Space-Time Geometry of Quark and Strange Quark Matter

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Abstract We study quark and strange quark matter in the context of general relativity. For this purpose, we solve Einstein's field equations for quark and strange quark matter in spherical symmetric space-times. We analyze strange quark matter for the different equations of state (EOS) in the spherical symmetric space-times, thus we are able to obtain the space-time geometries of quark and strange quark matter. Also, we discuss the features of the obtained solutions. The obtained solutions are consistent with the results of Brookhaven Laboratory, i.e. the quark-gluon plasma has a vanishing shear (i.e. quark-gluon plasma is perfect).

Key words: cosmology — theory, early universe

1 INTRODUCTION

It is still a challenging problem to know the exact physical situation at very early stages of the formation of our universe, at which it is generally assumed that during the phase transition (as the universe passes through its critical temperature) the symmetry of the universe is broken spontaneously (Yavuz & Yilmaz 1996).

Spontaneous symmetry breaking is a classical idea, described within the particle physics context in terms of the Higgs field. The symmetry is said to be spontaneously broken if the ground state is not invariant under the full symmetry of the Lagrangian density. Thus, the vacuum expectation value of the Higgs field is nonzero. In quantum field theories, broken symmetries are restored at high enough temperatures (Sakellariadou 2005).

One of the most important phase transitions of the universe is the Quark Gluan Plasma (QGP) \rightarrow hadron gas transition (called quark-hadron phase transition) when the cosmic temperature was $T \sim 200 \text{ MeV}$ (Xu 2005).

Soon after the discovery of Quantum Chromodynamics (QCD), it was conjectured that at high temperature T the color charge is screened (Shuryak 1978) and the corresponding phase of matter was named Quark-Gluon Plasma (QGP). It is a special kind of plasma in which the electric charges are replaced by the color charges of quarks and gluons, mediating the strong interaction among them. Such a state of matter is expected to exist at extreme temperatures, above 150 MeV, or densities, above about 10 times normal nuclear matter density (Mustafa 2006). Also, it is well known that the QGP is actually a strongly coupled plasma (Shuryak 2006).

The aim of the ongoing relativistic heavy-ion collision experiments is to explore the possible QGP phase of QCD (Mustafa 2006).

Robust results from Au+Au at the BNL RHIC experiments have shown collective effects known as radial (Adler et al. 2004) and elliptic (Adcox et al. 2002; Adler et al. 2003) flows, and a suppression of

high- p_T hadron spectra (Adcox et al. 2002; Adler et al. 2003), which could possibly indicate the quenching of light quark and gluon jets (Gyulassy & Plümer 1990).

The hydrodynamical description of the observed collective flow indicates that the matter produced at RHIC *behaves like a perfect fluid* (Teaney et al. 2001; Mustafa 2006).

Let us also consider the definitions of perfect fluid. A perfect fluid is defined also in two different ways. First, a fluid which looks isotropic in its rest frame is called a perfect fluid. Secondly, a fluid which has no shear stresses, viscosity or heat conduction is a perfect fluid. It turns out that these two different definitions of the perfect fluids are equivalent. Perfect fluids are frequently used in general relativity to construct the model as idealized matter. This terminology has also been adopted by the Particle Data Groups (Csanad et al. 2006).

The possibility of the existence of quark matter dates back to early seventies (Gondek et al. 2006). Itoh (1970), Bodmer (1971) and Witten (1984) proposed two ways of formation of quark matter: the quark-hadron phase transition in the early universe and the conversion of neutron stars into strange ones at ultra-high densities (Mak & Harko 2004).

In the theories of strong interaction, quark bag models suppose that breaking of physical vacuum takes place inside hadrons. As a result, vacuum energy densities inside and outside a hadron become essentially different and the vacuum pressure on the bag wall equilibrates the pressure of quarks, thus stabilizing the system. If the hypothesis of the quark matter is true, then some neutron stars could actually be strange stars, build entirely of strange matter (Alcock et al. 1986; Haensel et al. 1986). Yilmaz & Baysal (2005) have studied the rigidly strange quark star. For a review of strange quark matter and star properties see (Weber 2005; Cheng et al. 1998; Harko & Cheng 2002).

Typically, quark matter is modeled with an equation of state (EOS) based on the phenomenological bag model of quark matter, in which quark confinement is described by an energy term proportional to the volume. In the framework of this model the quark matter is composed of massless u, d quarks, massive s quarks and electrons (Gondek et al. 2006). In the simplified version of the bag model, assuming the quarks are massless and non interacting, we have the quark pressure $p_q = \rho_q/3$ (ρ_q is the quark energy density); the total energy density is

$$\rho = \rho_{\rm q} + B \,, \tag{1}$$

while the total pressure is

$$p = p_{\rm q} - B \,. \tag{2}$$

One then obtains the EOS for strange quark matter (Kapusta 1994; Sontani et al. 2004; Xu 2003),

$$p = \frac{1}{3}(\rho - 4B).$$
(3)

where B is the difference between the energy density of the perturbative and non-perturbative QCD vacuum (the bag constant). Equation (3) is essentially the EOS of a gas of massless particles with corrections due to the QCD trace anomaly and perturbative interactions. These corrections are always negative, reducing the energy density at given temperature by about a factor of two (Farhi & Jaffe 1984). For quark stars obeying the bag model EOS (3) the Chandrasekhar limit has been evaluated from simple energy balance relations in Bannerjee et al. (2000). In addition to the fundamental constants, the maximum mass also depends on the bag constant (Harko & Cheng 2002).

Recently, Dey et al. (1998) have obtained new sets of EOSs for strange matter based on a model of interquark potential which has the following features: (a) asymptotic freedom, (b) confinement at zero baryon density and deconfinement at high baryon density, (c) chiral symmetry restoration and (d) gives stable uncharged β -stable strange matter. These EOSs have later been approximated to the following linear form by Gondek et al. (2000),

$$p = \epsilon(\rho - \rho_0), \qquad (4)$$

where ρ_0 denotes the energy density at zero pressure and ϵ is a constant (Sharma et al. 2006).

Also, recent investigations show that a number of pulsars like Her X–1 (Li et al. 1995), 4U 1820–30 (Bombaci 1997; Dey et al. 1998), SAX J 1808.4–3658 (Li et al. 1999), 4U 1728–34 (Li et al. 1999), PSR 0943+10 (Xu et al. 1999; Xu et al. 2001) and RX J185635–3754 (Xu 2002; Pons et al. 2002), earlier thought to be neutron stars, are actually good strange star candidates (Sharma 2006).

So, it will be interesting to study quark matter and strange quark matter in the spherically symmetric space - time. In this study, we examine the quark and strange quark matter in the general spherically symmetric space-time.

This paper is outlined as follows. In Section 2, the Einstein field equations are solved for quark matter in spherical symmetric space-times by using the feature of perfect fluid of quark matter. In Section 3, the Einstein field equations are obtained in the spherical symmetric space-times by using different equations of state for strange quark matter. Section 4 gives some concluding remarks.

2 SPACE-TIME GEOMETRY OF QUARK-GLUON MATTER

We choose general spherically symmetric metric for two reasons. One is that the geometry of stars is generally assumed to be spherically symmetric in the ideal case, and the other is that the FRW universe which describes our universe is spherically symmetric.

A symmetric metric can generally be written in the form of

$$ds^{2} = -e^{2\nu}dt^{2} + e^{2\psi}dr^{2} + R^{2}d\Omega^{2}, \qquad (5)$$

where ν, ψ and R are functions of t and r, $d\Omega^2 = d\theta^2 + \sin^2 \theta d\phi^2$ is the usual line element on a unit 2-sphere (Joshi 2000).

Due to experimental results of Brookhaven Laboratory (*http://www.bnl.gov/bnlweb/pubaf/pr*), i.e., the perfect fluid of quark-matter, the form of the energy momentum tensor of quark matter is taken as (Yilmaz 2006)

$$T_{ik} = (\rho + p)u_i u_k + pg_{ik} , \qquad (6)$$

where $\rho = \rho_q + B$ and $p = p_q - B$. Now the explicit form of the Einstein's field equations for the metric of Equation (5) with matter field given by Equation (6) are (choosing $8\pi G = c = 1$)

$$\frac{1}{R^2} - e^{-2\psi} \left(\frac{R'^2}{R^2} + 2\nu' \frac{R'}{R} \right) + e^{-2\nu} \left(2\frac{\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} - 2\dot{\nu}\frac{\dot{R}}{R} \right) = -p, \tag{7}$$

$$e^{-2\psi} \left(-\nu'' - \nu'^{2} + \psi'\nu' - \frac{R''}{R} - \nu'\frac{R'}{R} + \psi'\frac{R'}{R} \right) + e^{-2\nu} \left(\ddot{\psi} + \dot{\psi}^{2} - \dot{\psi}\dot{\nu} + \frac{\ddot{R}}{R} + \dot{\psi}\frac{\dot{R}}{R} - \dot{\nu}\frac{\dot{R}}{R} \right) = -p,$$
(8)

$$\frac{1}{R^2} + e^{-2\psi} \left(2\psi' \frac{R'}{R} - 2\frac{R''}{R} - \frac{R'^2}{R^2} \right) + e^{-2\nu} \left(\frac{\dot{R}^2}{R^2} + 2\dot{\psi}\frac{\dot{R}}{R} \right) = \rho, \tag{9}$$

and

$$\dot{R}' - \dot{\psi}R' - \nu'\dot{R} = 0, \qquad (10)$$

where over dot(.) and prime(l) stand for partial derivatives with respect to t and r, respectively.

The physical variables, namely the expansion and shear which is responsible for dissipation of the matter, have the following expressions for the metric of Equation (5):

$$\Theta(t,r) = \frac{1}{e^{\nu}} \left[\dot{\psi} + 2\frac{\dot{R}}{R} \right] , \qquad (11)$$

and

$$\sigma_{\phi}^{\phi} = \sigma_{\theta}^{\theta} = -\frac{1}{2}\sigma_r^r = \frac{1}{3}e^{-\nu} \left(\frac{\dot{R}}{R} - \dot{\psi}\right). \tag{12}$$

According to the experimental results of Brookhaven Lab (http://www.bnl.gov/bnlweb/pubaf/pr) the quark-gluon plasma has a very small viscosity, i.e., a nearly zero shear viscosity, and so it behaves like a

perfect fluid (Back et al. 2005; Adams et al. 2005; Adcox et al. 2005). We consider the situation when the matter shear vanishes identically. This means

$$\frac{R}{R} = \dot{\psi} \,, \tag{13}$$

or

$$R = q(r)e^{\psi} \,. \tag{14}$$

We can use available freedom of scaling to rescale the radial coordinate r so that we always have q(r) = r. We then have

$$R = r e^{\psi} \,. \tag{15}$$

It is now possible to integrate Equation (10) using Equation (15), to obtain

$$e^{\nu} = a(t)\dot{\psi}\,,\tag{16}$$

where a(t) is an arbitrary function of integration.

If we use Equation (16), then the Einstein Field Equations (EFEs) read

$$\frac{3}{a^2} - 2\frac{\dot{a}}{a^3\dot{\psi}} - \left(2\frac{\dot{\psi}'}{r\dot{\psi}} + 2\frac{\psi'}{r} + 2\frac{\dot{\psi}'\psi'}{\dot{\psi}} + \psi'^2\right)e^{-2\psi} = -p\,,\tag{17}$$

$$\frac{3}{a^2} - 2\frac{\dot{a}}{a^3\dot{\psi}} - \left(\frac{\dot{\psi}'}{r\dot{\psi}} + \frac{\psi'}{r} + \frac{\dot{\psi}''}{\dot{\psi}} + \psi''\right)e^{-2\psi} = -p\,,\tag{18}$$

$$\frac{3}{a^2} - \left(4\frac{\psi'}{r} + 2\psi'' + \psi'^2\right)e^{-2\psi} = \rho.$$
(19)

From Equations (17) and (18), we obtain

$$-\frac{\psi'}{r} - \psi'^2 - \frac{\dot{\psi}'}{r\dot{\psi}} - 2\frac{\dot{\psi}'\psi'}{\dot{\psi}} + \psi'' + \frac{\dot{\psi}''}{\dot{\psi}} = 0.$$
 (20)

To solve this equation, we choose $\psi(r, t)$ to be separable:

$$e^{\psi(r,t)} = f(t)g(r),$$
 (21)

with $f(t) = c_1 t^n$, and $g(r) = \frac{1}{c_2} r^{-m}$, with m and n constants. Substituting Equation (21) into Equation (20), we obtain

$$\psi(r,t) = \ln(c_1 t^n) - \ln(c_2 r^m).$$
(22)

Since Equation (22) has to satisfy Equation (20), m in Equation (22) should take 0 and 2. However, there is no any restriction on constant n.

So, we have found the metric potentials ($\nu(r,t)$, $\psi(r,t)$ and R(r,t)) exactly. This means that we can determine the space-time geometry of the matter according to Einstein's General Relativity Theory.

In this case, the space-time geometry of quark-gluon matter is given by

$$ds^{2} = -\frac{n^{2}a^{2}}{t^{2}}dt^{2} + \frac{c_{1}^{2}t^{2n}}{c_{2}^{2}r^{2m}}\left(dr^{2} + \frac{d\Omega^{2}}{r^{2}}\right),$$
(23)

where m = 0 and 2. From Equations (17), (18), (19), (22) and (11), we obtain the following solutions

$$p = -\frac{3}{a^2} + \frac{2t\dot{a}}{na^3},$$
(24)

$$\rho = \frac{3}{a^2} \,, \tag{25}$$

and the expansion is

$$\Theta = \frac{3}{a(t)}.$$
(26)

3 SPACE-TIME GEOMETRY OF STRANGE QUARK MATTER

Since strange quark matter, i.e. strange quark stars, are governed by EOSs (3) and (4), we will particularize these equations for the strange quark matter. So, we obtain the following strange quark model depending on EOSs.

Case (i) Strange Quark Matter in the BAG Model:

From Equations (3), (24) and (25) we obtain

$$a(t) = \pm \frac{3}{\sqrt{3B + 9c_3 t^{-4n}}}.$$
(27)

Using Equations (24), (25) and (27), we obtain

$$p = -B + \frac{c_3}{t^{4n}},$$
 (28)

and

$$\rho = B + \frac{3c_3}{t^{4n}}.$$
(29)

In this case, the expansion and the space-time geometry of the strange quark matter are respectively given,

$$\Theta = \pm \sqrt{3B + 9c_3 t^{-4n}},\tag{30}$$

and

$$ds^{2} = -\frac{3n^{2}}{\left(B + 3c_{3}t^{-4n}\right)t^{2}}dt^{2} + \frac{c_{1}^{2}t^{2n}}{c_{2}^{2}r^{2m}}\left(dr^{2} + \frac{d\Omega^{2}}{r^{2}}\right),$$
(31)

where m = 0 and 2.

Case (ii) Strange Quark Matter in the case of linear equation of state:

From Equations (4), (24) and (25), we obtain

$$a(t) = \pm \frac{\sqrt{3\sqrt{\epsilon+1}}}{\sqrt{3(\epsilon+1)c_3t^{-3n-3n\epsilon} + \epsilon\rho_0}}.$$
(32)

Substituting Equation (32) into Equations (24) and (25) we obtain

$$p = \epsilon \frac{3(\epsilon+1)c_3 t^{-3n-3n\epsilon} - \rho_0}{\epsilon+1}, \qquad (33)$$

and

$$\rho = \frac{3(\epsilon+1)c_3t^{-3n-3n\epsilon} + \epsilon\rho_0}{\epsilon+1}.$$
(34)

In this case, from Equations (23) and (26) we obtain the following expressions for the expansion and the space-time geometry of the strange quark matter,

$$\Theta = \pm \frac{\sqrt{3}\sqrt{3(\epsilon+1)c_3t^{-3n-3n\epsilon} + \epsilon\rho_0}}{\sqrt{\epsilon+1}}, \qquad (35)$$

$$ds^{2} = -\frac{3(\epsilon+1)n^{2}}{[3(\epsilon+1)c_{3}t^{-3n-3n\epsilon} + \epsilon\rho_{0}]t^{2}}dt^{2} + \frac{c_{1}^{2}t^{2n}}{c_{2}^{2}r^{2m}}\left(dr^{2} + \frac{d\Omega^{2}}{r^{2}}\right),$$
(36)

where m = 0 and 2.

4 CONCLUSIONS

General relativity provides a rich arena to understand the natural relation between geometry and matter furnished by the Einstein equations. Field equations mean that any two field configurations connected by a diffeomorphism are empirically indistinguishable and thus physically identical.

Our solutions are consistent with the results of Brookhaven Laboratory, i.e. vanishing shear of quarkgluon plasma (i.e. quark-gluon plasma is perfect). Also, our solutions do not contain the r coordinate. Because when quark-gluon plasma exists the radius of the universe is very small. Also, since the radius of quark stars is very small, the density and pressure of quark stars are dependent on t but independent of r. So, our solutions are physically meaningful.

In the cases of quark and strange quark matter, the geometries of these matter have no shear that is responsible for the dissipation of the matter. These results are consistent with the result of Brookhaven Laboratory (*http://www.bnl.gov/bnlweb/pubaf/pr*).

In the three cases, the difference in the space-time geometries comes only in the metric potential in front of dt^2 (see Equations (23), (31) and (36)).

Let us check the mass in the case of strange quark matter. The total mass M of the star is defined as $M = 4\pi \int \rho r^2 dr = \frac{4\pi}{3} \rho r^3$. So, the mass function M(t, r) for the metric of Equation (5) is given by

$$M(t,r) = R(1 + e^{-2\nu}\dot{R}^2 - e^{-2\psi}{R'}^2), \qquad (37)$$

which can be interpreted as the total mass inside the comoving radius r at the time t.

(i) In the case of strange quark matter in the BAG Model, the mass is given as follows:

For m = 0 and m = 2, from Equations (15), (16), (22) and (29) we obtain respectively,

$$M(r,t) = \frac{c_1^3 r^3 t^{3n}}{3c_2^3} (B + 3c_3 t^{-4n}) = \frac{c_1^3 r^3 t^{3n}}{3c_2^3} \rho,$$
(38)

and

$$M(r,t) = \frac{c_1^3 t^{3n}}{3c_2^3 r^3} (B + 3c_3 t^{-4n}) = \frac{c_1^3 t^{3n}}{3c_2^3 r^3} \rho.$$
(39)

From Equations (38) and (39), it is easily seen that in addition to the fundamental constants, mass also depends on the bag constant. This result is consistent with the results of Bannerjee et al. (2000).

(ii) In the case of strange quark matter in the linear equation of state, the mass is given as follows:

For m = 0 and m = 2, from Equations (15), (16), (22) and (34) we obtain respectively,

$$M(r,t) = \frac{c_1^3 r^3 t^{3n}}{3c_2^3} \left(\frac{3(\epsilon+1)c_3 t^{-3n\epsilon-3n} + \epsilon\rho_0}{\epsilon+1}\right) = \frac{c_1^3 r^3 t^{3n}}{3c_2^3}\rho,$$
(40)

and

$$M(r,t) = \frac{c_1^3 t^{3n}}{3c_2^3 r^3} \left(\frac{3(\epsilon+1)c_3 t^{-3n\epsilon-3n} + \epsilon\rho_0}{\epsilon+1} \right) = \frac{c_1^3 t^{3n}}{3c_2^3 r^3} \rho.$$
(41)

In this case, one obtains the values $\epsilon = 0.463$, $\rho_0 = 1.15 \times 10^{15} \text{ g cm}^{-3}$ and $\epsilon = 0.455$, $\rho_0 = 1.33 \times 10^{15} \text{ g cm}^{-3}$, respectively. Also, the standard bag model (case (i)), corresponds to $\epsilon = 0.333$ and $\rho_0 = 4 \times 10^{14} \text{ g cm}^{-3}$ (Cheng et al. 1998).

From Equations (38) and (40), we have concluded that the mass for strange stars, m, is proportional to r^3 . In contrast, neutron stars have radii that decrease with increasing mass. This is a consequence of the underlying interaction between the stellar constituents which makes "low" mass strange stars self-bound objects, contrary to the case of neutron stars which are bound by gravity. These results are also consistent with the results of Bannerjee et al. (2000) and Cheng et al. (2000).

Also, One may obtain many numerical results for strange quark star by using our exact solutions.

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