle $n-1+l+1 = n+l$ into this equation, and a prediction of R_{m} in cycle n can be made, $R_{\mathrm{p}}(n)$; (iii) estimate $r_l^w(n)$ and the corresponding predicted value $r_{\mathrm{p}}(n)$ using $R_{\mathrm{p}}(n)$, then check if this prediction is reasonable using Equation (6); (iv) increase n by 1 each step and repeat the process above until suitable number of predictions are made.

For example, for $l = -7$ and $w = 6$, to predict R_{m} in cycle $n = 21$, we use six pairs of $R_{\mathrm{m}}(i)$ and $P_{\mathrm{m}}(i-7)$, $i = 15, 16, \cdots, 20$, to calculate the regression equation,

$$R_{\mathrm{m}} = 46.4 + 0.624 P_{\mathrm{m},-7}, \quad \sigma(n) = 32.2, \tag{7}$$

where $P_{\mathrm{m},-7}$ means the value of P_{m} seven cycles earlier, and $\sigma(n) = 32.2$ is the standard deviation used to estimate the prediction deviation in (the next) cycle n. Substituting $P_{\mathrm{m}}(14) = 145$ (month) of cycle $n + l = 14$ into Equation (7), the next $R_{\mathrm{m}}(n)$ value can be predicted as $R_{\mathrm{p}}(21) = 136.9$. Its prediction error, $\Delta R_{\mathrm{p}}(21) = \|R_{\mathrm{p}}(21) - R_{\mathrm{m}}(21)\| = 17.2$, is small compared with the estimated deviation $\sigma(n)$. The reason is that the predicted value, $R_{\mathrm{p}}(21)$, will make the extended correlation coefficient, $r_{\mathrm{p}}^{w+1}(21) = 0.45$ (coming from the formula of $r_l^{w+1}(21)$ after replaced $R_{\mathrm{m}}(21)$ by $R_{\mathrm{p}}(21)$), even greater, $r_{\mathrm{p}}^{w+1}(21) > r_{-7}^{6}(20) = 0.44 > 0$ (see Appendix A). As a result, the predicted running correlation coefficient, $r_{\mathrm{p}}(21) = 0.47$ (coming from $r_{-7}^{6}(21)$ after replaced $R_{\mathrm{m}}(21)$ by $R_{\mathrm{p}}(21)$), is possibly near to the observed $r_{-7}^{6}(21) = 0.50$, both having the same increasing trend, $r_l^w(n) > r_l^w(n-1)$, then the condition (Eq. (6)) is satisfied and the prediction is good. Notice that $r_{-7}^{6}(20)$ is not high, and that it is even insignificant at 95% confidence level.

Therefore, if the current $r_l^w(n-1)$ is positive and the next $r_l^w(n)$ is likely higher estimated from its varying trend, $r_l^w(n) > r_l^w(n-1) > 0$, a good prediction (with a relatively small error) of R_{m} can be made from the linear regression equation of R_{m} versus P_{m} in the ascending phase of $r_l^w(n)$ (e.g., for $n = 21-22$ in Fig. 1).

On the other hand, if the current $r_l^w(n-1)$ is negative and $r_l^w(n) > r_l^w(n-1)$ (e.g., for n= 17–19 in Fig. 1), it can not yield a good prediction of $R_{\mathrm{m}}(n)$ because the predicted value will make the extended $r_{\mathrm{p}}^{w+1}(n)$ even more negative, $r_{\mathrm{p}}^{w+1}(n) < r_l^w(n-1) < 0$ (see Appendix A). Thus it can hardly make $r_{\mathrm{p}}(n)$, i.e., the predicted $r_l^w(n)$, satisfy the ascending varying trend of $r_l^w(n)$.

Table 2 Predictions of R_m in Cycles 9 – 22 for $l = -7$ and $w = 6$

Cycle		Current cycle			Prediction					
n	Used	Regression	$R_\mathrm{m}(n)$	$r_l^w(n)$	$R_\mathrm{p}(n)$[a]	$r_\mathrm{p}(n)$[b]	$\sigma(n)$[c]	ΔR_p[d]	$r_\mathrm{p}^{w+1}(n)$[e]	$\Delta r(n)$[f]
8	$-4-8$	$23.4+0.368P_{\mathrm{m},-7}$	116.8	0.29						
9	$-3-9$	$58.0+0.128P_{\mathrm{m},-7}$	93.2	0.11	60.2	0.35	28.7	32.9	0.33	$-$
10	$-2-10$	$123.7-0.421P_{\mathrm{m},-7}$	85.8	-0.20	72.5	-0.18	31.0	13.3	0.11	$-$
11	$-1-11$	$136.7-0.475P_{\mathrm{m},-7}$	99.9	-0.25	79.9	-0.16	30.2	20.0	-0.22	$+$
12	$0-12$	$103.7-0.130P_{\mathrm{m},-7}$	68.2	-0.17	58.3	-0.31	29.1	9.9	-0.39	$-$
13	$1-13$	$109.1-0.119P_{\mathrm{m},-7}$	96.0	-0.25	80.0	-0.51	19.5	16.0	-0.23	$+$
14	$2-14$	$111.4-0.195P_{\mathrm{m},-7}$	64.6	-0.47	90.2	-0.34	15.5	25.6	-0.26	$+$
15	$3-15$	$127.9-0.298P_{\mathrm{m},-7}$	111.3	-0.63	94.4	-0.49	13.2	16.9	-0.52	$+$
16	$4-16$	$133.2-0.331P_{\mathrm{m},-7}$	81.6	-0.66	86.1	-0.67	14.4	4.5	-0.63	$+$
17	$5-17$	$141.4-0.344P_{\mathrm{m},-7}$	125.1	-0.47	85.8	-0.60	13.9	39.3	-0.66	$-$
18	$6-18$	$161.4-0.414P_{\mathrm{m},-7}$	145.2	-0.46	99.7	-0.40	21.3	45.5	-0.48	$-$
19	$7-19$	$110.6+0.062P_{\mathrm{m},-7}$	186.1	0.04	95.1	-0.50	26.1	91.0	-0.47	$-$
20	$8-20$	$46.4+0.624P_{\mathrm{m},-7}$	109.3	0.44	117.9	0.43	43.9	8.7	0.04	$+$
21	$9-21$	$-25.3+1.153P_{\mathrm{m},-7}$	154.2	0.50	136.9	0.47	32.2	17.2	0.45	$+$
22	$10-22$	$-29.0+1.270P_{\mathrm{m},-7}$	153.4	0.76	133.8	0.75	31.7	19.7	0.50	$+$
23					137.3	0.90	17.4		0.76	$+$
good[g]							σ_g =25.1	δ_g =16.1		$+$
bad[h]							σ_b =25.0	δ_b =38.7		$-$

[a] The predicted value of $R_\mathrm{m}(n)$ from the regression equation in cycle $n-1$.
[b] $r_\mathrm{p}(n)$: The predicted value of $r_l^w(n)$ after replaced $R_\mathrm{m}(n)$ by $R_\mathrm{p}(n)$.
[c] The standard deviation of regression equation in cycle $n-1$, used to estimate the prediction deviation in cycle n.
[d] $\Delta R_\mathrm{p} = \|R_\mathrm{p} - R_\mathrm{m}\|$: prediction error.
[e] $r_\mathrm{p}^{w+1}(n)$: the extended value $r_l^{w+1}(n)$ after replaced $R_\mathrm{m}(n)$ by $R_\mathrm{p}(n)$.
[f] The '+' or '−' means whether or not condition (6) is satisfied.
[g] When condition (6) is satisfied, the mean prediction error is $\delta_g = 16.1$, and the corresponding mean estimated deviation σ_g =25.1.
[h] When condition (6) is not satisfied, the mean prediction error is $\delta_b = 38.7$, and the corresponding mean estimated deviation σ_b =25.0.

For example, $r^6_{-7}(16) = -0.6611$ is highly negative for $n-1 = 16$ (its minus peak), but it is inferred from Figure 1 that $r^6_{-7}(17) > r^6_{-7}(16)$. With data up to cycle 16, the regression equation can be calculated as

$$R_\mathrm{m} = 133.2 - 0.331 P_{\mathrm{m},-7}, \quad \sigma(n) = 13.9. \tag{8}$$

The value of $R_\mathrm{m}(n)$ in $n = 17$ is predicted as $R_\mathrm{p}(17) = 85.8$. The prediction error, $\Delta R_\mathrm{p}(17) = \|R_\mathrm{p}(17) - R_\mathrm{m}(17)\| = 39.3$, is large compared with the estimated deviation $\sigma(n)$. The reason is that the predicted value $R_\mathrm{p}(17)$ will make the negative value of $r^6_{-7}(16)$ even more negative, $r_\mathrm{p}^{6+1}(17) = -0.6614 < r^6_{-7}(16) < 0$, opposite to its ascending trend. As a result, the predicted $r_\mathrm{p}(17) = -0.60$ is unlikely near to the observed $r^6_{-7}(17) = -0.47$, and Equation (6) is not satisfied.

With this technique, the predictions of $R_\mathrm{m}(n)$ for cycles 9 – 22 can be tested. The results are listed in Table 2, and the values of $r^6_{-7}(n)$, $r_\mathrm{p}(n)$, $\Delta R_\mathrm{p}(n)$, and $\sigma(n)$ are shown in Figure 4.

From Table 2 and Figure 4, we note:

a) When the predicted $r_\mathrm{p}(n)$ (dotted line) is close to the observed $r_l^w(n)$ (solid line), the predicted value, $R_\mathrm{p}(n)$, is near to the observed $R_\mathrm{m}(n)$, e.g., for $n = 10$, 16, 20 – 22.
b) When Equation (6) is satisfied [$\Delta r(n) > 0$, 8 cases], the prediction error $\Delta R_\mathrm{p}(n)$ (dashed line) usually (6/8 cases) falls below the estimated deviation $\sigma(n)$ (the dash-dot-dot line), and below 20 (*triangles*, 7/8 cases). The mean prediction error, δ_g =16.1, is less than the corresponding mean estimated deviation, σ_g =25.1. This situation occurs where $r_l^w(n-1) > 0$ in the ascending phase (e.g., for $n = 20-22$) and $r_l^w(n-1) < 0$ in the descending phase (e.g.,for $n = 11$, 13, 15 – 16).
c) When Equation (6) is not satisfied [$\Delta r(n) < 0$, 6 cases], the prediction error $\Delta R_\mathrm{p}(n)$ is usually above (4/6 cases) the estimated deviation $\sigma(n)$, and above 20 (4/6 cases). The mean prediction error,

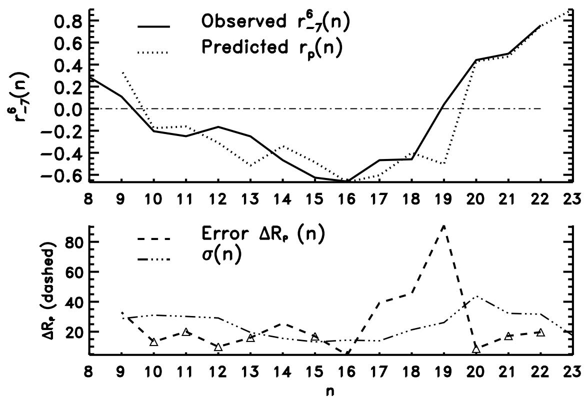

Fig. 4 Predictions of R_{m} from regression equation. Upper panel: observed $r^6_{-7}(n)$ (solid line), and predicted $r_{\mathrm{p}}(n)$ (dotted line). Lower panel: the prediction error $\Delta R_{\mathrm{p}} = \|R_{\mathrm{p}} - R_{\mathrm{m}}\|$ (dashed line), and the estimated deviation $\sigma(n)$ (dash-dot-dot-line). ΔR_{p} is usually less than $\sigma(n)$, and less than 20 (triangles) when Equation (6) is satisfied.

δ_b =38.7, is greater than the corresponding mean estimated deviation, σ_b =25.0. This situation occurs where $r^w_l(n-1) < 0$ in the ascending phase (e.g., for $n = 17-19$) and $r^w_l(n-1) > 0$ in the descending phase (e.g., for n = 9).

An exception occurs in cycle 12. Here, Equation (6) is not satisfied, but its prediction error, 9.9, is small, because $r^6_{-7}(n)$ is in its descending phase and Equation (6) should have been satisfied. In fact, Equation (6) will be satisfied when using the predicted $r_p(11) = -0.16$ (Table 2 and Fig. 4), thus this good prediction seems reasonable. Another exception is in cycle 14, the condition is satisfied, but its prediction error (25.6) is larger than $\sigma(14) = 15.5$, which may be caused by the very small $R_{\mathrm{m}}(14) = 64.6$ in the modern era data since cycle 8. Considering that Equation (6) will not be satisfied when using the predicted $r_{\mathrm{p}}(13) = -0.51$ (Table 2 and Fig. 4), this bad prediction seems reasonable. In summary, in most cases, the predictions are reasonable when Equation (6) is considered.

Vitinsky et al. (1986) pointed out that there are systematic uncertainties of about 25% in monthly sunspot numbers due to observational conditions in the definition of groups (and individual sunspots). A prediction is rather good if its prediction error can be limited within the mean estimated deviation, $\overline{\sigma} =$ 25, in the modern era data. In the case $r^6_{-7}(n)$, the mean prediction error is only about δ_g =16.1 when Equation (6) is satisfied, much smaller than δ_b =38.7 when Equation (6) is not satisfied. Therefore, a prediction is called 'good' when Equation (6) is satisfied, and 'bad' otherwise.

However, if Equation (6) is not satisfied, suitable l and w can always be selected to satisfy this condition and so to yield a better prediction. For example, the condition is not satisfied in cycles 17 – 19. So the cases $r^6_{-12}(n)$ and $r^4_{-13}(n)$ have been considered, and similar works have been repeated as before. Their results are shown in Table 3 and Figure 5.

Similar conclusions can be obtained, (i) in the case $r^6_{-12}(n)$, the mean good prediction error, $\delta_g = 12.4$, is smaller than the mean bad prediction error, $\delta_b = 31.2$; (ii) in the case $r^4_{-13}(n)$, the mean good prediction error, $\delta_g = 16.3$, is smaller than the mean bad prediction error, $\delta_b = 40.1$. Here attentions are focused on the prediction errors in cycles 17 – 19. In Figure 5a, Equation (6) is satisfied in cycles 17 – 18, and the prediction errors (0.6, 5.4) are now both small. In Figure 5b, when Equation (6) is satisfied in cycle 19, the prediction error, 36.4, is much less than those when Equation (6) is not satisfied ($\sim$ 91 in Fig. 4, and $\sim$ 62 in Fig. 5a). It suggests that the prediction error can be greatly reduced when Equation (6) is satisfied (see also cycles 17–18 in Table 3). It should be pointed out that cycle 19 can not be accurately predicted even using other methods. For example, Kane (2007) has ever used a very high correlation coefficient ($r = +0.94$) in precursor method to 'predict' the size of cycle 19, but the prediction error, 52, is great

Table 3 Comparison of the Predictions of R_{m} in Cycles 9 – 23 for some l and w

		$l=-7, w=6$		$l=-12, w=6$		$l=-13, w=4$	
n	R_{m}	R_{p}	ΔR_{p}[a]	R_{p}	ΔR_{p}[a]	R_{p}	ΔR_{p}[a]
9	93.2	60.2	32.9^{-}	66.5	26.6^{-}	68.1	25.1^{+}
10	85.8	72.5	13.3^{-}	74.4	11.4^{+}	59.3	26.4^{-}
11	99.9	79.9	20.0^{+}	73.6	26.3^{+}	88.6	11.3^{-}
12	68.2	58.3	9.9^{-}	67.6	0.7^{-}	99.2	31.0^{-}
13	96.0	80.0	16.0^{+}	88.4	7.5^{+}	92.4	3.6^{+}
14	64.6	90.2	25.6^{+}	87.6	23.0^{+}	87.6	23.0^{+}
15	111.3	94.4	16.9^{+}	77.9	33.4^{-}	107.4	3.9^{+}
16	81.6	86.1	4.5^{+}	76.1	5.5^{+}	91.7	10.1^{-}
17	125.1	85.8	39.3^{-}	125.7	0.6^{+}	99.8	25.2^{-}
18	145.2	99.7	45.5^{-}	139.8	5.4^{+}	10.9	134.3^{-}
19	186.1	95.1	91.0^{-}	124.2	61.9^{-}	149.7	36.4^{+}
20	109.3	117.9	8.7^{+}	69.4	39.9^{-}	151.7	42.4^{-}
21	154.2	136.9	17.2^{+}	129.7	24.4^{-}	104.1	50.1^{-}
22	153.4	133.8	19.7^{+}	135.8	17.6^{+}	148.0	5.5^{+}
23	120.8[b]	137.3	16.5^{+}	135.3	14.5^{+}	150.9	30.1^{-}
24[c]		99.2	$\pm 13.7^{-}$	157.6	$\pm 19.0^{-}$	140.4	$\pm 15.7^{+}$
good[d]	δ_g		16.1		12.4		16.3
bad[e]	δ_b		38.7		31.2		40.1

[a] '+' means Equation (6) is satisfied, and '$-$' means Equation (6) is not satisfied.
[b] The amplitude of Wolf sunspot number in cycle 23.
[c] Results when using 120.8 as the amplitude in cycle 23. The values of ΔR_{p} in cycle 24 are the estimated deviations from cycle 23.
[d] The mean prediction error when Equation (6) is satisfied, except for cycle 24.
[e] The mean prediction error when Equation (6) is not satisfied, except for cycle 24.

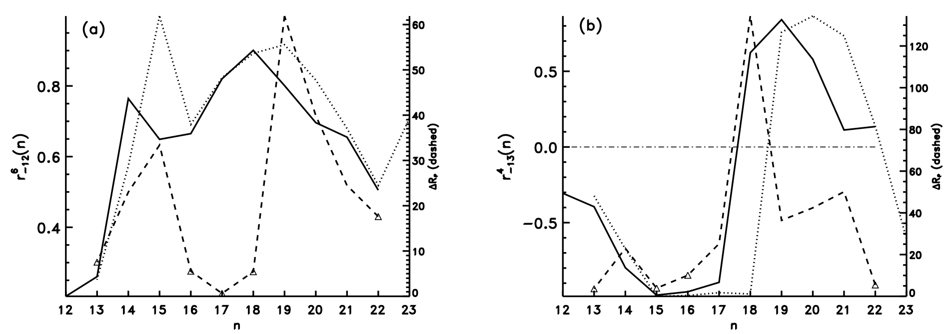

Fig. 5 Observed $r_l^w(n)$(solid line), predicted $r_{\mathrm{p}}(n)$(dotted line) and prediction error $\Delta R_{\mathrm{p}} = \|R_{\mathrm{p}} - R_{\mathrm{m}}\|$ (dashed line, right-hand scaled). (a) For $r^{6}_{-12}(n)$, Equation (6) is satisfied in cycles 17 – 18, and the prediction errors are now 0.6 and 5.4. (b) For $r^{4}_{-13}(n)$, Equation (6) is satisfied in cycle 19, and the prediction error is now reduced to about 36.

— much larger than 36.4 when using a not-high correlation (~ 0.6) in Figure 5b. The reason is that cycle 19 was an abnormally largest cycle and "most of the predictions about this cycle have proved grossly erroneous" (Kane 2007). Apart from this special cycle, the prediction errors are all less than about 26 under the condition of Equation (6), and the mean is only about 15.

In general, the prediction error is usually less when Equation (6) is satisfied than it is not satisfied (e.g., cycles 17–18 in Table 3), and large prediction errors always appear in the cases when the condition is not satisfied, especially when $r_l^w(n)$ changes sign (e. g., cycle 19 in Fig. 4 and cycle 18 in Fig. 5b).

Therefore, Equation (6) can provide a better prediction than that without this condition. If only one correlation coefficient is to be used, its varying trend (and whether the condition satisfies) is not known. When the correlation coefficient is at its positive maximum (or negative minimum), the prediction will destroy its varying trend. In this case, the prediction should be treated with caution and may likely be unsuccessful because Equation (6) will not be satisfied. In summary, a good prediction depends on the varying trend of $r_l^w(n)$ under the condition of Equation (6) rather than just a high(est) correlation coefficient.

4.2 Predicting the Size of Cycle 24

For an application to cycle 24, the parameters have been extended to cycle 23 by taking the amplitude of smoothed monthly mean Wolf sunspot number (120.8 in May 2000) as the $R_{\rm m}$ value in cycle 23. Then the method in Section 4.1 was used in the cases of $r^6_{-7}(n)$, $r^6_{-12}(n)$ and $r^4_{-13}(n)$ to estimate the size of cycle 24. The results are listed in Table 3. Of the three predictions, only one may be reasonable.

a) From the regression equation of $r^6_{-7}(23)$, $R_{\rm m}(24)$ is predicted to be $R_{\rm p}(24) = 99.2 \pm 13.7$. It should be noted in Figure 4 that $r^6_{-7}(23)$ is most likely in its positive peak, thus Equation (6) would not be satisfied in the next cycle, so this prediction is bad.
b) From $r^6_{-12}(23)$, $R_{\rm m}(24)$ is predicted to be $R_{\rm p}(24) = 157.6 \pm 19.0$. It can be seen in Figure 5a that the next $r^6_{-12}(24)$ value should be positive and in the descending phase, thus Equation (6) would not be satisfied in the next cycle, so this prediction is also bad.
c) From $r^4_{-13}(23)$, $R_{\rm m}(24)$ is predicted to be $R_{\rm p}(24) = 140.4 \pm 15.7$. In view of the varying trend of $r^4_{-13}(n)$ in Figure 5b, its negative value is just in the descending phase, thus Equation (6) would be satisfied in the next cycle, so this prediction is reasonable.

According to the above analysis, the size of cycle 24 should be reasonably predicted as $R_{\rm p}(24) = 140.4 \pm 15.7$ — it is unlikely to be less than 100 or greater than 158. The predicted value is near to the 136 predicted by Li et al. (2005a), to the 142 ± 24 by Kane (2007), to the 145 ± 30 by Hathaway & Wilson (2004), to the 149.5 ± 27.6 by Du et al. (2006), and to the 150.3 ± 22.4 by Du (2006a), but it is lower than the prediction (~ 168) by Dikpati et al. (2006), and higher than that (~ 76) by Choudhuri et al. (2007), both based on a flux-transport dynamo model.

Recently, Kane (2007) used the precursor method to predict the size of cycles 19–24. From his table 2, one notes that the prediction errors are small (less than 20) when Equation (6) is satisfied (in cycles 21–22), and large (larger than 20) when Equation (6) is not satisfied (in cycles 19–20, and 23), although the correlations he used are all very high (~ 0.9). His results testified the rationale of Equation (6) in the precursor method. Since the correlation (0.89) he used to predict cycle 24 is his lowest one, its varying trend should satisfy Equation (6) and thus his prediction value, 142, should also be reasonable.

Certainly, there may be many different pairs of w and l satisfying (or not satisfying) the Equation (6) for a certain n. Our study gives only three special cases in order to show how to improve solar cycle predictions using this condition. The only reasonable prediction is the one that satisfies Equation (6).

4.3 Predicting the Peak Date of Cycle 24

To estimate the peak date of cycle 24, we use $r^9_{-7}(n)$ of $P_{\rm m}$ - $R_{\rm m}$ (similar to using $r^9_7(n)$ of $R_{\rm m}$ - $P_{\rm m}$ in Fig. 3a) on repeating the process above. The results are shown in Figure 6. It should be noted that, when Equation (6) is satisfied (cycles 10, 13, 16, 18 and 23), the mean prediction error ($\delta_g = 10.4$) is much less than when Equation (6) is not satisfied (cycles 8, 9, 11, 12, 15, 17, 20 and 21, $\delta_b = 28.8$), and that the larger prediction errors always occur in the cases when Equation (6) is not satisfied (cycles 8, 9, 11, 12, 17 and 21). The behaviour of $r^9_{-7}(n)$ in Figure 6 shows that its positive value is in an ascending phase near $n = 23$, so it is most likely that Equation (6) will be satisfied in the next cycle. Therefore, the prediction for cycle 24, $P_{\rm p}(24) = 144 \pm 11$ months, is reasonable, and the peak of cycle 24 should occur in May 2012 $\pm$ 11 months. This is later than the predictions by some authors (Sello 2003; Li et al. 2005a; Du et al. 2007; Xu et al. 2008) that the peak of cycle 24 may occur in 2011.

5 DISCUSSION AND CONCLUSIONS

The 11 - yr (Schwabe) cycle is fundamental, while the 80 - yr Gleissberg cycle is often described as an amplitude modulation of the solar cycle (Cole 1973). Apart from these two cycles, some other periodicities

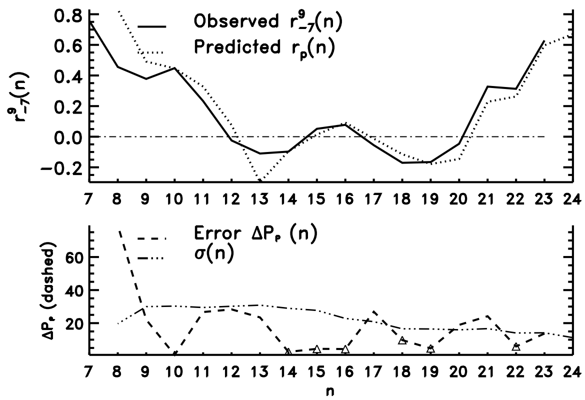

Fig. 6 Predictions of $P_{\rm m}$ from the regression equation, $P_{\rm m}$ - $R_{\rm m}$ at lag -7. Upper panel: the observed $r^{9}_{-7}(n)$ (solid line) and the predicted $r_{\rm p}(n)$ (dotted line). Lower panel: the prediction error $\Delta P_{\rm p} = \|P_{\rm p} - P_{\rm m}\|$ (dashed line), and the estimated deviation $\sigma(n)$ (dash-dot-dot line). $\Delta R_{\rm p}$ is usually less than $\sigma(n)$, and less than 12 months (triangles) when Equation (6) is satisfied.

seem insignificant in the sunspot numbers (Li et al. 2005b). Studies of the correlations between cycle amplitudes and cycle lengths are important for understanding the mechanism of the solar cycle (Bracewell 1988; Dicke 1988). A high and stable correlation coefficient between two parameters is often searched after when the regression equation is used to study and/or predict solar activities.

However, a high correlation coefficient does not mean a successful prediction, such as the high correltion coefficients in the so-called precursor methods for predicting the size of cycle 23 (Li et al. 2001). One reason may be that solar activities are modulated by long-term trends, for which a single correlation coefficient can hardly incorporate.

The present work shows that the running correlation coefficient ($r^{w}_{l}(n)$) between $R_{\rm m}$ and $P_{\rm m}$ at a certain cycle lag varies in a cyclical way with the cycle number following a long-term periodicity rather than keeping to a constant value. This cyclical behavior may partly reflect the long-term trends in the solar activity. The same width, w, aims to make the values has the same accuracy of measurement for comparison. If $r^{w}_{l}(n)$ could be estimated from its varying behavior, then the $R_{\rm m}(n)$ value may be predicted. A good prediction of $R_{\rm m}$ does not depend so much on a high value of the current correlation coefficient as on the next $r^{w}_{l}(n)$ value to be predicted.

A running average method is proposed to predict the size and length of a solar cycle. Through the varying trend of $r^{w}_{l}(n)$, the prediction may be improved. Various values of l and w have been examined with the result that the prediction error under the proposed condition is usually much less than when Equation (6) is not satisfied. This condition reflects some long-term trends in sunspot cycles and is based on the fact that the prediction, as an extrapolation, must be on the regression line and increase the strength of correlation. Therefore, a reasonable prediction can be made through selecting suitable l and w to satisfy the condition.

From the analysis above, we can summarize the conclusions as follows:

a) The running correlation coefficient between solar cycle amplitudes and max-max cycle lengths at a certain cycle lag varies in a cyclical way on the cycle number.
b) A reasonable prediction can be made under the condition that the running correlation coefficient becomes stronger.
c) Using the proposed condition, it is predicted that cycle 24 will have a peak sunspot number of 140.4 ± 14.7 and that the peak will occur in May 2012 ± 11 months.

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Appendix A: THE CORRELATION COEFFICIENT AFTER ADDING A PAIR OF DATA ON THE REGRESSION LINE

For two time series, X=$\{x_1, x_2, \cdots, x_w\}$ and Y=$\{y_1, y_2, \cdots, y_w\}$, their correlation coefficient is given by

$$r = \frac{\sum_{i=1}^{w} \Delta x_i \Delta y_i}{\sqrt{\sum_{i=1}^{w} \Delta x_i^2 \sum_{i=1}^{w} \Delta y_i^2}} = \frac{\sum_{i=1}^{w} \Delta x_i \Delta y_i}{S_x S_y}, \tag{A.1}$$

where $\overline{x} = \frac{1}{w}\sum_{i=1}^{w} x_\mathrm{i}$, $\overline{y} = \frac{1}{w}\sum_{i=1}^{w} y_\mathrm{i}$, $\Delta x_i = x_\mathrm{i} - \overline{x}$, $\Delta y_i = y_\mathrm{i} - \overline{y}$, $S_x = \sqrt{\sum_{i=1}^{w} \Delta x_i^2}$, and $S_y = \sqrt{\sum_{i=1}^{w} \Delta y_i^2}$. Suppose that the regression equation is $y = kx + c$ with k, c constants, then $\overline{y} = k\overline{x} + c$, and k has the same sign with r.

For a given point (x_{w+1}, y_{w+1}) on the regression line, let $x_{w+1} = \overline{x} + \Delta x$, then $y_{w+1} = kx_{w+1} + c = k\overline{x} + k\Delta x + c = \overline{y} + k\Delta x$. The "extended" correlation coefficient is given by

$$r' = \frac{\sum_{i=1}^{w+1} \Delta x_i' \Delta y_i'}{\sqrt{\sum_{i=1}^{w+1} {\Delta x_i'}^2 \sum_{i=1}^{w+1} {\Delta y_i'}^2}}, \tag{A.2}$$

where

$$\begin{aligned}
\overline{x'} &= \frac{\sum_{i=1}^{w} x_\mathrm{i} + x_{w+1}}{w+1} = \overline{x} + \frac{\Delta x}{w+1}, & \text{(A.3)}\\
\overline{y'} &= \frac{\sum_{i=1}^{w} y_\mathrm{i} + y_{w+1}}{w+1} = \overline{y} + \frac{k\Delta y}{w+1}, & \text{(A.4)}\\
\Delta x_i' &= x_i - \overline{x'} = \Delta x_i - \Delta x/(w+1), & \text{(A.5)}\\
\Delta y_i' &= y_i - \overline{y'} = \Delta y_i - k\Delta x/(w+1), & \text{(A.6)}\\
\Delta x_{w+1}' &= (\overline{x} + \Delta x) - \overline{x'} = w\Delta x/(w+1), & \text{(A.7)}\\
\Delta y_{w+1}' &= (\overline{y} + k\Delta x) - \overline{y'} = wk\Delta x/(w+1). & \text{(A.8)}
\end{aligned}$$

It follows,

$$\begin{aligned}
r' &= \frac{\sum_{i=1}^{w} \Delta x_i' \Delta y_i' + \Delta x_{w+1}' \Delta y_{w+1}'}{\sqrt{\left(\sum_{i=1}^{w} {\Delta x_i'}^2 + {\Delta x_{w+1}'}^2\right)\left(\sum_{i=1}^{w} {\Delta y_i'}^2 + {\Delta y_{w+1}'}^2\right)}}\\
&= \frac{rS_xS_y + k\delta^2}{\sqrt{(S_x^2 + \delta^2)\left(S_y^2 + k^2\delta^2\right)}}, & \text{(A.9)}
\end{aligned}$$

where $\delta^2 = w\Delta x^2/(w+1)$.

Now we consider

$$\begin{aligned}
\theta(r) &= (r'^2 - r^2)\left(S_x^2 + \delta^2\right)\left(S_y^2 + k^2\delta^2\right)/\delta^2\\
&= k^2\delta^2 + 2rkS_xS_y - r^2(k^2S_x^2 + S_y^2 + k^2\delta^2). & \text{(A.10)}
\end{aligned}$$

Because for the quadratic function $\theta(r)$,

(a) $\theta''(r) = -2(k^2 S_x^2 + S_y^2 + k^2\delta^2) < 0$;
(b) if $r = 0$, then $\theta(0) = k^2\delta^2 > 0$;
(c) if $r = 1$, then $S_y = kS_x$, $\theta(1) = (1 - r^2)k^2\delta^2 - r^2(S_y - kS_x)^2 + 2rkS_xS_y(1 - r) = 0$;
(d) if $r = -1$, then $S_y = -kS_x$, $\theta(-1) = (1 - r^2)k^2\delta^2 - r^2(S_y + kS_x)^2 + 2rkS_xS_y(1 + r) = 0$.

The function $\theta(r)$ must have a maximum $\theta_{\rm m} \geq \theta(0)$ at a certain $-1 < r_{\rm m} < 1$. (i) If $-1 < r_{\rm m} < 0$, $\theta(r)$ will increase from $\theta(-1) = 0$ to its maximum $\theta_{\rm m}$ as r increases from -1 to $r_{\rm m}$, and then decrease from $\theta_{\rm m}$ to $\theta(0)$ and to $\theta(1) = 0$ as r increases from $r_{\rm m}$ to 0 and to 1. (ii) If $0 < r_{\rm m} < 1$, $\theta(r)$ will increase from $\theta(-1) = 0$ to $\theta(0)$ and to $\theta_{\rm m}$ as r increases from -1 to 0 and to $r_{\rm m}$, and then decrease from $\theta_{\rm m}$ to $\theta(1) = 0$ as r increases from $r_{\rm m}$ to 1. Therefore, for any r, we have always $\theta(r) > 0$, i.e., ${r'}^2 > r^2$ or $\|r'\| > \|r\|$.

If $r > 0$, then $r' > r$, the point on the regression line will increase the positive correlation coefficient. If $r < 0$, then $-r' > -r$ or $r' < r$, the point on the regression line will also strengthen the negative correlation coefficient.

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