Are Type Ia Supernovae Reliable Distance Indicators? *

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Abstract Recent applications of type Ia supernovae (SNe Ia) in cosmology have successfully revealed the accelerating expansion of the universe. However, as distance indicators used in measuring the expansion history of the universe and probing the nature of dark energy, these objects must pass more strict tests. We propose a K-S test to investigate if there exists any systematic bias when deriving the luminosity distances under the standard candle assumption. Two samples, one comprising 71 high-redshift SNe Ia and the other, 44 nearby ones, are used in our investigation. We find that it is likely there exists a bias in the adopted samples, which is probably caused by a systematic error, e.g. in the color parameter used in the luminosity calibration and a bias may be caused by the SN evolution or by varying properties of the dust surrounding the SNe Ia.

Key words: cosmology: observations — distance scale — galaxies: distances and redshifts — methods: statistical — supernovae: general

1 INTRODUCTION

Type Ia supernovae (SNe Ia), as standard candles on cosmic scales up to $z > 1$, revealed the accelerating expansion of the universe from several high redshift SN survey programs (Riess et al. 1998; Perlmutter et al. 1999; Knop et al. 2003; Tonry et al. 2003; Barris et al. 2004; Riess et al. 2004; Astier et al. 2006). With more distant SNe Ia up to $z \sim 1.7$ from HST observations, the universe is found to have experienced a stage from deceleration to acceleration around $z \sim 0.7$ (Riess et al. 2004). The cosmic acceleration was confirmed independently by the observations of the cosmic microwave background anisotropy (WMAP, Bennett et al. 2003) and the large scale structure in the distribution of galaxies (SDSS, Tegmark et al. 2004a,b).

To account for the observed acceleration of the universe, a dark energy component with negative pressure was generally suggested to be some exotic energy that derives the accelerating expansion of the universe. Many theoretical models describing the nature of the dark energy or non-Friedmann cosmological models have been proposed in recent years. They include a cosmological constant Λ (Carroll et al. 1992), an evolving scalar field (referring to quintessence, Ratra & Peebles 1988; Caldwell et al. 1998), phantom energy, in which the sum of the pressure and energy density is negative (Caldwell 2002), the so-called "X-matter" (Turner & White 1997; Zhu 1998; Zhu, Fujimoto & Tatsumi 2001; Zhu, Fujimoto & He 2004b), the Chaplygin gas (Kamenshchik et al. 2001; Bento et al. 2002; Zhu 2004), the Cardassion model (Freese & Lewis 2002; Zhu & Fujimoto 2002, 2003, 2004; Cao 2003; Zhu, Fujimoto & He 2004a), and the brane world model (Randall & Sundrum 1999a,b; Deffayet, Dvali & Gabadadze 2002).

Recently, the SN Ia data were combined with gamma-ray burst (GRB) data to constrain the cosmological model. The latter could extend the redshift range up to $z > 2$ (Dai et al. 2004). Based on the $E_p - E_\gamma$ relation found recently by Ghirlanda et al. (2004b), GRB sources obeying this relation may be hopefully used for measuring the universe (Dai et al. 2004). This issue has been currently investigated by many groups (see Ghirlanda et al. 2004a; Firmani et al. 2005; Friedman & Bloom 2005; Liang & Zhang 2005; Qin et al. 2006; Xu 2005; Xu et al. 2005). Current GRB data could be used to marginalize some parameters in their reasonable ranges (Xu et al. 2005), or they could be employed to constrain the cosmological model with a new Bayesian method (Firmani et al. 2005). It was found that, when combining the SN Ia sample and the GRB sample to constrain the cosmological model, the SN Ia data play a very important role. Without the SN Ia sample, the GRB sample alone could only give a poor constraint, while including the SN Ia sample the constraint improves significantly (compare figs. 2 and 4 in Qin et al. 2006).

When employing distance indicators such as SNe Ia or GRBs to measure the universe, the Chi-square minimization method was generally applied. The best fit of the theoretical curve to the luminosity distance data obtains when the statistic χ^2 reaches its minimum. In addition, based on this χ^2 minimization method, the confidence level as well as the confidence contour would be determined. However, according to its definition, the statistic χ^2 could be influenced by a biased data set which could lead to a smaller value of the statistic in some particular cases (for example, when parts of the data are altered so that they probably happened to be closer to the expected curve). Obviously, the statistic itself could not guarantee the reliability of the data themselves. Thus, while we need statistics such as χ^2 to describe how close the data approach the expected curve, we need other statistics as well to test if the data are biased. This motivates our analysis below.

2 METHODS FOR CHECKING STANDARD CANDLES AND THEIR APPLICATION TO SNE IA

Objects serving as standard cosmological candles are assumed to have the same luminosity. Under this assumption, the scatter of the observational values of these objects must be due to their measurement uncertainties. This gives rise to methods for checking if the objects concerned could fit the condition that standard candles would satisfy. The condition is: the observational values of the standard candles should belong to a parent population that is created from the real values of the standard candles plus random chance deviations, with the chance probability determined by the measurement uncertainty.

The SN sample adopted in Astier et al. (2006) contains 71 Supernova Legacy Survey (SNLS) high-z SNe Ia and 44 nearby ones from other experiments. The parameter used to assess the likelihood of the cosmological parameters is the statistic χ^2 defined as (Astier et al. 2006)

$$\chi^2 = \sum_{\text{objects}} \frac{(\mu_B - 5\log_{10}(d_L(\theta, z)/10\,\text{pc}))^2}{\sigma^2(\mu_B) + \sigma_{\text{int}}^2}\,, \tag{1}$$

where θ represents the cosmological parameters that define the fitting model (with the exception of H_0), $\sigma(\mu_B)$ is the measurement variance, and σ_{int} is the intrinsic dispersion of SN absolute magnitudes. The measurement uncertainty is determined by $\sqrt{\sigma^2(\mu_B) + \sigma_{\text{int}}^2}$. The statistic χ^2 defined by Equation (1) is essential for yielding the likelihood function which would not only give the best estimated cosmological parameters but also the confidence intervals for these parameters.

Statistically, Equation (1) implies that the following statements for the SN Ia sample: a) the objects concerned are standard cosmological candles; b) the scatter of the observational values of these objects, which follows a Gaussian distribution, is produced by chance; c) the width of the Gaussian distribution is determined by the corresponding measurement uncertainty. Obviously, if one of these statements does not hold, the defining Equation (1) as well as its application to SNe Ia data will not be appropriate. The above statements a, b and c form the basic assumption. If this assumption is true then the cosmological model deduced from a fit to the objects should be reliable.

Under this assumption, one can create a set of observation data (μ'_B) by simulation of a very large size serving as the parent population. Comparing these simulation data with the observed sample, one will be able to tell if the assumption is acceptable. This is done below.

Note that the redshifts of the sources of the sample are not the same. We thus consider the relative value of the luminosity distance $\mu_B/\mu_{\rm th}$, where μ_B is the deduced luminosity distance moduli from observation and $\mu_{\rm th} = 5\log_{10}(d_L(\theta, z)/10\,{\rm pc})$ is the theoretical luminosity distance moduli derived from the adopted cosmological model. Obviously, the distribution of $\mu_B/\mu_{\rm th}$ should peak at unity under the given assumption. We label the set of $\mu_B/\mu_{\rm th}$ data of the 71 SNe Ia as sample 1a. According to the null hypothesis, the observed value of $\mu_{\rm th}$ for each source is obtained by chance from a parent population of μ'_B whose distribution obeys a Gaussian distribution with standard deviation equal to the measurement uncertainty. For each source one can create a μ'_B via simulation as long as the expected value $\mu_{\rm th}$ and the measurement uncertainty are known. In this way, from the 71 $\mu_{\rm th}$ and the corresponding measurement uncertainties, one can create a set of 71 μ'_B data by a Monte-Carlo simulation and then obtain a set of 71 $\mu'_B/\mu_{\rm th}$ data. We performed the simulation 100 times and acquired 100 sets of 71 $\mu'_B/\mu_{\rm th}$ data. Combining these simulated data sets we obtain a large sample of size 7100, which we label as sample 1b.

The cosmological parameters $\Omega_{\rm m}$ (mass density) and Ω_Λ (vacuum energy density) adopted for calculating $\mu_{\rm th}$ are taken from Astier et al. (2006). According to Astier et al. (2006), the cosmological parameters $\Omega_{\rm m}$ and Ω_Λ obtained from sample 1a are 0.263 and 0.737 respectively for the flat universe. In this paper, our discussion will focus on the most popular flat universe model. Panel (a) of Figure 1 shows the distributions of $\mu_B/\mu_{\rm th}$ and $\mu'_B/\mu_{\rm th}$ of samples 1a and 1b for the flat universe model.

Performing a K-S test to the pairs of distributions of the two samples we obtain the probability value $P_{\rm KS}$ for the flat universe presented in Table 1. We find that under the basic assumption the probability is only 0.037. The probability for sample 1a being drawn from sample 1b is too small to be acceptable.

Table 1 Probabilities of the K-S test for the adopted cosmological models between three pairs of SNe Ia samples

Samples	$\Omega_{\rm m}$	$P_{\rm KS}$
1a, 1b	0.263	0.037
2a, 2b	0.287	0.020
3a, 3b	0.260	0.0018

If we adopt the generally used standard model $(\Omega_{\rm m}, \Omega_\Lambda) = (0.3, 0.7)$ (e.g., Friedman & Bloom 2005), then we obtain a worse result: $P_{\rm KS} = 6.38 \times 10^{-4}$. We therefore tend to believe that the basic assumption does not hold for the flat universe.

To examine whether the discrepancy also exists for the nearby SNe Ia, we performed a similar simulation analysis to the 44 nearby SNe Ia sample used in Astier et al. (2006). For this sample, We label the set of $\mu_B/\mu_{\rm th}$ data of the 44 SNe Ia as sample 2a and the corresponding simulation data set as sample 2b. The adopted cosmological parameters $\Omega_{\rm m}$ and Ω_Λ for calculating $\mu_{\rm th}$ are obtained by applying the conventional χ^2 minimization method as was previously done. According to Astier et al. (2006), the intrinsic dispersion is taken as $\sigma_{\rm int} = 0.15$, and $H_0 = 70\,{\rm km\ s^{-1}\ Mpc^{-1}}$ is adopted. From the 44 nearby SNe Ia sample of Astier et al. (2006) we obtained the following cosmological parameters, $(\Omega_{\rm m}, \Omega_\Lambda) = (0.287, 0.713)$ $(\chi^2/{\rm dof} = 1.00)$ for the flat universe. Panel (b) of Figure 1 shows distributions of $\mu_B/\mu_{\rm th}$ and $\mu'_B/\mu_{\rm th}$ for samples 2a and 2b for the flat universe model. The result of the K-S test between the two samples is again listed in Table 1.

We also combined the high-redshift SNe Ia with nearby ones to check the basic assumption. In a similar way, we label the set of $\mu_B/\mu_{\rm th}$ data of the 115 SNe Ia as sample 3a (containing samples 1a and 2a) and the corresponding simulation data set as sample 3b. In this case, the cosmological parameters adopted in the calculations are taken from Astrier et al. (2006), with $\Omega_{\rm m} = 0.26$ and $\Omega_\Lambda = 0.74$. Our analysis shows that the probability of the K-S test is only 0.0018 (see Table 1). The basic assumption does not hold for the flat universe with this assembled sample (see also panel (c) of Fig. 1). The Hubble diagram of the 71 high-z and 44 nearby SNe Ia is shown in Figure 2, where the average distances from 100 sets of simulation data (μ'_B) are also plotted. As revealed by the figure, the observational data are more widely scattered than the simulation data, although they share the same measurement uncertainties with respect to the same theoretical values, $\mu_{\rm th}$.
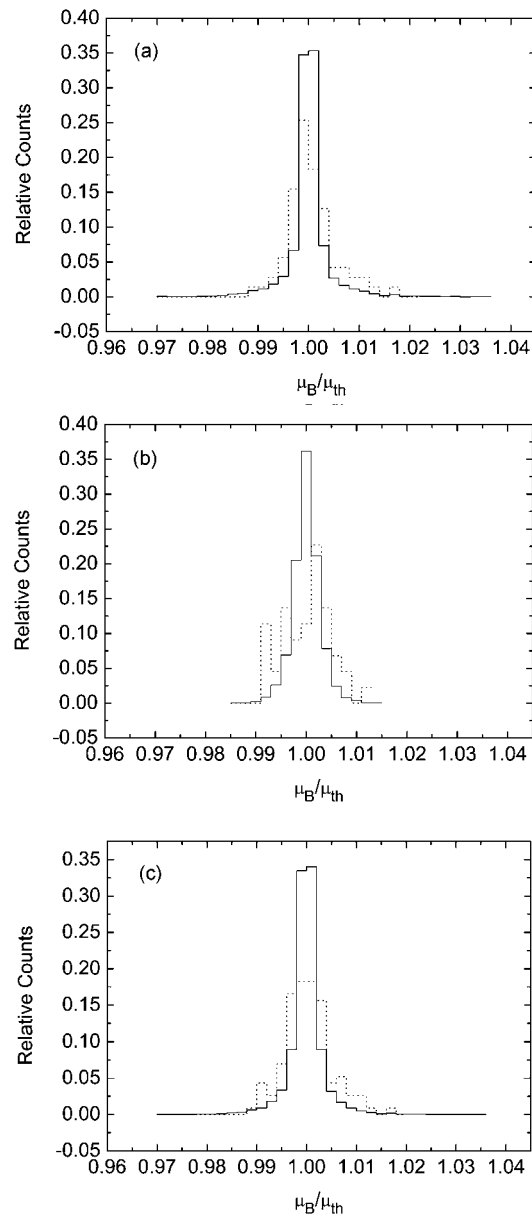
Fig. 1 Distributions of $\mu_B/\mu_{\rm th}$ (samples 1a, 2a and 3a, dotted lines) and $\mu'_B/\mu_{\rm th}$ (samples 1b, 2b and 3b, solid lines) for the 71 high-z SNe Ia sample (panel a), 44 nearby SNe Ia sample (panel b) and 115 assembled SNe Ia sample (panel c). Here the count in each bin is normalized to the total count of the corresponding sample.

To provide an intuitive view of the discrepancy between the distributions of $\mu_B/\mu_{\rm th}$ and $\mu'_B/\mu_{\rm th}$ of samples 1a and 1b, 2a and 2b, and 3a and 3b, we present their cumulative distributions in Figure 3. The cumulative distribution function of $\mu_B/\mu_{\rm th}$ is defined as the count of sources with their $\mu_B/\mu_{\rm th}$ not larger than a given value (say, $\mu_B/\mu_{\rm th} \leq a$, with a taking any value between the minimum and maximum values of $\mu_B/\mu_{\rm th}$), divided by the total count of the sample. The difference between the observational data and the
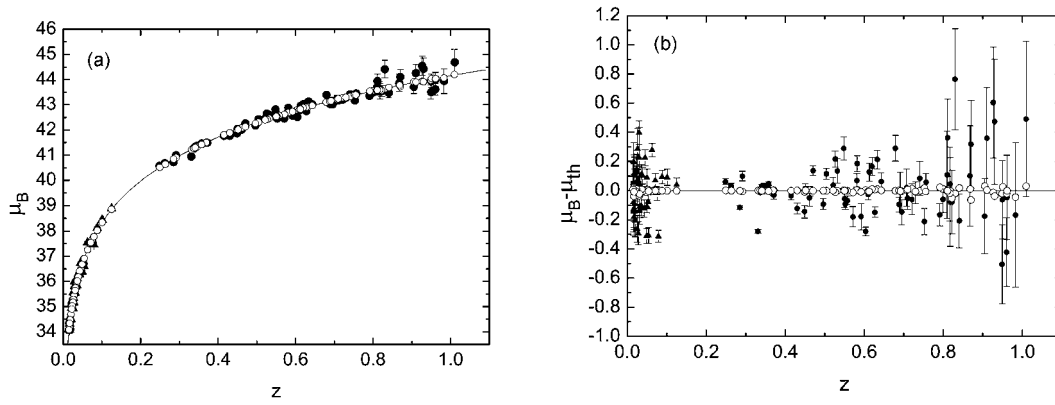
Fig. 2 Panel (a) shows the Hubble diagram of 71 high-z (solid circles) and 44 nearby (solid triangles) SNe Ia. The open circles are the corresponding simulation data (μ'_B), averaged over 100 individual values. The solid line is the best-fit to the observational data. Panel (b) shows the residuals for the best fit to a flat universe.

simulated data is clearly seen in the figure. The scatter of the observational values of SNe Ia does not follow a Gaussian distribution which is presented by the solid line in the figure. Note that the simulated data are created by assuming that the expected observed value of μ_{th}, μ'_B, follows a Gaussian distribution around μ_{th} with its σ being the provided measurement uncertainty of the source (see the null hypothesis presented above). Following this assumption, we obtain the solid lines in Figure 3. However, what the real distribution of the observational data, μ_B, follows is unknown. As shown by the dotted lines in Figure 3, at least the distribution does not agree with a Gaussian distribution. Otherwise it will be closed to the solid lines (a detailed analysis of how the distribution deviates from a Gaussian distribution is performed above by a K-S test). The deviation seen in Figure 3 is in agreement with the results of the K-S test presented in Table 1. In terms of statistics, distributions of μ_B/μ_{th} and μ'_B/μ_{th} for the adopted samples are very different. Its interpretation is that the basic assumption does not hold for the flat universe.

3 POSSIBLE EFFECTS

According to the above analysis, we suspect that there might be a bias in the SN Ia sample. In this section, we discuss some of the effects that are potentially related to the bias inferred from current SN Ia data.

3.1 Effect of Measurement Errors

One might notice that, as part of the basic assumption, the measurement uncertainty is assumed to stand for the probability distribution of the observational value of a quantity following a Gaussian distribution with width equal to the measurement uncertainty. However, some of the errors adopted might not be so perfect to serve as a measurement of a Gaussian probability distribution. This would happen, for instance, when a measurement uncertainty is determined via the fit of a curve to a few data points. Thus, it could be that some of the measurement errors do not really reflect the probability arising from a Gaussian distribution. Astier et al. (2006) discussed the systematic uncertainty in detail. They found that the dominant systematic uncertainty arises from the photometric calibration. Since the distribution of μ_B/μ_{th} does not follow a Gaussian distribution (see Figs. 1 and 3), we are wondering if there exists any systematic bias in estimating the measurement uncertainty. This needs a more detailed discussion which will be carried out below.

3.2 Effect of Intrinsic Distributions

Due to fluctuation, it would not be surprising if the distance indicator has itself a distribution (see the relevant discussions in Qin et al. 2006), albeit an extremely narrow one. The best method of SN Ia luminosity calibration with appropriate absorption correction can achieve an intrinsic dispersion of $\sigma < 0.12\,\text{mag}$ (Wang et al. 2006) in the optical bands. Astier et al. (2006) derived that the intrinsic dispersions of the
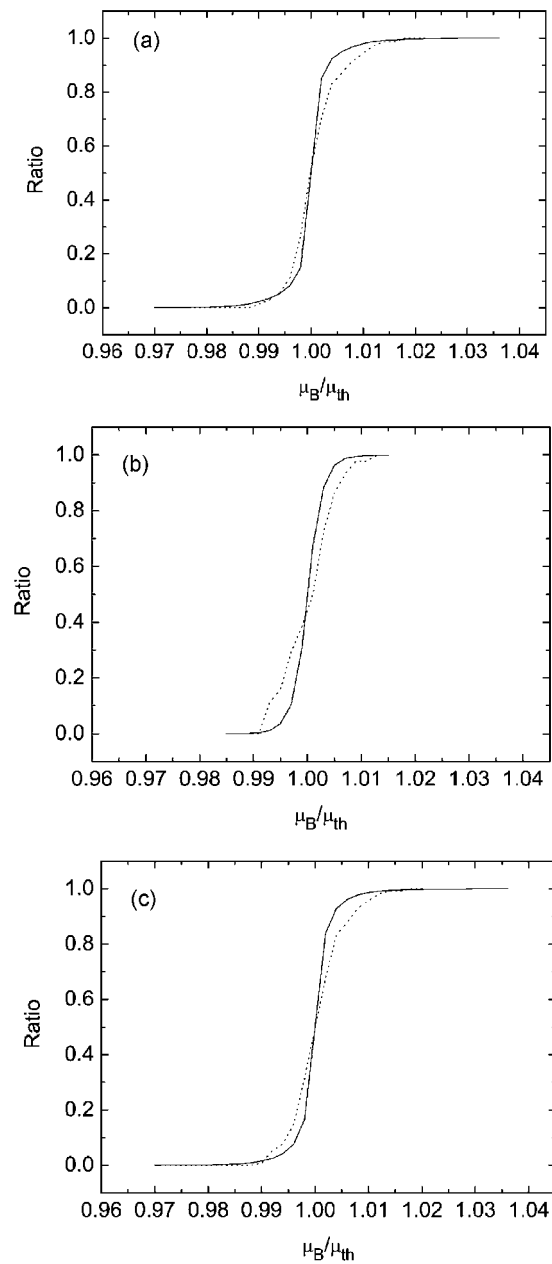
Fig. 3 Cumulative distribution functions of $\mu_B/\mu_{\rm th}$ (samples 1a, 2a and 3a, dotted lines) and $\mu'_B/\mu_{\rm th}$ (samples 1b, 2b and 3b, solid lines) for the 71 high-z SN Ia sample (panel a), 44 nearby SN Ia sample (panel b) and 115 assembled SN Ia sample (panel c).

nearby and high-redshift SNe Ia are 0.15 ± 0.02 and 0.12 ± 0.02, respectively. However, the luminosity calibration method used by them did not separate the host galaxy extinction from the intrinsic color correlation which gives larger dispersion and may cause bias in the luminosity distances if the dust around SNe Ia evolves with redshift. According to Wang et al. (2006), the dust around the nearby SNe Ia may be different from the dust in the Milky Way due to the impact of the dramatic explosion. So the bias may be

probably due to such unknown systematic effects in the color parameter used in the luminosity calibration, the luminosity evolution or the variance of the dust properties surrounding SNe Ia.

4 DISCUSSION AND CONCLUSIONS

As discussed by Kim et al. (2004), the uncertainty of a source must include both the systematic uncertainty and the magnitude dispersion. Thus the culprit for the bias seen in Figure 1 is likely due to the systematic effects as we discussed in Section 3. This is what we try to propose for current SN cosmology.

Probably the following issues are far beyond the scope of this paper, but we are presenting them in the hope that they might be relevant to further investigations. 1) Is it possible that the probability function is not a Gaussian distribution? Currently we have no answer to this. For a non-Gaussian error distribution of the luminosity distances, the basic assumption of SNe Ia's being standard candles would not hold. In this case, whether Equation (1) works for the estimations of the cosmological parameters is questionable. Even if the χ^2 statistic method is assumed to be approximately applicable to SNe Ia, we are not clear what reasonable errors should be expected for the distances. This must be answered. 2) As revealed recently by Wang et al. (2005), there is clear evidence for a tight linear correlation between peak luminosities of SNe Ia and their $B - V$ colors at ~ 12 days after the B maximum. They found that this empirical correlation allows one to reduce the scatter from ~ 0.5 mag to the levels of 0.18 and 0.12 mag in the V and I bands, respectively. Wang et al. (2001) gave similar results, too. The bias might also be related to improper corrections for the intrinsic color correlation. If so, could the bias be eased? Or, would it make the situation worse? This deserves a detailed analysis.

According to the above analysis and discussion, we come to the following conclusion: assuming that the three items of the basic assumption hold, then it is likely that there exists a bias in the SN Ia sample which is probably caused by a systematic error.

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