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A Model for Contact Binary Systems *

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Abstract A model for contact binary systems is presented, which incorporates the following special features: a) The energy exchange between the components is based on the understanding that the energy exchange is due to the release of potential, kinetic and thermal energies of the exchanged mass. b) A special form of mass and angular momentum loss occurring in contact binaries is losses via the outer Lagrangian point. c) The effects of spin, orbital rotation and tidal action on the stellar structure as well as the effect of meridian circulation on the mixing of the chemical elements are considered. d) The model is valid not only for low-mass contact binaries but also for high-mass contact binaries. For illustration, we used the model to trace the evolution of a massive binary system consisting of one $12M_\odot$ and one $5M_\odot$ star. The result shows that the start and end of the contact stage fall within the semi-detached phase during which the primary continually transfers mass to the secondary. The time span of the contact stage is short and the mass transfer rate is very large. Therefore, the contact stage can be regarded as a special part of the semi-detached phase with a large mass transfer rate. Both mass loss through the outer Lagrangian point and oscillation between contact and semi-contact states can occur during the contact phase, and the effective temperatures of the primary and the secondary are almost equal.

Key words: stars: binaries — stars: evolution

1 INTRODUCTION

Contact binaries are binaries in which both components have filled their Roche lobes. A number of complicated physical processes may occur in the contact binary systems. Due to the short orbital distance, the gravitational force from the primary or the secondary is strong enough to accrete matter from the surface of the other. Thus, the system has a common envelope and the processes of mass and energy exchange between the components occur in the common envelope. The mass and energy exchange between the components can influence the masses, luminosities, and effective temperatures of the components. When the common envelope spills over the outer Lagrangian point, a special form of mass and angular momentum loss occurs through the outer Lagrangian point, which can influence the period of the system significantly. For the components of contact binaries, there exist not only the spin of the components, but also the orbital rotation and the tidal action from the companion star. The spin and orbital rotation of the components, and the tidal action from the companion star can change the structure of the components from spherical symmetric to non-spherical symmetric. In addition, a special form of mass flow, the meridian circulation, occurs in the components caused by the effects of rotation. The meridian circulation drives the transport of chemical elements in the components. The structure and evolution of a contact binary system depend

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mainly on the initial masses, initial chemical composition of the components, and the initial orbital separation between the components, but they are also significantly affected by the physical processes listed above. Among those processes the effects of mass and energy exchange between the components, the losses of mass and angular momentum via the outer Lagrangian point, and the effects of rotation and tide are the most important. Hence, the understanding and treatment of those processes are the important contents for the model of contact binary systems.

There have been a large number of studies on the structure and evolution of low-mass contact binaries (e.g., W UMa-type contact binaries) with different models for those processes (e.g., Halzhrust & Meyer-Hofmeister 1973; Robertson & Eggleton 1977; Halzhrust & Refsdal 1980,1984; Eggleton 1983; Kähler et al. 1986; Li et al. 2004, 2005). From these previous studies the following points emerge: 1) There are different ways to approximate the energy exchange between the components. Some studies estimated the energy exchange by using the mass-luminosity relation that is valid for the W UMa-type contact binaries (e.g. Robertson & Eggleton 1977; Rahunen 1981; Li et al. 2004a,b, 2005). Another approach to treat the energy exchange is based on the understanding that the energy exchange is related to the entropy difference and the contact depth (e.g., Halzhrust & Meyer-Hofmeister 1973; Kähler et al. 1986). 2) The effect of angular momentum loss due to magnetic braking on the evolution of W UMa-type contact binaries was studied by Li et al. (2004a). 3) The effect of spin of the components on the stellar structure equations was examined (e.g. Eggleton 1983; Li et al. 2004a,b, 2005).

The purpose of the present paper is to present a model of contact binary systems with the following emphases: a) The energy exchange between the components is studied based on the understanding that the energy exchange is due to the release of the potential, kinetic and thermal energies of the exchanged mass. b) As a special form of the mass and angular momentum loss occurring in contact binaries, the losses of mass and angular momentum via the outer Lagrangian point are checked. c) The effects of spin, orbital rotation, and tide on the stellar structure equations, as well as the effect of the meridian circulation on the mixing of the chemical elements are considered. d) The model is valid not only for low-mass contact binaries but also for the more massive contact binaries. In addition, to show the special features of contact binaries, the evolution of a massive binary system consisting of a $12M_\odot$ and a $5M_\odot$ star is followed using of this model. In Sections 2 and 3, the physical processes of mass and energy exchange are discussed respectively. In Section 4 the mass and angular momentum loss via the outer Lagranging point is discussed. In Section 5 the effects of rotation and tide on the stellar structure equations, and the mixing of chemical elements due to the meridian circulation are discussed. Section 6 analyzes the orbital evolution of binary systems with consideration of theses processes. In the last section, the evolution of a binary system consisting of a $12M_\odot$ and a $5M_\odot$ star is monitored from the zero age main sequence to the later stage after the contact phase.

2 MASS EXCHANGE BETWEEN THE COMPONENTS

2.1 The Condition for Mass Exchange

The potential at a point P for a synchronous rotating component in the Roche model is given by

$$\Psi_P = -\frac{GM_1}{r_1} - \frac{1}{2}\omega^2 r_1^2 \sin^2\theta - \frac{1}{2}\omega^2 (X_\omega - r_1\sin\theta)^2 - \frac{GM_2}{r_2}, \tag{1}$$

where r_1 and r_2 are the distances between the point P and the centers of the primary and the secondary respectively, M_1 and M_2 are the masses of the primary and the secondary, X_ω is the distance between the primary and the rotation axis of the system, ω is the orbital angular velocity of the system. The angle θ and the coordinate system are shown in Figure 1. The first and second terms on the right side of Equation (1) correspond to the respective contributions from the gravitation and spin of the primary. The third and fourth terms respectively correspond to the contributions from the orbital rotation of the system and the gravitation of the secondary. Using Equation (1), one obtains the average potentials at the surfaces of the two components denoted by $\bar{\Psi}_1$ and $\bar{\Psi}_2$.

Mass flow occurs in the common envelope of a contact binary, when the average potential of the primary $\bar{\Psi}_1$ differs from that of the secondary $\bar{\Psi}_2$, and the mass exchange will start off a process of changing the inner state of the components (include transfer of mass and energy between the inner and outer layers and the expansion or contraction of the components). As the result of the mass transfer in the common envelope

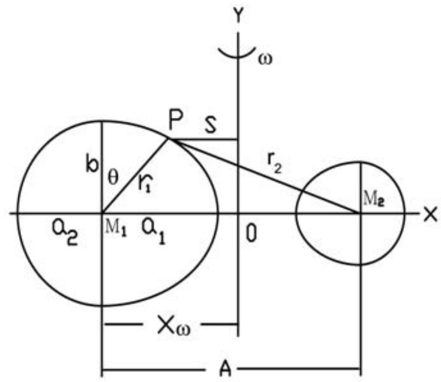

Fig. 1 Geometry for the calculation of the potential at the point P.

and the change of the inner state of the components, the average potential of the primary and the secondary may be changed. If the changed average potentials of the components are not equal, then mass exchange between the components will resume. Mass exchange between the primary and the secondary can take place in two directions:

If $\bar{\Psi}_1 > \bar{\Psi}_2$, mass flows from the primary to the secondary;

If $\bar{\Psi}_1 < \bar{\Psi}_2$, mass flows from the secondary to the primary.

The condition to terminate the mass exchange can be expressed as:

$$\bar{\Psi}_1 = \bar{\Psi}_2. \tag{2}$$

The time scale of the mass transfer process (i.e., from the beginning of mass transfer in the common envelope to when the condition $\bar{\Psi}_1 = \bar{\Psi}_2$ is satisfied) is very short, however, compared to that of the contact phase.

2.2 Rate of Mass Transfer

We approximate the rate of mass exchange by the requirement that the average potentials of the primary and the secondary are equal after the mass exchange. Thus, the rate of mass transfer is related to the difference between the average potentials of the two components before the mass transfer. Furthermore, the effect of the total mass on the rate of mass exchange is also considered in the equation. So the mass transfer rate is approximated as:

$$\frac{dM_{1c}}{dt} = -C \cdot M_1(\bar{\Psi}_1 - \bar{\Psi}_2), \tag{3}$$

$$\frac{dM_{2c}}{dt} = -\frac{dM_{1c}}{dt}, \tag{4}$$

where constant C is an adjustable parameter. If the value of C is chosen too small, then the condition of $\bar{\Psi}_1 > \bar{\Psi}_2$ will exist both before and after the mass transfer, and it will need several transfers from the primary to the secondary before the condition $\bar{\Psi}_1 = \bar{\Psi}_2$ is satisfied. On the other hand, if C is chosen too large, then we may have $\bar{\Psi}_1 > \bar{\Psi}_2$ before the transfer and $\bar{\Psi}_1 < \bar{\Psi}_2$ after, and again it needs several transfers back and forth before the condition $\bar{\Psi}_1 = \bar{\Psi}_2$ is satisfied. An extreme case occurs when the transferred mass is so large that the potential of the donor component after the transfer becomes smaller than that of the inner Lagrangian point, and the binary system changes from a contact system to a semi-contact one.

3 ENERGY EXCHANGE

A model to treat the energy exchange is presented based on the understanding that the energy exchange is due to the release of potential, kinetic, and thermal energies of the exchanged mass.

3.1 Release of the Potential Energy

It is assumed that the exchanged mass ΔM_{1c} is distributed within a shell on the surface of the primary before the transfer, and within a shell on the surface of the secondary after the transfer. The release of the potential energy due to the flow of ΔM_{1c} from the surface of the one to that of the other can be obtained as:

$$\Delta E_P = \Delta M_{1c} \cdot (\bar{\Psi}_1 - \bar{\Psi}_2), \tag{5}$$

where $\bar{\Psi}_1$ and $\bar{\Psi}_2$ are the average potentials at the surfaces of the primary and the secondary, respectively.

3.2 Release in Kinetic Energy

As assumed above, the exchanged mass ΔM_{1c} is distributed within a shell on the surface of the primary before the transfer, and within a shell on the surface of the secondary after the transfer. If, as an approximation, the shells are regarded as rigid bodies, the release of the rotational kinetic energy due to the flow of ΔM_{1c} can be obtained as

$$\Delta E_K = \Delta M_{1c}\{\tfrac{1}{2}(\tfrac{2}{3}R_1^2 + X_\omega^2) - \tfrac{1}{2}[\tfrac{2}{3}R_2^2 + (A - X_\omega)^2]\}\omega^2, \tag{6}$$

where X_ω is the distance between the primary and the axis through the center of mass of the system, A is the orbital separation between the two components, and ω is the angular velocity of the binary system. The terms $(\frac{2}{3}R_1^2 + X_\omega^2) \cdot \Delta M_{1c}$ and $[\frac{2}{3}R_2^2 + (A - X_\omega)^2] \cdot \Delta M_{1c}$ are the moments of inertia of the respective shells involved.

3.3 Release of Thermal Energy

There are two physical processes that can affect the thermal condition of the outer layers of the components. The first is the transfer of mass and energy in the common envelope. As the result of this process, the effective temperatures of the components tend to be equalized. The second is the transfer of mass and energy between the inner and outer layers of the components when there is mass exchange between the components. As the result of this process, the thermal conditions of the outer layers of the components are further changed and a new difference between the effective temperatures of the components comes into being. The difference of the timescales between these two processes is important whether or not the effective temperatures of the components are equal. As mentioned before, the exchanged mass ΔM_{1c} is distributed within a shell on the surface of the primary before the transfer, and within a shell on the surface of the secondary after, and the effective temperatures of the primary and the secondary are different. The release of thermal energy in such case can be expressed as:

$$\Delta E_T = \Delta M_{1c}\Big(\frac{3kT_{\rm eff1}}{2\mu_1 m_p} - \frac{3kT_{\rm eff2}}{2\mu_2 m_p}\Big), \tag{7}$$

where $T_{\rm eff1}$ and $T_{\rm eff2}$ are the effective temperatures of the components, and μ_1 and μ_2 are their mean molecular weights, and m_p is the mass of the proton. Equation (7) admits the case of no thermal energy exchange ($\Delta E_T = 0$), when the effective temperatures of the components are equal.

3.4 The Total Energy Exchange

The total energy exchange due to the release of different forms of energy is:

$$\Delta E_c = \Delta E_P + \Delta E_K + \Delta E_T. \tag{8}$$

According to the virial theorem, half of the exchanged energy will be converted into a change of the luminosity and the rest into an exchange of thermal energy of the components.

4 MASS AND ANGULAR MOMENTUM LOSS FROM THE OUTER LAGRANGIAN POINT

4.1 Potential at the Outer Lagrangian Point L_2

Figure 2 shows the Roche equipotential surface, passing through the outer Lagrangian point L_2. From Equation (1) the potential at the outer Lagrangin point L_2 is obtained as:

$$\Psi_{L2} = \begin{cases} -\frac{GM_1}{X_{L_2}+X_\omega} - \frac{GM_2}{X_{L_2}+X_\omega-A} - \frac{1}{2}\omega^2 \cdot X_{L_2}^2, & \text{if } M_1 > M_2, \\ -\frac{GM_1}{X_{L_2}-X_\omega} - \frac{GM_2}{X_{L_2}-X_\omega+A} - \frac{1}{2}\omega^2 \cdot X_{L_2}^2, & \text{if } M_1 < M_2, \end{cases} \tag{9}$$

where X_ω is the distance between the primary and the the mass center of the system (Fig. 2), $X_\omega = \frac{M_2 A}{(M_1+M_2)}$), and X_{L2} is the distance between the outer Lagrangian point L_2 and the mass center, which is determined by condition that the sum of the gravitational forces and the centrifugal force at the point L_2 are zero:

$$\frac{GM_1}{(X_{L_2}+X_\omega)^2} - \frac{GM_2}{(X_{L_2}+X_\omega-A)^2} - \omega^2 \cdot X_{L_2} = 0, \quad \text{when} \quad M_1 > M_2, \tag{10}$$

$$\frac{GM_1}{(X_{L_2}-X_\omega)^2} - \frac{GM_2}{(X_{L_2}+A-X_\omega)^2} - \omega^2 \cdot X_{L_2} = 0, \quad \text{when} \quad M_1 < M_2. \tag{11}$$

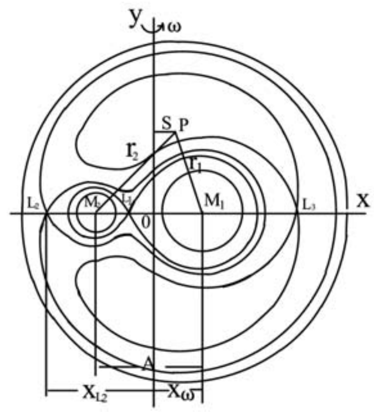

Fig. 2 Geometry for the calculation of the potentials at various points.

4.2 Mass Loss via the Outer Lagrangian Point

The condition for the occurrence of mass loss via the outer Lagrangian point is:

$$\bar{\Psi}_2 \geq \Psi_{L_2}, \quad \text{when} \;\; M_1 > M_2, \tag{12}$$

$$\bar{\Psi}_1 \geq \Psi_{L_2}, \quad \text{when} \;\; M_1 < M_2. \tag{13}$$

The rates of mass loss through the outer Lagrangian point can be written as:

$$\frac{\Delta M_L}{\Delta t} = \begin{cases} -C_L \frac{M_1+M_2}{2}(\bar{\Psi}_2 - \Psi_{L_2}), & M_1 \geq M_2, \\ -C_L \frac{M_1+M_2}{2}(\bar{\Psi}_1 - \Psi_{L_2}), & M_1 < M_2, \end{cases} \tag{14}$$

where the constants C_L is an adjustable parameter. The physical background of Equation (14) is that the mass loss rate is changed by the requirement that the average potentials of the component should be changed simultaneously with that of the outer Lagrangian point. Thus, the rate of mass loss is related to the difference between the average potential of the component and the potential of the outer Lagrangian point before the start of mass loss. Furthermore, the effect of the total mass of the components on the rate of mass loss should also be considered in this equation. Two cases can occur when the mass loss rate is calculated with Equation (14): If the chosen value of the parameter C_L is very small, then the mass loss from the outer Lagrangian point need to occur several times before the common envelope losses enough mass to stop the mass loss process. On the other hand, if C_L is chosen very large, then the potential of the component after the mass transfer drops below that of the outer Lagrangian point, so terminating the mass loss.

4.3 Angular Momentum Loss via the Outer Lagrangian Point

It is assumed that the components are in synchronous rotation and a part of the lost mass ΔM_L was distributed within a shell on the surface of the secondary before the escape, while the other part of ΔM_L was distributed within a shell on the surface of the primary. If the shells are approximated as rigid bodies, the angular momentum carried away by the escaped mass ΔM_L can be written as:

$$\Delta J_L = (\tfrac{2}{3}R_1^2 + X_\omega^2)\tfrac{\Delta M_L \cdot M_1}{M_1+M_2}\omega + [\tfrac{2}{3}R_2^2 + (A - X_\omega)^2]\tfrac{\Delta M_L \cdot M_2}{M_1+M_2}\omega, \tag{15}$$

where X_ω is the distance between the primary and the mass center of the system, and ω is the angular velocity of the binary system. The terms $(\frac{2}{3}R_1^2 + X_\omega^2)\frac{\Delta M_L \cdot M_1}{M_1+M_2}$ and $[\frac{2}{3}R_2^2 + (A - X_\omega)^2]\frac{\Delta M_L \cdot M_2}{M_1+M_2}$ are the moments of inertia of the shells concerned, and R_1 and R_2 are the radii of the primary and secondary. The first and the second terms on the right side of Equation (15) correspond, respectively, to the angular momentum carried away by the lost mass from the primary and the secondary.

5 EFFECTS OF ROTATION AND TIDE

5.1 The Equipotentials of the Components

From Equation (1), the equipotentials of the components defined by the function Ψ =constant are asymmetric ellipsoids with two semi-major axes a_1 and a_2 ($a_1 > a_2$) and one semi-minor axis b (see fig. 1 and Huang 2004).

5.2 The Stellar Structure Equations

Due to the effect of tide, the rotation of the individual component is synchronous with the orbital motion of the system. Such synchronous rotation exists also in the interior of the components. Thus, the rotation of the components is solid-body rotation and may be called "conservative rotation". Kippenhahn & Thomas (1970) introduced a method to simplify the two-dimensional model with conservative rotation to a one-dimensional model and gave the structural equations as follows:

$$\frac{dr_\Psi}{dM_\Psi} = \frac{1}{4\pi r_\Psi^2 \rho}, \tag{16}$$

$$\frac{dP}{dM_\psi} = -\frac{GM_\Psi}{4\pi r_\Psi^4} f_P, \tag{17}$$

$$\frac{dL}{dM_\Psi} = \varepsilon_N - \varepsilon_\nu + \varepsilon_g, \tag{18}$$

$$\frac{d\ln T}{d\ln P} = \begin{cases} \nabla_{\mathrm{R}} f_{\mathrm{T}}/f_{\mathrm{p}} \\ \nabla_{\mathrm{con}} \end{cases}, \tag{19}$$

where

$$f_P = \frac{4\pi r_\Psi^4}{GM_\Psi S_\Psi}\frac{1}{\langle g_{\mathrm{eff}}^{-1}\rangle}, \tag{20}$$

$$f_T = \left(\frac{4\pi r_\Psi^2}{S_\Psi}\right)^2 \frac{1}{\langle g_{\mathrm{eff}}\rangle\langle g_{\mathrm{eff}}^{-1}\rangle}, \tag{21}$$

$$\nabla_R = \frac{3\kappa LP}{4acGM_\Psi T^4}. \tag{22}$$

Here, $\langle g_{\mathrm{eff}}\rangle$, $\langle g_{\mathrm{eff}}^{-1}\rangle$ are the mean values of the effective gravity and its inverse over the equipotential surface, ∇_R the radiative temperature gradient.

5.3 Rotational Mixing

The effect of rotation results in an outward mass-flow along the rotational axis and an inward mass flow along the equatorial plane. Such outward and inward mass flows in a star are called meridian circulation (Kippenhahn & Weigert 1990; Maeder & Meynet 2000). As a result of the meridian circulation and the shear turbulence, a radial mass exchange occurs that can drive a transport of chemical elements in rotating stars. For components with solid-body rotation there exists no differential rotation which can drive shear turbulence. Hence, the rotational mixing in the components of a binary system can be driven only by the meridian circulation. Since the transport of chemical elements is caused by the effect of rotation, and the effect of rotation of a mass layer can be approximated by the ratio of the mean effective gravity $\langle g_{\rm eff}\rangle$ to the gravity g_i of this layer, so the change of the composition of the element α via a diffusion-advection process can be approximated by:

$$\left(\frac{\partial X_\alpha}{\partial t}\right)_r = \left(\frac{\partial}{\partial r}\right)_t \left[-D_{\rm adv}\frac{\langle g_{\rm eff}\rangle_i}{g_i} - D_{\rm diff}\left(\frac{\partial X_\alpha}{\partial r}\right)_t\right] + \left(\frac{dX_\alpha}{dt}\right)_{\rm nuc}, \tag{23}$$

where $D_{\rm adv}$ and $D_{\rm diff}$ are the relevant coefficients ($D_{\rm adv} = \frac{K_1}{\tau_{\rm adv}}$, $D_{\rm diff} = \frac{K_2}{\tau_{\rm th}}$), K_1 and K_2 being two dimensionless parameters. The thermal and the advection time-scales can be given as (see Maeder & Meynet 2000; Huang & Yu 1998): $\tau_{\rm th} = \frac{1}{2}q\frac{GM^2}{RL}$, $q = 0.5$ and $\tau_{\rm adv} = \tau_{\rm th}\frac{GM}{R^3\omega^2}$. The last term of Equation (23) is the change of the composition due to nuclear reactions.

At the inner and outer boundary the reflecting conditions are used:

$$\left(\frac{\partial X_\alpha}{\partial r}\right)_t |_{r=0} = 0 = \left(\frac{\partial X_\alpha}{\partial r}\right)_t |_{r=R}\ . \tag{24}$$

5.4 Mass and Angular Momentum Loss due to Stellar Winds

Among the numerous observational and theoretical investigations on the mass loss rate via stellar winds (see Chiosi & Maeder 1986) we adopt the observational formula for the mass loss rate introduced by Waldron (1984), according to which, the rate of mass loss (in $M_\odot$ yr^{-1}) is dependent upon both the stellar luminosity and the stellar radius, and is given by

$$\log(\dot{M}) = 1.07\log(L/L_\odot) + 1.77\log(R/R_\odot) - 14.3. \tag{25}$$

The rate of mass loss from a rotating component star in a binary system may be different from a rotating single star of the same spectral type and luminosity class, due to the effects of tide, and the centrifugal forces caused by the rotation. From Friend & Castor (1982), the mass loss rate is related to the gravitational potential. Therefore, the rate of mass loss for a component star is enhanced by a factor ζ, given by

$$\zeta = \frac{g}{\langle g_{\rm eff}\rangle}, \tag{26}$$

where g is the gravitational acceleration for a single star with the same mass.

It is assumed that the mass (ΔM_{1w}) lost by the winds from the primary was originally distributed in a shell on the surface of the primary, and similarly the mass loss of ΔM_{2w} by the secondary. If the shells are approximated as rigid bodies, then the angular momentum carried away by the escaped masses can be written as:

$$\Delta J = (\tfrac{2}{3}R_1^2 + X_\omega^2)\Delta M_{1w}\omega + [\tfrac{2}{3}R_2^2 + (A - X_\omega)^2]\Delta M_{2w}\omega, \tag{27}$$

where X_ω is the distance between the primary and the center of mass of the system, ω is the angular velocity of the binary system. The terms $(\frac{2}{3}R_1^2 + X_\omega^2)\Delta M_{1w}$ and $[\frac{2}{3}R_2^2 + (A - X_\omega)^2]\Delta M_{2w}$ are the moments of inertia of the shells, and R_1 and R_2 are the radii of the primary and the secondary. The first and the second terms on the right side of Equation (27) correspond to the angular momentum carried away by the lost mass from the primary and the secondary, respectively.

6 THE ORBITAL EVOLUTION

The angular velocity of the system and the orbital separation between the two components vary due to a number of physical processes as the binary system evolves, including the loss of mass and angular momentum via stellar winds, the exchange of mass between the components, the loss of mass and angular momentum via the outer Lagrangian point and the changes in the moments of inertia of the components. According to Huang & Taam (1990), the changes in the angular velocity of the system and the orbital separation between the two components can be obtained as:

$$\frac{\Delta\omega}{\omega} = \frac{1}{2}\frac{\Delta M_1 + \Delta M_2}{M_1 + M_2} - \frac{3}{2}\frac{\Delta A}{A}, \tag{28}$$

$$\begin{aligned}\frac{\Delta A}{A} = \frac{1}{\frac{1}{2}-2\beta}\Big[&\frac{\Delta J}{J} - \frac{\Delta M_1}{M_1}(1-\beta) - \frac{\Delta M_2}{M_2}(1-\beta)\\ &- \frac{\Delta M_1+\Delta M_2}{M_1+M_2}(\beta - \tfrac{1}{2}) - \frac{\Delta I_1+\Delta I_2}{I_1+I_2}\beta\Big],\end{aligned} \tag{29}$$

where M_1, M_2 and I_1, I_2 are the masses and moments of inertia of the primary and the secondary, respectively.

The changes in the masses of the primary and secondary can be written as:

$$\Delta M_1 = \Delta M_{1c} + \Delta M_{1\omega} + \Delta M_{1L}, \tag{30}$$

$$\Delta M_2 = \Delta M_{2c} + \Delta M_{2\omega} + \Delta M_{2L}. \tag{31}$$

The several terms on the right hand side of Equations (30) and (31) correspond to the exchange of mass between the components (include the Roche lobe overflow in the semi-detached phase and the mass exchange in the contact phase), the mass losses via stellar winds, and the mass losses via the outer Lagrangian point, respectively.

In Equation (29), J is the total angular momentum of the system, which can be written as

$$J = J_0 + (I_1 + I_2)\,\omega, \tag{32}$$

where

$$J_0 = \frac{G^{1/2}A^{1/2}M_1M_2}{(M_1+M_2)^{1/2}}. \tag{33}$$

Here, J_0 is the orbital angular momentum of the system, $(I_1 + I_2)\,\omega$ is the rotational angular momentum, $\beta = \frac{(I_1+I_2)\omega}{J}$ is the ratio of rotational angular momentum to the total angular momentum of the system. The loss of angular momentum is given by

$$\Delta J = \Delta J_\omega + \Delta J_L, \tag{34}$$

where ΔJ_w and ΔJ_L are the angular momentum losses due to stellar winds and via the outer Lagrangian point.

7 COMPUTATIONS AND RESULTS

As an example, the evolution of a massive binary system consisting of a $12\,M_\odot$ and a $5\,M_\odot$ stars was computed with a modified stellar evolution code for the evolution of rotating binaries developed by Huang (2004). The code was updated to include mass and energy exchange between the components and loss of mass and angular momentum through the outer Lagrangian point during the contact phase. Both components of the system were treated simultaneously including the effect of mass loss due to stellar winds, the effect of convective overshooting, and the effects of rotation and mass transfer between the components. The evolution was followed from the zero age main sequence to the later stage after the contact phase. An initial chemical composition of $X = 0.70$, $Z = 0.02$ was adopted for both stars. The initial orbital separation between the components was taken to be $17.013R_\odot$, so that both stars filled their Roche lobes during the central hydrogen-burning phase of the primary. Owing to the short timescale of the contact phase, the losses of mass and angular momentum due to stellar winds can be neglected in this phase.

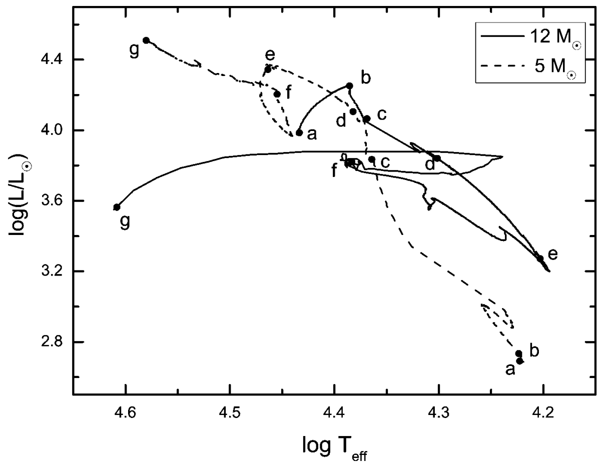

Fig. 3 Evolutionary tracks in the HR diagram of the 12 $M_\odot$ primary (solid curve) and the 5 $M_\odot$ secondary (dashed).

The evolutionary tracks of the primary and the secondary in the HR diagram are displayed in Figure 3, by the solid and dashed curves, respectively. The points a, b, c, d, e, f and g on the tracks represent, in order, the zero age main sequence, the beginning of the Roche lobe overflow, the beginning of the contact phase, the starting of the mass loss from the outer Lagrangian point, the end of the contact phase, the end of the central hydrogen-burning phase and the end of the calculations. Table 1 lists the ages, orbital periods, masses, luminosities, and effective temperatures of the primary and secondary, as well as the central hydrogen and helium content and the surface hydrogen content of the primary at different evolutionary points. From Table 1 it can be found that the binary system evolves first into a semi-detached system and begins the Roche lobe overflow (point b). After 1.55×10^5 yr, the system evolves into a contact binary and begins the mass transfer in the common envelope (point c). From point b to c, the primary transfers an amount of mass of $1.219 M_\odot$ to the secondary, hence an average mass transfer rate of $7.87 \times 10^{-6} M_\odot$ yr^{-1}. The contact phase (from point c to e) lasts 1.34×10^4 yr, during which the primary transfers continuously to the secondary a mass of $4.358 M_\odot$. Hence, for the contact phase, we have an average mass transfer rate of $3.25 \times 10^{-4} M_\odot$ yr^{-1}, which is from one to two orders of magnitude greater than in the semi-detached phase. At the end of the contact phase (point e), the binary changes back to a semi-detached system. Owing to the fact that the start and end of the contact stage fall within the semi-detached phase and the primary transfers continuously mass to the secondary, the contact stage can be regarded as a special part of the semi-detached phase characterized by a short time span, a large transfer of mass at a rate much greater than that through the Roche lobe overflow. Table 1 shows that the total mass of the system ($M_1 + M_2$) is decreased by $0.005 M_\odot$ during the contact phase (from point c to e). This decrease is due to the loss of mass via the outer Lagrangian point. From the ratio of the mass loss and the time interval of the contact phase, an average mass loss rate via the outer Lagrangian point of $3.73 \times 10^{-7} M_\odot$ yr^{-1} can be found, which of via the outer Lagrangian point is comparable with that of a red giant star due to stellar wind.

Figure 4 illustrates the time variation of the parameter IOVER. IOVER= 1, 2, 3 according as the binary is semi-detached ($\bar{\Psi}_1 > \Psi_{L1}$ and $\bar{\Psi}_2 < \Psi_{L1}$), in contact ($\bar{\Psi}_1 > \Psi_{L1}$ and $\bar{\Psi}_2 > \Psi_{L1}$), or starting to lose mass via the outer Lagrangian point ($\bar{\Psi}_1 > \Psi_{L1}$ and $\bar{\Psi}_2 > \Psi_{L2}$). Figure 4 shows that IOVER takes the value 3 several times during the middle part of the contact phase, and oscillates between 2 and 1 frequently during the later part. This shows that both the process of mass loss via the outer Lagrangian point and oscillation between contact and semi-contact states can occur in contact binaries.

Figure 5 displays the time variation of the total mass change of the primary during the contact phase. The total mass change ΔM_1 of the primary consists of the mass loss due to stellar winds ΔM_{1w}, the mass exchange between the components ΔM_{1c}, and the mass loss via the outer Lagrangian point. ΔM_1 is positive or negative according as mass flows from the primary to the secondary, or from the secondary to

Table 1 Model Parameter Values at the Different Evolutionary Points a, b, c, d, e, f and g, Marked in Fig. 3

Seq.	$t(10^7\,\mathrm{yr})$	P(d)	M_1	M_2	$\log L_1/L_\odot$	$\log L_2/L_\odot$	$\log T_{1\mathrm{eff}}$	$\log T_{2\mathrm{eff}}$	$X_1(c)$	$Y_1(c)$	X_1
a											
MOD	0.000000	1.870	12.000	5.000	3.981	2.679	4.435	4.218	0.70	0.28	0.7
b											
MOD	1.4292167	1.979	11.879	4.999	4.255	2.725	4.386	4.221	0.2606	0.7194	0.6999
c											
MOD	1.4447047	1.804	10.660	6.216	4.053	3.823	4.371	4.362	0.2524	0.7276	0.7000
d											
MOD	1.4449207	1.782	9.943	6.933	3.856	4.054	4.307	4.372	0.2524	0.7276	0.70
e											
MOD	1.4460467	2.042	6.302	10.569	3.236	4.369	4.198	4.461	0.2521	0.7279	0.6998
f											
MOD	2.3088047	1.982	4.341	12.530	3.807	4.219	4.389	4.453	0.00	0.98	0.4570
g											
MOD	2.34176	8.887	1.723	15.147	3.55	4.509	4.6100	4.578	0.00	0.98	0.1511

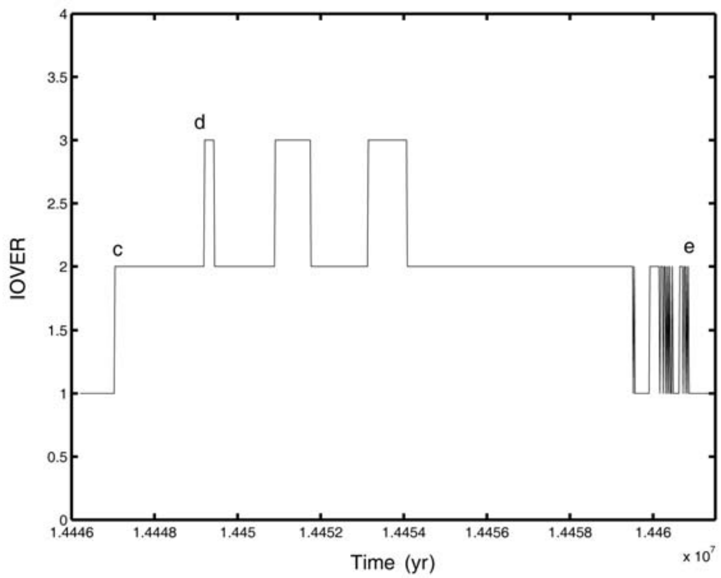

Fig. 4 Time variation of the parameter IOVER during the contact phase. The values of 1, 2 and 3 of the parameter IOVER correspond to the semi-detached state, the contact state and the start of mass loss via the outer Lagrangian point, respectively.

the primary. Figure 5 shows that ΔM_1 is negative (or mass flows from the secondary to the primary) on two occasions in the time interval from $t = 1.4446 \times 10^7$ to $t = 1.4450 \times 10^7$ yr.

Figure 6 illustrates the time variation of the total energy exchange during the contact phase. The total energy exchange ΔE includes the release in potential energy ΔE_P, in kinetic energy ΔE_K and in thermal energy ΔE_T. ΔE is positive or negative according as the energy goes from the primary to the secondary, or from the secondary to the primary. Figure 6 shows that ΔE is negative (energy goes from the secondary to the primary) on two occasions in the time interval between $t = 1.4446 \times 10^7$ and $t = 1.4450 \times 10^7$ yr. The figure also shows that the total energy transfer mainly consists of release of potential energy ΔE_P and kinetic energy ΔE_K, while the release of thermal energy ΔE_T is close to zero. This means that the effective temperatures of the primary and the secondary are almost equal.

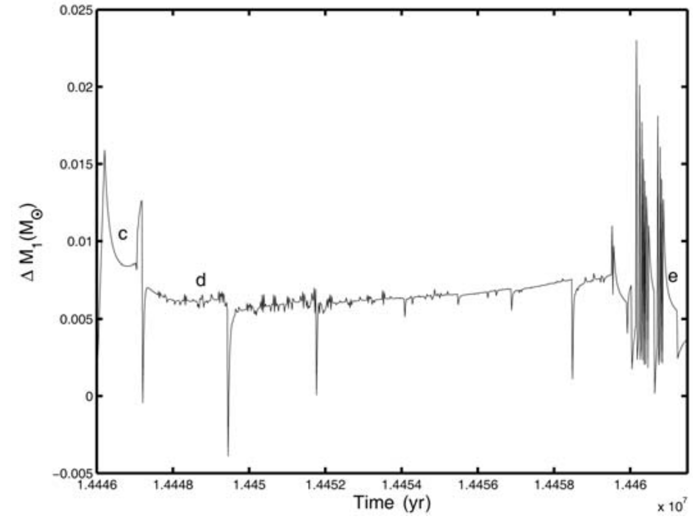

Fig. 5 Time variation of the mass flow during the contact phase.

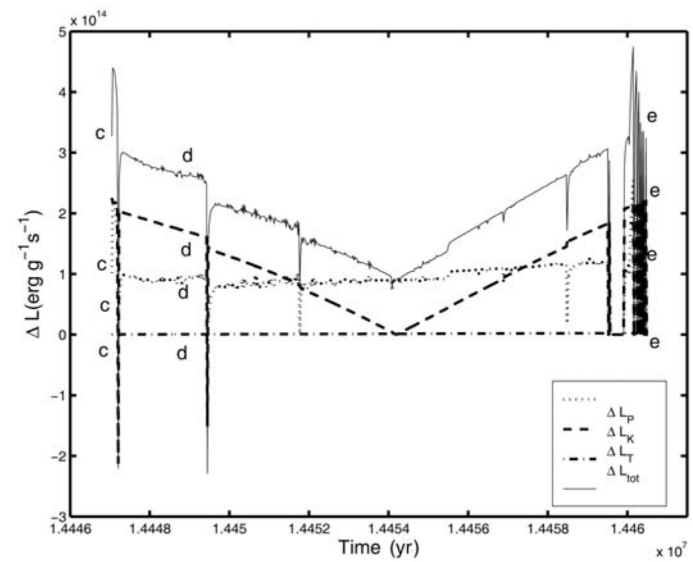

Fig. 6 Time variation of energy exchange during the contact phase. The solid, dotted, dashed and dotted-dashed curves correspond to the exchanges in the total, potential, kinetic and thermal energies, respectively.

8 SHORT CONCLUSIONS

For a massive contact binary system consisting of a $12\,M_\odot$ and a $5\,M_\odot$ star, the following results are found: 1) Owing to the fact that the start and end of the contact stage fall within the semi-detached phase and the primary transfers continuously mass to the secondary, the contact stage can be regarded as a special part of the semi-detached phase characterized by a short time interval, a large total amount of mass transfer and a rate of mass transfer much greater than that via the Roche lobe overflow. 2) A special feature of the contact system is the occurrence of mass and angular momentum loss via the outer Lagrangian point. It can occur intermittently and can affect the orbital period of the system significantly. Thus, it was found that some short variations could occur in the slow time variation of the period. The average rate of mass loss via the outer Lagrangian point is comparable with that of a red giant star due to stellar wind. 3) There exists vibration between contact and semi-contact states in the contact phase. 4) The effective temperatures of the primary and the secondary are almost equal.

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