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GRB Energies and $E_\gamma - E_{\rm peak}$ Correlation with the Jet Expanding Laterally at the Sound Speed *

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Abstract A Gamma-ray burst (GRB) is generally believed to be a jet with a small opening angle, this opening angle is usually derived with the afterglow light curve break time using an analytical method. Here we show that the method is not accurate. Using the set of equations of hydrodynamic evolution with the sideways expansion at the local sound speed derived by previous authors and the observed light curve break times, we numerically derive the initial opening angles. Then the collimation-corrected energies (E_γ) for a sample of GRBs are calculated. They are found to show a wide spread, suggesting that the previously declared clustering by some authors may not exist. Also, the $E_{\rm peak} - E_\gamma$ relation, claimed by some other authors ($E_{\rm peak}$ is the spectral peak energy), is found still to hold, with a slightly stronger correlation.

Key words: gamma-ray: bursts — methods: numerical — ISM: jets

1 INTRODUCTION

It is widely believed that gamma-ray bursts (GRBs) are collimated. There are two main arguments for this belief: one is that if GRB is isotropic, then the emission energy will be too high to understand. This forced some theorists to believe that GRB is jetted, at lest for some bursts. The other is the break in the observed light curve. When the bulk Lorentz factor of a jet slows to $\gamma \sim \theta_0^{-1}$ (θ_0 is the initial half opening angle of the jet), the jet edge will be seen and the sideways expansion will become significant (Rhoads 1997, 1999; Sari, Piran & Halpern 1999; Panaitescu & Mészáros 1999), and thus a break in the light curve will naturally be expected. This is consistent with some observed results.

Based on the analytical hydrodynamic evolution of GRB jet and the observed jet break time, the initial opening angle of a jet can be derived with the following formula (Sari, Piran & Halpern 1999)

$$\theta_0 = 0.161\Big(\frac{t_{j,d}}{1+z}\Big)^{3/8}\Big(\frac{n\zeta}{E_{\gamma,\rm iso}}\Big)^{1/8}, \tag{1}$$

where $t_{j,d}$ is the light curve break time in days, z is the redshift, n is the number density of circumburst medium, $E_{\gamma,\rm iso}$ is the initial isotropic equivalent emission energy in γ-rays and ζ is the GRB radiative efficiency. With this angle, the actual emitted energy in γ-rays, namely the collimation-corrected energy, is $E_\gamma = E_{\gamma,\rm iso}$(1-cos θ_0). With a sample of 15 bursts, Frail et al. (2001) found that E_γ is strongly clustered at about 5×10^{50} erg. Bloom et al. (2003) confirmed this conclusion with a larger sample of 24 bursts, but found the clustering occurs at energy 1.3×10^{51} erg. These seem to suggest that GRBs might be standard energy reservoirs. Recently, a tight correlation between the collimation-corrected energy E_γ and the νF_ν peak

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energy $E_{\rm peak}$ of GRBs was reported by Ghirlanda et al. (2004b), which has been widely used to constrain the cosmological parameters (e.g. Dai, Liang & Xu 2004; Ghirlanda et al. 2004a; Xu, Dai & Liang 2005). Both of these results depend on the determination of θ_0. However, usually θ_0 derived with Equation (1) is only approximately valid. It is assumed that the jet expands laterally at the speed of light, which is only valid for an ultra relativistic jet (Sari, Piran & Halpern 1999). At a later time when the jet enters a phase of moderate relativistic expansion, this assumption is invalid. A more reasonable assumption is that the jet expands laterally at the comoving sound speed c_s, which should apply during both the ultra relativistic phase and the moderately relativistic phase, and even the nonrelativistic phase (Huang et al. 2000; Huang & Cheng 2003). The hydrodynamic evolution of the jet expanding sideways at the comoving sound speed has been studied analytically and numerically (Rhoads 1997, 1999; Huang, Dai & Lu 1999; Huang et al. 2000).

In this paper, different from the previous analytical approach, we numerically derive the initial GRB jet opening angle θ_0 with the hydrodynamic evolution of the jet expanding laterally at the sound speed. Our motivation is to obtain more precisely the initial GRB jet opening angles, hence the GRB energies, E_γ. With a sample of bursts, we have recalculated E_γ and examined if they are clustered and if the $E_{\rm peak}-E_\gamma$ relation still holds. In the second section, we introduce our numerical approach to calculate the initial jet opening angle, derive the angles for a sample of bursts and carry out a statistical investigation. In the third section, we derive the collimation-corrected energies E_γ and then make their distribution and the $E_{\rm peak}-E_\gamma$ distribution. A discussion and our conclusions then follow in the last section.

2 DISTRIBUTION OF THE JET OPENING ANGLE

2.1 Calculation of the Initial Opening Angle θ_0

The hydrodynamic evolution of jets has been studied numerically by a number of authors (e.g. Panaitescu & Mészáros 1999; Moderski et al. 2000; Huang et al. 2000; Kumar & Granot 2003). We adopt the set of equations in Huang et al. (2000) that describe the overall evolution of relativistic GRB ejecta from the ultrarelativistic phase to the nonrelativistic phase. The evolution of the bulk Lorentz factor is given by (Huang, Dai & Lu 1999; Huang et al. 2000)

$$\frac{d\gamma}{dm} = -\frac{\gamma^2-1}{M_{\rm ej}+\epsilon m+2(1-\epsilon)\gamma m}. \tag{2}$$

where m is the swept-up mass and ϵ is the radiation efficiency. $M_{\rm ej}$ is the ejection mass which is defined by $E_{k,\rm iso}(1-\cos\theta_0) = \gamma_0 M_{\rm ej}c^2$, where θ_0 is the initial half opening angle of the jet and γ_0, the initial Lorentz factor. If the lateral expansion velocity of the outflow is taken to equal the co-moving sound speed, then (Huang et al. 2000)

$$\frac{d\theta}{dt} = \frac{c_s(\gamma+\sqrt{\gamma^2-1})}{R}, \tag{3}$$

where θ is the half opening angle of the jet, $c_s = \frac{\sqrt{\hat{\gamma}(\hat{\gamma}-1)(\gamma-1)}}{\sqrt{1+\hat{\gamma}(\gamma-1)}}c$ is the comoving sound speed, $\hat{\gamma} = (4\gamma+1)/(3\gamma)$ is the adiabatic index and R is the radial coordinate in the burst source frame. For deriving the initial half opening angle of jet, we need to solve the set of equations of hydrodynamic evolution. We set the parameters as follows: $E_{k,\rm iso} = E_{\gamma,\rm iso}/\zeta$, $\gamma_0 = 300$, $\epsilon = 0$. Following Ghirlanda et al. (2004b), we put the radiative efficiency $\zeta = 0.2$. For a GRB jet, when $t = t_j$, $\gamma = \theta^{-1}$. We take this as a boundary condition, and adjust the initial jet opening angle θ_0 over an appropriate range until this condition is met. Then we derive θ_0.

2.2 Sample Selection and the Distribution of θ_0

We consider a sample of 30 bursts, with 24 from Ghirlanda et al. (2004b), three from Bloom et al. (2003), two from Firmani et al. (2006) and one from Perri et al. (2005). In our method, we need to know the light curve jet break time t_j. We thus include in our analysis only those bursts whose break times (or upper/lower limit) were measured. Table 1 lists the parameters of these bursts and the derived θ_0 and E_γ. For estimating the uncertainty of θ_0, a Monte Carlo simulation is carried out. We consider each of the observables, z, $E_{\gamma,\rm iso}$, n and t_j to follow a normal distribution. The parameters of these bursts characterized by $E_{\gamma,\rm iso}\pm\sigma_{E_{\gamma,\rm iso}}$, $n\pm\sigma_n$, $t_j\pm\sigma_{t_j}$ and $z\pm\sigma_z$ are generated randomly according to the normal distributions. We generate 100

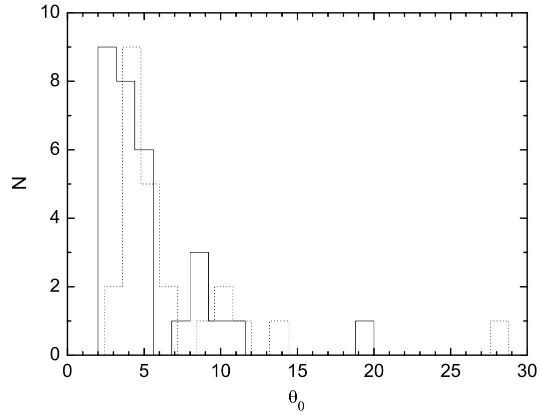

Fig. 1 Distribution of the opening angles θ_0 of GRB jets. Solid histogram for our data; dotted histogram for the data of Ghirlanda et al. (2004b).

sets of parameters and thus derive $100\,\theta_0$. Hence we can find the error of θ_0. Figure 1 plots the distribution of θ_0. A comparison with the distribution derived by Ghirlanda et al. (2004b) shows that our values are systematically smaller. The jet angles that we derived vary from 2° to 20° and are more concentrated, with a peak angle at about 2.5°, while those derived by Ghirlanda et al. (2004b) vary from 2° to 30° and concentrate around 4°.

3 THE DISTRIBUTION OF E_γ AND THE $E_{\rm PEAK} - E_\gamma$ RELATION

The true equivalent isotropic gamma-ray energy is given by

$$E_\gamma = E_{\gamma,\rm iso}(1 - \cos\,\theta_0). \tag{4}$$

For the sample listed in Table 1, the energies are calculated and their errors derived with the error propagation technique. The histogram of their distribution (Fig. 2) shows that the energies span about three orders of magnitude (from 10^{49} to 10^{51} erg) and no evident clustering is seen. A bimodal distribution seems indicated. These results are inconsistent with the results obtained by Bloom et al. (2003). The plot of $E_\gamma - E_{\rm peak}$ is shown in Figure 3, showing that the two quantities are still strongly correlated. Excluding bursts for which only upper/lower limit of $E_{\rm peak}$ and/or E_γ are given, the Spearman's rank correlation coefficient is $r_s = 0.94$ with a probability of being uncorrelated of $P = 4.8 \times 10^{-9}$ (18 data points). For comparison, Ghirlanda et al. (2004b) derived $r_s = 0.94$ and $P = 1.4 \times 10^{-7}$ (15 data points). This shows that the correlation that we derived is slightly stronger than that derived by Ghirlanda et al. (2004b). The best power law, obtained considering the errors in both coordinates (routine "fitexy" of Press et al. 1992), is

$$E_{\rm peak} = 565.6 \left(\frac{E_\gamma}{6.8 \times 10^{50}\ \rm erg} \right)^{0.636 \pm 0.034} \rm keV, \tag{5}$$

where 6.8×10^{50} erg is the mean collimation-corrected energy for all the bursts in our sample. The power-law index in this correlation is 0.636, which is smaller than that derived by Ghirlanda et al. (2004b).

4 DISCUSSION AND CONCLUSIONS

In this paper, instead of the usually employed analytical method, we derive the jet opening angle with a numerical method. With a latest GRB sample, we calculated the half opening angles of the GRB jets and found that they are clustered around 2.5°, less than the 4° derived by Ghirlanda et al. (2004b) and much less than the widely used value 6°. This is due, not only to the different methods used but also to the different speeds of sideways expansion adopted. We argue that it is more appropriate for the jet to expand laterally at the comoving speed of sound rather than at the constant speed of light (as was done in the previous papers) and so the jet opening angles that we derived should be more accurate.

Table 1 Sample of GRBs

GRB	z^{a}	t_j (day)	n^{b} (cm^{-3})	$E_{\gamma,\mathrm{iso}}$ (erg)	$E_{\mathrm{peak}}{}^{\mathrm{c}}$ (keV)	θ_0^{d} (deg)	E_γ^{e} (erg)	Ref
970828	0.957	2.2 [0.4]	3.0 [1]	2.96E53 [0.35]	583 [116]	4.87 [0.43]	1.07E51 [0.23]	(1)
971214	3.42	>2.5 [0.4]	3.0 [1]	2.11E53 [0.24]	685 [133]	>3.9	>4.89E50	(1)
980613	1.096	>3.1	3.0 [1]	6.9E51 [0.95]	194 [89]	>8.61	>7.78E49	(1)
980703	0.966	3.4 [0.5]	28 [1]	6.9E52 [0.82]	502 [100]	9.03 [0.71]	8.55E50 [1.68]	(1)
990123	1.6	2.04 [0.46]	3.0 [1]	2.39E54 [0.28]	2030 [161]	3.22 [0.36]	3.77E51 [0.95]	(1)
990510	1.619	1.6 [0.2]	0.29 [0.1]	1.78E53 [0.19]	423 [42]	3.02 [0.23]	2.47E50 [0.46]	(1)
990705	0.843	1.0 [0.2]	3.0 [1]	1.82E53 [0.23]	348 [28]	3.91 [0.36]	4.24E50 [0.95]	(1)
990712	0.43	1.6 [0.2]	3.0 [1]	6.72E51 [1.29]	93 [15]	7.8 [0.44]	6.22E49 [1.38]	(1)
991216	1.02	1.2 [0.4]	4.7 [2.3]	6.75E53 [0.81]	641 [128]	3.62 [0.58]	1.35E51 [0.46]	(1)
000131	4.5	<3.5	3.0 [1]	1.84E54 [0.22]	714 [142]	<3.06	<2.62E51	(1)
000210	0.8463	>1.7	3.0	1.69E53 [0.14]	...	>4.85	>6.05E50	(2)
000926	2.0369	1.8	27 [3]	2.8E53 [0.99]	...	5.09	1.10E51	(2)
000911	1.058	<1.5	3.0 [1]	8.8E53 [1.05]	1190 [238]	<3.57	<1.71E51	(1)
010222	1.473	0.93 [0.1]	1.7 [0.18]	1.33E54 [0.15]	>886	2.39 [0.14]	1.16E51	(1)
010921	0.45	<33.	3.0 [1]	9.0E51 [1.0]	153 [31]	<19.46	<5.14E50	(1)
011121	0.36	>7	3.0 [1]	4.55E52 [0.54]	>952	>10.7	>7.91E50	(1)
011211	2.14	1.5 [0.15]	3.0 [1]	6.3E52 [0.7]	186 [24]	4.27 [0.27]	1.75E50 [0.29]	(1)
020124	3.2	3. [0.4]	3.0 [1]	3.02E53 [0.36]	503 [100]	4.07 [0.31]	7.62E50 [1.47]	(1)
020405	0.69	1.67 [0.52]	3.0 [1]	1.1E53 [0.13]	612 [122]	5.26 [0.82]	4.63E50 [1.54]	(1)
020813	1.25	0.43 [0.06]	3.0 [1]	8.0E53 [0.96]	474 [95]	2.09 [0.15]	5.32E50 [1.00]	(1)
021004	2.3320	7.6	30.	5.56E53 [0.72]	...	10.29	8.94E50	(2)
021211	1.01	>1	3.0 [1]	1.1E52 [0.13]	94 [19]	>5.43	>4.94E49	(1)
030226	1.98	0.84 [0.1]	3.0 [1]	1.2E53 [0.13]	322 [64]	3.19 [0.23]	1.86E50 [0.34]	(1)
030328	1.52	0.8 [0.1]	3.0 [1]	2.8E53 [0.33]	277 [55]	2.98 [0.23]	3.79E50 [0.74]	(1)
030329	0.1685	0.5 [0.1]	1 [0.11]	1.8E52 [0.21]	79 (3)	4.18 [0.35]	4.79E49 [0.98]	(1)
030429	2.66	1.77 [1.0]	3.0 [1]	2.19E52 [0.26]	128 [26]	4.92 [1.19]	8.07E49 [4.02]	(1)
030723X	< 2.1	1.67 [0.3]	3.0 [1]	<2.16E50	<15	9.07 [0.70]	<2.70E48	(1)
041006	0.716	0.16 [0.04]	3.0 [1]	7.1E52 [0.8]	109 [22]	2.17 [0.30]	5.09E49 [1.52]	(3)
050318	1.44	0.212[0.12]	3.0 [1]	2.2E52 [0.16]	115 [37]	2.48 [0.61]	2.06E49 [1.02]	(4)
050525	0.606	0.15 [0.01]	3.0 [1]	3.5E52 [0.5]	126.6 [6]	2.4 [0.15]	3.07E49 [0.58]	(3)

Notes- The isotropic energies of the three bursts from Bloom et al. (2003) are in the range of 20–2000 keV. For the burst GRB 050318, the light curve breaks at $t_j = 0.212^{+0.04}_{-0.12}$ day and the peak energy is $E_{\mathrm{peak}} = 115^{+37}_{-20}$ keV (Perri et al. 2005), and we used $t_j = 0.212 \pm 0.12$ day and $E_{\mathrm{peak}} = 115 \pm 37$ keV in the simulation calculation of the error of θ_0.

[a] We have assumed that the error is 10%.

[b] The circumburst medium density is taken as $n = 3.0 \pm 1.0$ when unavailable.

[c] E_{peak} is the burst source frame peak energy.

[d] The error of θ_0 is obtained with Monte Carlo simulation.

[e] The error of E_γ is derived with the error propagation formula.

References- (1) Ghirlanda et al. 2004b (references therein); (2) Bloom et al. 2003 (references therein); (3) Firmani et al. 2006 (references therein); (4) Perri et al. 2005.

We calculated the collimation-corrected energies and drew the histogram of their distribution. It seems to show a bimodal distribution, with peaks around 5×10^{49} erg and 7×10^{50} erg. The higher peak at energy 7×10^{50} erg corresponds to that derived by Bloom et al. (2003), and is ~ 0.5 dex lower. The lower peak has not been seen previously. We do not know if the results are due to selection effects or other reasons: the limited available GRB data precludes a further study. However, our results seem to suggest that clustering may not exist at all in the collimation-corrected energies, but more data are needed to confirm this suggestion. We also re-examined the $E_{\mathrm{peak}} - E_\gamma$ relation and found that this relation is slightly improved, with the power-law index changing from ∼0.71 to ∼0.64. This may provide a constraint on models attempting to interpret this relation.

There are several uncertainties in our method. The assumed efficiency ζ converting the kinetic energy into gamma-rays is a highly uncertain factor. Many authors make efforts to determine the GRB ef-

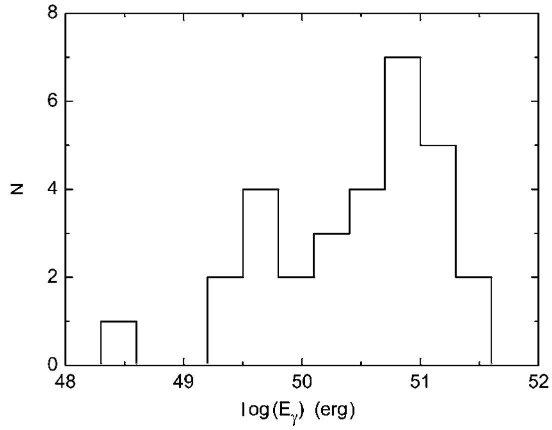

Fig. 2 Distribution of the collimation-corrected energy E_γ for the sample presented in Table 1.

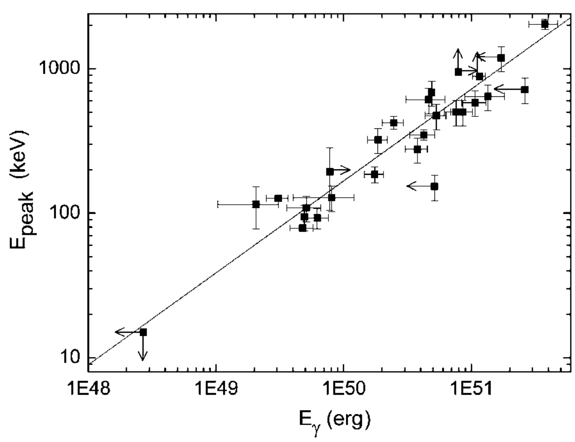

Fig. 3 $E_{\rm peak} - E_\gamma$ relation. $E_{\rm peak}$ errors are available from Ghirlanda et al. (2004b), and E_γ errors are derived with the error propagation formula.

ficiency, but to date, it is poorly constrained due to many effects (see e.g. Lloyd-Ronning & Zhang 2004; Zhao & Bai 2006; Zhang et al. 2007). However, in this paper, we focus on some statistical properties, and the statistical properties of some quantities, such as the collimation-corrected energy, seem to be independent of the distribution of the GRB efficiency (Xu & Dai 2004). Thus, the assumption of a constant efficiency may be justified.

Another uncertainty comes from the time of the light curve break, t_j. In fact, this quantity is difficult to determine precisely. Most of the t_j in our sample have large uncertainties. This affects the precision in the derived initial jet opening angles. For the burst GRB 030429, the error of t_j is even comparable with t_j itself: $t_j = 1.77 \pm 1.0$ day (i.e. $t_j = 0.77 - 2.77$ day), while the resulting error of the initial jet opening angle of the burst is also quite large: $\theta_0 = 4.29 \pm 1.19$. However, this problem is hard to overcome at present due to the limited observational precision and its inherent complexity.

The circumburst environment is also an uncertain factor. It is difficult to determine whether the GRB is in a wind circumburst environment, or interstellar medium (ISM) or a dense medium (Mészáros, Rees & Wijers 1998; Dai et al. 1999). Here, we only consider the simplest case of ISM. As done previously by some authors (e.g. Bloom et al. 2003; Ghirlanda et al. 2004b), for some bursts where the number density is unavailable, we simply assumed the typical value. Uncertainty in the number density of the circumburst medium will increase the uncertainty in the derived initial jet opening angle and hence in the GRB energy, which may impact on the conclusions we have drawn. However, here, attention is concentrated on the statistical properties, and the effect from the uncertainty in the number density of the circumburst medium may not be so severe.

In addition, the initial bulk Lorentz factor, which is assumed by us in calculation, is also unknown. To date only a lower limit is estimated (see e.g. Lithwick & Sari 2001). The initial bulk Lorentz factor determines the initial mass of GRB outflow. At a later time, the mass of the jet is dominated by the swept-up mass, so the assumed initial Lorentz factor weakly affects its subsequent evolution and hence the derived jet opening angle. Our calculation also proves that if instead of the value 300, the initial Lorentz factor is taken as 100 (or 500), then the resulting jet opening angles will only decrease by $\leq 0.4^\circ$ (or increase by $\leq 0.1^\circ$). Thus, it seems that the effect from this factor is unimportant.

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