Chin. J. Astron. Astrophys. Vol. 8 (2008), No. 3, 269–276
(*http://www.chjaa.org*)

Chinese Journal of
Astronomy and
Astrophysics

Microscopic Magnetic Dipole Radiation in Neutron Stars *

Hao Tong, Qiu-He Peng and Hua Bai

Department of Astronomy, Nanjing University, Nanjing 210093, China; *htong_2005@163.com*

Received 2007 July 19; accepted 2007 October 31

Abstract There is a $^3P_2$ neutron superfluid region in NS (neutron star) interior. For a rotating NS the $^3P_2$ superfluid region is like a system of rotating magnetic dipoles. It will give out electromagnetic radiation, which may provide a new heating mechanism of NSs. This mechanism plus some cooling agent may give a sound explanation to NS glitches.

Key words: stars: neutron — pulsars: general — dense matter — magnetic fields

1 INTRODUCTION

Since the discovery of the NS (neutron star) and glitches in late 1960s, it is generally believed that there is superfluidity in NS's interior (Ruderman 1976; Shapiro et al. 1983; Elgarøy et al. 1996). Cooling mechanism associated with superfluidity was first proposed by Flowers et al. (1976). Not until recently, is the importance of superfluidity recognized seriously in the "Minimal model" (Gusakov et al. 2004; Page et al. 2004; Kaminker et al. 2006).

One may ask: Since the cooling agent associated with superfluidity must be considered in the "Minimal model", what about heating mechanisms? A heating mechanism accompanying superfluidity has been presented by Alpar et al. (1989), and was taken into the NS cooling model by Umeda et al. (1994), Page et al. (2006) and Tsuruta (2006). Here we investigate another possible heating mechanism: microscope magnetic dipole radiation (MMDR).

Our basic consideration is as follows: there is $^3P_2$ superfluid region in the interior of an NS. We calculate its paramagnetic properties in the presence of a background magnetic field. Since the neutron superfluid is in a vortex state, the $^3P_2$ superfluid region is like a system of rotating magnetic dipoles, which will induce magnetic dipole radiation. Because the emitted photons can not penetrate the NS matter, a heating mechanism would result. The origin of this mechanism is the microscopic magnetic dipole radiation: we call it MMDR heating of NSs. Moreover, this heating mechanism plus some cooling agent may give an adequate explanation of the NS glitches (Bai et al. 2006; Peng et al. 2006; Peng 2008, in preparation).

2 SUPERFLUIDITY IN NS

First, we check the superfluidity in NSs. There are two relevant regimes of neutron superfluid inside the NS interior: one is the isotropic $^1S_0$ neutron superfluid within the density range $1 \times 10^{10} < \rho(\mathrm{g\,cm^{-3}}) < 1.6 \times 10^{14}$. The critical temperature is $T_\mathrm{c}(^1S_0) \approx 1 \times 10^{10}\,\mathrm{K}$.

Another regime is the anisotropic $^3P_2$ neutron superfluid within a wide density range: $1.3 \times 10^{14} < \rho\,(\mathrm{g\,cm^{-3}}) < 7.2 \times 10^{14}$. The critical temperature is

$$T_\mathrm{c}(^3P_2) = \Delta_\mathrm{max}(^3P_2)/2k \approx 2.78 \times 10^8\,\mathrm{K}\,. \tag{1}$$

We note that the energy gap $\Delta(^3P_2)$ is almost a constant at the maximum with an error less than 3% over a rather wide density range, $3.3 \times 10^{14} < \rho\,(\mathrm{g\,cm^{-3}}) < 5.2 \times 10^{14}$ (see fig. 2 of Elgarøy et al. 1996, but we neglect the F state of neutron Cooper pair here).

* Supported by the National Natural Science Foundation of China.

It is well known that a rotational superfluid must be in superfluid vortexes. In general the vortex filaments are arranged in a symmetrical lattice, almost exactly parallel to the axis of rotation of the NS. The circulation of every vortex filament, Γ, is quantized:

$$\Gamma = \oint \boldsymbol{v} \cdot \mathrm{d}\boldsymbol{l} = n\Gamma_0, \tag{2}$$

$$\Gamma_0 = \frac{2\pi\hbar}{2m_{\mathrm{n}}}. \tag{3}$$

Here n is a circulation quantum number of the vortex, m_{n} the neutron mass, $\hbar$ the Planck's constant and Γ_0 the unit vortex quantum (Feynman 1955; Lifshitz et al. 1999).

It is supposed that the core of the superfluid vortex is a cylindrical region of normal neutron fluid immersed in the superfluid neutron sea. Normally, the radius of the vortex core, a_0, is taken to be the coherent length of the neutron superfluid,

$$a_0 = \frac{E_{\mathrm{F}}}{k_{\mathrm{F}}\,\Delta} \approx (3\pi^2)^{1/3} \frac{\hbar^2}{2\,m_{\mathrm{n}}^{4/3}} \frac{\rho^{1/3}}{\Delta}. \tag{4}$$

Here E_{F} is the Fermi energy of the neutrons, k_{F} the corresponding Fermi wave number, and ρ the total density of the neutron superfluid region (Ruderman 1976). Outside the core of the vortex, the neutrons are in a superfluid state.

The kinetic properties of superfluid vortexes follow directly from the Feynman circulation theorem, Equation (2). The superfluid neutrons revolve around the vortex line with velocity

$$v_s(r) = \frac{n\,\hbar}{2m_{\mathrm{n}}r}, \tag{5}$$

where r is the distance from the field point to the axis of the vortex filament. The distribution of angular velocity in the neutron superfluid revolving around the vortex filament is

$$\omega_s(r) = \frac{n\hbar}{2m_{\mathrm{n}}r^2}. \tag{6}$$

Therefore, the revolution of the superfluid neutrons around the vortex filament is in a differential state: the closer to the center of the vortex, the faster the superfluid neutrons rotate. In places near $r \approx a_0$, the angular velocity reaches its maximum

$$\omega_c = \frac{n\hbar}{2m_{\mathrm{n}}a_0^2}, \tag{7}$$

and inside the core of the vortex, $r < a_0$, the normal neutron fluid revolves rigidly at angular velocity of ω_c.

According to Feynman (1955), the number of superfluid vortex filament per unit area is $2\Omega/\bar{n}\Gamma_0$, where Ω is the macroscopic angular velocity, $\bar{n}$ is the mean circulation quantum number. Then the average separation b between vortex filaments and the total number of superfluid vortexes are, respectively,

$$b = \left(\frac{\bar{n}\hbar}{2m_{\mathrm{n}}\Omega}\right)^{1/2}, \tag{8}$$

$$N_{\mathrm{vort}} = \left(\frac{R}{b}\right)^2 = \frac{2m_{\mathrm{n}}\Omega}{\bar{n}\hbar} R^2. \tag{9}$$

Here R is the radius of the $^3P_2$ superfluid region.

In this paper, we only consider the thermodynamic condition, $n = \bar{n} = 1$. Here the deviation from equilibrium has been checked by Yakovleve et al. (2001) and Reisenegger et al. (1995, 2006).

For a $^3P_2$ neutron superfluid vortex in the interior of an NS, $a_0 \sim 10^{-10}$ cm and $b \sim 10^{-3}$ cm. Therefore, the vortex core is very tiny, and the distribution of the vortex filament is exceedingly sparse. The separation between vortex filaments reaches a macroscopic scale, and so the superfluidity is a macroscopic quantum phenomenon.

3 INDUCED PARAMAGNETIC MOMENT OF THE $^3P_2$ NEUTRON SUPERFLUID IN THE B-PHASE

3.1 Two Phases of the $^3P_2$ Neutron Superfluid

A $^3P_2$ neutron Cooper pair has spin angular momentum with a spin quantum number, $s = 1$. The magnetic moment of the $^3P_2$ neutron Cooper pair is twice that of a neutron, $2\mu_\mathrm{n}$ in magnitude, where $\mu_\mathrm{n} = -0.966 \times 10^{-23}\,\mathrm{erg\,G^{-1}}$ is the anomalous neutron magnetic moment. Its projection on an external magnetic field (z-direction) is $s_z \times 2\,\mu_\mathrm{n}$, $s_z = 1, 0, -1$. It is interesting to note that the behavior of the $^3P_2$ neutron superfluid is very similar to that of the liquid ^{3}He at very low temperature (Leggett 1975):

1. The projection distribution for the magnetic moment of the $^3P_2$ neutron Cooper pairs in the absence of external magnetic field is stochastic, or "Equal Spin Pair" (ESP) phase. The $^3P_2$ neutron superfluid is basically isotropic and with no significant magnetic moment in the absence of external magnetic field. We call it the A-phase of the $^3P_2$ neutron superfluid, similar to the A-phase of the liquid ^{3}He at very low temperature (Leggett 1975).
2. However, the projection distribution for the magnetic moment of the $^3P_2$ neutron Cooper pairs in the presence of an external magnetic field is not stochastic. There are more $^3P_2$ neutron Cooper pairs with paramagnetic moment than with diamagnetic moment. Therefore, the $^3P_2$ neutron superfluid has a net induced paramagnetic moment and its behavior is anisotropic in the presence of an external magnetic field. We call it the B-phase of the $^3P_2$ neutron superfluid, similar to the B-phase of the liquid ^{3}He at very low temperature (Leggett 1975).

3.2 Induced Paramagnetic Moment of the $^3P_2$ Neutron Superfluid in the B-phase

We now consider the paramagnetism of $^3P_2$ neutron superfluid regions. Following standard treatment of magnetism (Pathria 2003; Feng et al. 2005), the Hamiltonian of the system in the presence of an external field is

$$H = -2\boldsymbol{\mu}_\mathrm{n} \cdot \boldsymbol{B} = -2\mu_{\mathrm{n}z} B\,. \tag{10}$$

Here B is the external field (in the z direction), $2\boldsymbol{\mu}_\mathrm{n}$ is the magnetic moment of the $^3P_2$ neutron Cooper pair, and $2\mu_{\mathrm{n}z}$ is its projection on the z direction.

The ensemble average only gives an additional thermal factor,

$$\langle 2\mu_\mathrm{n} \rangle = 2\mu_\mathrm{n} f(\mu_\mathrm{n} B/k\,T), \tag{11}$$

$$f(\mu_\mathrm{n} B/k\,T) = \frac{2\sinh \beta 2\mu_\mathrm{n} B}{1 + 2\cosh \beta 2\mu_\mathrm{n} B}\,. \tag{12}$$

In the limit of high temperature

$$f(\mu_\mathrm{n} B/kT) = \frac{4}{3}\frac{\mu_\mathrm{n} B}{kT}\,. \tag{13}$$

The $\propto \frac{1}{T}$ behavior is just Curie's law of paramagnetism in terrestrial laboratory. The qualitative behavior is as follows: as the NS cools down, its internal temperature T decreases, while the thermal factor increases, and the $^3P_2$ neutron Cooper tends to align in the same direction. This is the mathematical formalism of the B-phase of the $^3P_2$ neutron superfluid.

As shown by Lifshitz et al. (1999), there is a finite probability for two neutrons to combine into a Cooper pair. Since only the particles in the vicinity of the Fermi surface make a contribution, only a finite fraction q of the Fermi sphere can be in the condensate state,

$$q = \frac{4\pi p_\mathrm{F}^2 \Delta k}{\frac{4\pi}{3} p_\mathrm{F}^3} = 3\sqrt{\frac{\Delta}{E_\mathrm{F}}} \sim 0.087\,. \tag{14}$$

We have used the relation: $\Delta k = \sqrt{2m_\mathrm{n}\Delta}, p_\mathrm{F} = \sqrt{2m_\mathrm{n}E_\mathrm{F}}$. Here $\Delta \sim 0.05\,\mathrm{MeV}$ is the energy gap of the $^3P_2$ superfluid region, $E_\mathrm{F} \sim 60\,\mathrm{MeV}$ is the Fermi energy of the neutron system, p_F the corresponding Fermi momentum and Δk the thickness of the shell which will combine to Cooper pairs.

In conclusion, in any given volume, ΔV, of ${}^3P_2$ neutron superfluid region, only a small fraction of the Fermi sphere can combine into ${}^3P_2$ Cooper pairs. The ensemble average gives another thermal factor. So the net magnetic moment of the specific volume is

$$M_{\Delta V} = \Delta V \rho N_{\rm A} \mu_{\rm n}\, q\, f(\mu_{\rm n} B/kT)\,, \tag{15}$$

where $N_{\rm A}$ is Avogadro's constant. The volume ΔV is specially chosen, so that it is large enough to contain a large number of Cooper pairs, but small enough compared with macroscopic scale: it is a mesoscopic volume. This is required by the coherence calculation of the microscopic magnetic dipole radiation (MMDR) below.

For a rotating NS, the ${}^3P_2$ superfluid region is in vortex state. In the presence of external field, it has a paramagnetic moment on the mesoscopic scale. So the ${}^3P_2$ neutron superfluid vortex is like a system of rotating magnetic dipoles, which will give magnetic dipole radiation. This radiation can not penetrate the NS matter, thus provides a heating mechanism of NS associated with superfluidity.

4 MICROSCOPIC MAGNETIC DIPOLE RADIATION HEATING

4.1 A Working Assumption

In a superfluid vortex, each superfluid neutron revolves round the axes of the vortex with angular velocity $\omega(r)$ (Eq. (6)). It is well known that a rotating magnetic dipole will give out magnetic dipole radiation (MDR). Therefore, the ${}^3P_2$ superfluid neutron will give electromagnetic radiation, and the frequency of the emitted photons is equal to the rotational frequency of the neutron.

We will use a phenomenological method to explore the radiation problem in this paper (see, e.g. Feynman 1955; Androdikashvili et al. 1966). The process is as follows: the rotational velocities of superfluid neutron decrease during the emission of the magnetic dipole radiation due to the dissipation of its energy. The superfluid neutrons, hence, will drift out according to Equation (6), and then the transverse pressure exerting on the normal neutrons in the vortex cores decrease. The normal neutrons move out to $r > a_0$ and become superfluid ones. At the end of the vortexes, other normal neutrons located in the normal neutron layer in the interior of neutron stars will flow into the vortex cores along the axes. At the same time, the density of superfluid neutrons at the boundaries of the vortex lattices increases, driving a flow into the normal neutron layer along the direction perpendicular to the axes. In this way, a "local circulation" is formed in the superfluid vortex region. This process is very similar to the Ekman pumping (Greenspan 1968; Anderson et al. 1978). The normal neutrons in the crust of the neutron star drift inward during the process of these local circulations, and the crust shrinks a little slowly. We may suppose that the energy of the magnetic dipole radiation really comes from the release of gravitational energy of the inward moving crust through the local circulation.

4.2 MMDR Heating

First, we calculate the magnetic dipole radiation by one single vortex. The power radiated by one neutron is (Huang et al. 1982)

$$W(\mathrm{n}) = \frac{2\omega^4}{3c^3}\sum_f |\langle f|\hat{M}_z|i\rangle|^2 \tag{16}$$

$$= \frac{2\omega^4}{3c^3}\sum_f \langle i|\hat{M}_z^\dagger|f\rangle\langle f|\hat{M}_z|i\rangle$$

$$= \frac{2\omega^4}{3c^3}\langle i|\hat{M}_z^\dagger\hat{M}_z|i\rangle, \tag{17}$$

where c is the speed of light, ω the angular velocity of the ${}^3P_2$ superfluid neutron, $|f\rangle$ the final state, $|i\rangle$ the initial state, and $\hat{M}_z$ the operator of magnetic moment. If we consider a coherent small volume ΔV, and take

$$\langle i|\hat{M}_z^\dagger\hat{M}_z|i\rangle = \langle i|\hat{M}_z^2|i\rangle \approx M_{\Delta V}^2\sin^2\theta\,, \tag{18}$$

where $M_{\Delta V}$ is the corresponding paramagnetic moment in a specific volume of ΔV, θ is the angle between the background field $\boldsymbol{B}$ and the rotational axis $\boldsymbol{\Omega}$, thus the required magnetic dipole radiation rate is

$$W_{\Delta V} = \frac{2\omega^4}{3c^3}|M_{\Delta V}|^2 \sin^2\theta \,. \tag{19}$$

The formula is similar to the classical case (Shapiro et al. 1983), because we have made approximations in obtaining Equation (18).

In the calculation of the radiation power, the coherence effect must be taken into consideration (Peng et al. 1982). This is like the coherence effect in electromagnetism. We first calculate the contribution of a specific volume ΔV, then make a summation of all the specific volumes. This is done for one superfluid vortex. The total power of MMDR is simply the sum of all single vortex contributions. See Appendix for details.

The total power of MMDR is

$$W_{\rm tot} = N_{\rm vort}\, \eta\, A \frac{1}{2} \log \frac{b}{a_0} \,. \tag{20}$$

Here $N_{\rm vort}$ is the total number of superfluid vortexes of Equation (9) and η the efficiency of coherence. Radiation of one single vortex is $A\frac{1}{2}\log\frac{b}{a_0}$, where the contribution factor A is

$$A = \frac{8\pi^4}{3c^2}\sin^2\theta |\rho N_{\rm A}\mu_{\rm n} q f|^2 R\omega_c^3 a_0^4 \,. \tag{21}$$

The factor A is proportional to R, and the vortex number $N_{\rm vort}$ is proportional to R^2, as can be seen in Equation (9). So the total power $W_{\rm tot}$ is proportional to R^3, that is proportional to the volume of the ${}^3P_2$ neutron superfluid region. This is what it should be.

5 MMDR vs. OTHER HEATING MECHANISMS

Heating mechanisms in NSs can be classified into different categories according to their energy input. There are several kinds of energy input: magnetic, rotational, chemical and confinement energy, etc. The corresponding heating mechanisms are respectively:

1. Ohm heating (Page et al. 2006 and reference therein);
2. Vortex creep heating (Alpar et al. 1989; Umeda et al. 1994);
3. Rotochemical heating (Reisenegger 1995; Reisenegger et al. 2006);
4. Deconfinement heating (Yuan et al. 1999; Kang et al. 2007).

MMDR heating has two distinguished features compared with the heating mechanisms stated above:

1. It is a heating mechanism associated with superfluidity, thus no superfluid suppression.
2. Its energy input is gravitational energy as stated in our working assumption.

6 CONCLUSIONS

Here we have presented another possible heating mechanism of NSs, associated with superfluidity. It can be compared with other heating mechanisms and cooling agents of NSs (Gusakov et al. 2004; Page et al. 2006; Tong et al. 2007).

As shown in Figure 1, MMDR heating may dominate only in the photon cooling stage. So it will not affect the cooling scenario of young and middle age NSs. For old NSs, e.g. PSR 1055–52, however, it may serve as a moderate heating agent (Tsurata 2006; Page 2006).

Using the toy model given by Yakovlev et al. (2003), we can make an illustrative calculation of NS cooling, including the MMDR heating.

Figure 2 shows that there is a plateau in the late stage of NS cooling curves. This plateau can be compared with that of Kang & Zheng (2007).

The MMDR heating must cease after some time. In our case the cease of MMDR heating has several possibilities.

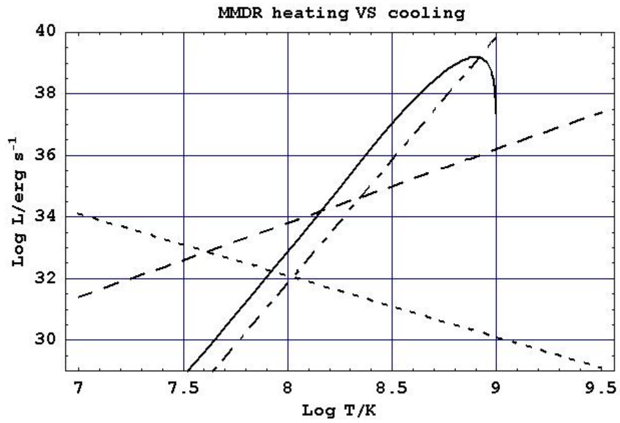

Fig. 1 MMDR Heating vs. Cooling. The dotted line represents MMDR heating. It increase with decreasing temperature, as a result of increasing thermal factor. The dashed, dot-dashed, and solid line corresponds to photon cooling, MUrca (Modified Urca) process, and PBF (Pair Breaking and Formation) process, respectively (Adapted from Gusakov et al. 2004).

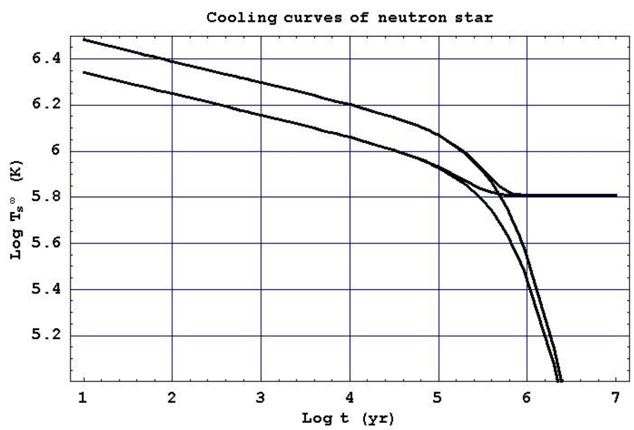

Fig. 2 Cooling curves including MMDR heating. The lower and upper curves correspond to MUrca- and bremsstrahlung- dominated cases, respectively, while the plateau at the late stage is due to MMDR heating. The physical input is the same as Yakovlev et al. (2003) except that we included the MMDR heating. Note, these cooling curves are for illustrative use only.

1. As shown by Huang et al. (1982), the energy input of MMDR heating is through the Ekman pumping. When this agent is inoperative, the MMDR heating has to pause.
2. When there is a phase transition in the core, e.g. deconfinement of hardrons, all the hadron processes disappear including the MMDR heating.
3. Due to complexities of the superfluidity gap (Elgarøy et al. 1996), the $^3P_2$ superfluidity region becomes slimmer when the core becomes more compact. The MMDR heating contribution can be ignored in this case.

We have presented another possible NS heating mechanism associated with superfluidity. The exact effect of MMDR heating needs accurate calculation of the cooling curves. This heating mechanism plus some cooling agent may give a sound explanation of the NS glitches. A detailed investigation can be found in Peng (2007).

Acknowledgements This work is supported by the National Natural Science Foundation of China, under Grants 10573011 and 10273006, and the Doctoral Program Foundation of the Ministry of Education of China.

Appendix A: CALCULATION OF MMDR POWER

We will follow the routine of Peng et al. (1982). At radius r from the center of the $^3P_2$ neutron superfluid vortex, the neutron rotational frequency is

$$\omega_s(r) = \frac{\hbar}{2m_{\rm n}r^2}. \tag{A.1}$$

As stated in the main text, the frequency of radiated photons is equal to the neutron rotational frequency $\omega_s(r)$. The wave length of the radiated photon is

$$\lambda_s(r) = \frac{2\pi c}{\omega_s(r)} = \frac{4\pi c m_{\rm n} r^2}{\hbar} \quad (r > a_0). \tag{A.2}$$

The length of one single superfluid vortex is $\bar{H} = \frac{\pi}{2}R$, where R is the radius of the superfluid region. One superfluid vortex can be separated into several segments, we need only consider the coherence effect inside each segment. If we assume a length scale $\lambda_s(r)$, then a cylindrical shell between the radius r and $r + {\rm d}r$ can be cut into $\bar{H}/\lambda_s(r)$ segments. Here the specific volume ΔV is

$$\Delta V = 2\pi r {\rm d}r \lambda_s(r). \tag{A.3}$$

Paramagnetic moment in the specific volume is

$$M_{\Delta V} = \Delta V \rho N_{\rm A} \mu_{\rm n} q f \left(\frac{\mu_{\rm n} B}{kT}\right). \tag{A.4}$$

Using Equation (19), the differential power of one single superfluid vortex is,

$$\begin{aligned}
{\rm d}W^{(1)} &= \frac{2\omega^4}{3c^3}\frac{\bar{H}}{\lambda_s(r)}|M_{\Delta V}|^2 \sin^2\theta & \text{(A.5)}\\
&= \frac{2\omega^4}{3c^3}\frac{\bar{H}}{\lambda_s(r)} \sin^2\theta |\rho N_A \mu_{\rm n} q f|^2 |2\pi r {\rm d}r \lambda_s(r)|^2 & \text{(A.6)}\\
&= \frac{8\pi^2\omega^4}{3c^3}\bar{H}\lambda_s(r) \sin^2\theta |\rho N_A \mu_{\rm n} q f|^2 r^2 {\rm d}r^2 & \text{(A.7)}\\
&= \frac{8\pi^4}{3c^2} \sin^2\theta |\rho N_A \mu_{\rm n} q f|^2 R\omega^3 r^2 {\rm d}r^2. & \text{(A.8)}
\end{aligned}$$

In obtaining the final expression, we have used the definition of $\lambda_s(r)$ and $\bar{H}$. Using non-dimensional quantities,

$$\begin{aligned}
\omega_s(r) &= \omega_c \omega'(r),\\
r &= a_0 r',\\
\omega' &= \frac{1}{r'^2},
\end{aligned} \tag{A.9}$$

the differential power is

$$\begin{aligned}
{\rm d}W^{(1)} &= \frac{8\pi^4}{3c^2} \sin^2\theta |\rho N_A \mu_{\rm n} q f|^2 R\omega_c^3 a_0^4 \omega'^3 r'^2 {\rm d}r'^2 & \text{(A.10)}\\
&= A\omega'^3 r'^2 {\rm d}r'^2 & \text{(A.11)}\\
&= A{\rm d}I. & \text{(A.12)}
\end{aligned}$$

Integrating $\mathrm{d}I$, a δ-function is included automatically, $\delta(\omega' - 1/r'^2)$. Performing the integration from a_0 to b gives $\frac{1}{2}\log\frac{b}{a_0}$. The contribution factor A is

$$A = \frac{8\pi^4}{3c^2}\sin^2\theta|\rho N_{\mathrm{A}}\mu_{\mathrm{n}} q f|^2 R\omega_c^3 a_0^4. \tag{A.13}$$

The MMDR power of one superfluid vortex is

$$W^{(1)} = A\frac{1}{2}\log\frac{b}{a_0}. \tag{A.14}$$

In the above calculation, we assumed a length scale $\lambda_s(r)$. More generally, if the length scale is taken to be $\eta\lambda_s(r)$, where η is an efficiency factor, the MMDR power of one superfluid vortex is

$$W^{(1)} = \eta A\frac{1}{2}\log\frac{b}{a_0}. \tag{A.15}$$

In Section 2 the total number of superfluid vortexes has been presented,

$$N_{\mathrm{vort}} = \left(\frac{R}{b}\right)^2 = \frac{2m_{\mathrm{n}}\Omega}{\hbar}R^2. \tag{A.16}$$

In conclusion, the total power of MMDR in the $^3P_2$ neutron superfluid region in the interior of NS is

$$W_{\mathrm{tot}} = N_{\mathrm{vort}}\eta A\frac{1}{2}\log\frac{b}{a_0}. \tag{A.17}$$

References

Alpar M. A., Cheng K. S., Pines D., 1989, ApJ, 346, 823
Anderson P. M., Pines D., Ruderman M. et al., 1978, J. Low Temperature Phys., 30, 839
Androdikashvili E. L., Mamaladze Y. G., 1966, Rev. Mod. Phys., 38, 567
Bai H., Peng Q. H., Luo X. L., 2006, Chinese Phys. Lett., 23, 1057
Elgarøy Ø., Engvik L., Hjorth-Jensen M. et al., 1996, Phys. Rev. Lett., 77, 1428
Feng D., Jin G. J., 2005, Introduction to Condensed Matter Physics I, World Scientific, Singapore
Feynman R. P., 1955, Progress in low temperature physics 1, 17
Flowers E., Ruderman M., Sutherland P., 1976, ApJ, 205, 541
Greenspan H. P., 1968, The Theory of Rotating Fluid, ed. G. K. Bachlor and J. W. Miles, Cambridge University Press, Cambridge, 35
Gusakov M. E., Kaminker A. D., Yakovlev D. G. et al., 2004, A&A, 423, 1063
Huang J. H., Lingenfelter R. E., Peng Q. H. et al., 1982, A&A, 113, 9
Kaminker A. D., Gusakov M. E., Yakovlev D. G. et al., 2006, MNRAS, 365, 1300
Kang M., Zheng X. P., 2007, MNRAS, 375, 1503
Leggett A. J., 1975, Rev.Mod.Phys., 47, 331
Lifshitz E. M., Pitaevskii L. P., 1999, Statistical phyiscs (part 2), World Scientific, Beijing
Page D., Lattimer J., Prakash M. et al., 2004, ApJS, 155, 623
Page D., Geppert U., Weber F., 2006, Nucl. Phys. A, 777, 497
Pathria R. K., 2003, Statistical mechanics, 2nd ed., Elsevier, Singapore
Peng Q. H., Huang K. L., Huang J. H., 1982, A&A, 107, 258
Peng Q. H., Luo Z. Q., Chou C. G., 2006, Chin. J. Astron. Astrophys. (ChJAA), 6, 297
Reisenegger A., 1995, ApJ, 442, 749
Reisenegger A., Jofre P., Ferrandez R. et al., 2006, ApJ, 653, 568
Ruderman M., 1976, ApJ, 203, 213
Shapiro S. L., Teukolsky S. A., 1983, Block holes, white dwarfs, and nuetron stars, John Wiley & Sons, New York
Tong H., Peng Q. H., 2007, Chin. J. Astron. Astrophys. (ChJAA), 7, 809
Tsuruta S., 2006, On the Present and Future of Pulsar Astronomy, 26th meeting of the IAU, Joint Discussion 2, 16-17 August, 2006, Prague, Czech Republic, JD02, #25 (c.f. astro-ph/0602138)
Umeda H., Tsuruta S., Nomoto K., 1994, ApJ, 433, 256
Yakovlev D. G., Kaminker A. D., Gnedin O. Y. et al., 2001, Phys. Rept., 354, 1
Yakovlev D. G., Haensel P., 2003, A&A, 407, 259
Yuan Y. F., Zhang J. L., 1999, A&A, 344, 371