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Velocity Curve Analysis of Spectroscopic Binary Stars AI Phe, GM Dra, HD 93917 and V502 Oph by Nonlinear Regression

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Abstract We introduce a new method to derive the orbital parameters of spectroscopic binary stars by nonlinear least squares of $(o - c)$. Using the measured radial velocity data of the four double lined spectroscopic binary systems, AI Phe, GM Dra, HD 93917 and V502 Oph, we derived both the orbital and combined spectroscopic elements of these systems. Our numerical results are in good agreement with the those obtained using the method of Lehmann-Filhés.

Key words: stars: binaries: eclipsing — stars: binaries: spectroscopic

1 INTRODUCTION

Determining the orbital elements of binary stars help us to obtain the necessary information such as the mass and the radius of the stars which play important roles in the evolution of the stars' structures. Analyzing both the light and radial velocity curves deducing from the photometric and spectroscopic observations, respectively, leads to derivation of the orbital parameters. One of the usual methods to analyze the velocity curve is the method of Lehmann-Filhés, see Smart (1990). Here we introduce a new method to derive these parameters based on a nonlinear regression of the $(o - c)$. We test our method on four double-lined spectroscopic binary systems, AI Phe, GM Dra, HD 93917 and V502 Oph.

AI Phe is a detached binary with P=24.6 days. The spectral type is F7V+K0IV and the angle of inclination is 88.44°. The mean effective temperature is 6310±150 K and 5010±120 K for the primary and secondary components, respectively (Andersen et al. 1988). GM Dra is a contact system of the W-type, with spectral type F5V and P=0.338741 days (Rucinski et al.2002). HD 93917 also belongs to the W-type of contact binaries, its spectral type is F9.5V and P=0.443420 days (Rucinski et al. 2003). V502 Oph is an eclipsing binary with P=0.453390 days. The spectral types of the components are G1V for the primary and F9V for the secondary (Pych et al. 2004).

This paper is organized as follows. In Section 2, we reduce the problem to solving an equation which is a nonlinear function of the orbital parameters. In Section 3, the nonlinear regression technique for estimating the orbital elements is discussed. In Section 4, the numerical results obtained for the four binary systems are reported. Section 5 is devoted to conclusions.

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2 FORMULATION OF THE PROBLEM

The radial velocity of a star in a binary system is defined as

$$\mathrm{RV} = V_{\mathrm{cm}} + \dot{Z}, \tag{1}$$

where V_{cm} is the radial velocity of the center of mass of system with respect to the Sun and

$$\dot{Z} = K[\cos(\theta + \omega) + e\cos\omega] \tag{2}$$

is the radial velocity of the star with reference to the center of mass of the binary, see Smart (1990). In Equation (2), the dot denotes the time derivative and θ, ω and e are the angular polar coordinate (true anomaly), the longitude of periastron and the eccentricity, respectively. Note that θ and ω are measured from the periastron point and the spectroscopic reference line (plane of sky), respectively. Also

$$K = \frac{2\pi}{P}\frac{a\sin i}{\sqrt{1-e^2}}, \tag{3}$$

where P is the period of motion and inclination i is the angle between the line of sight and the normal of the orbital plane. The photometric phase, ϕ, measured from the photometric reference point (line of sight), is a measurable quantity. Hence, one has to try express θ appearing in Equation (2) in terms of ϕ, but this is not so easy in practice, except the two following cases:

i) Following Smart (1990) and Rucinski (2002), the photometric (orbital) phase, ϕ, is generally related to the eccentric anomaly, ψ, according to Kepler's equation as

$$\psi - e\sin\psi = 2\pi\Big(\frac{t - T_0}{P}\Big) = 2\pi\phi, \tag{4}$$

where T_0 is moment of the primary eclipse. Both T_0 and P are usually taken from the published sources and are fixed. Also θ and ψ satisfy the following relation,

$$\tan\frac{\theta}{2} = \sqrt{\frac{1+e}{1-e}}\tan\frac{\psi}{2}. \tag{5}$$

To obtain ψ for a given ϕ, one may solve Equation (4) numerically. Then θ is derived from Equation (5), but e should be fixed beforehand. So we see that deriving θ in terms of ϕ depends on knowing the eccentricity. For a small eccentricity, $e < 1$, expanding Equations (4) and (5) reduces to the following relation,

$$\begin{aligned}\theta = 2\pi\phi \;&+\; \Big(2e - \frac{e^3}{4}\Big)\sin(2\pi\phi) + \frac{5}{4}e^2\sin(4\pi\phi) \\ &+\; \frac{13}{12}e^3\sin(6\pi\phi) + O(e^4).\end{aligned} \tag{6}$$

Equation (6) shows that when $e \ll 1$, then one can set $\theta = 2\pi\phi$ in Equation (2).

ii) According to Budding (1993), for most normal eclipsing binaries, the line of sight is inclined at a low angle to the orbital plane. This yields the following relationship among θ, ϕ and ω :

$$\theta = 2\pi\phi - \omega \pm \pi/2, \tag{7}$$

where $\pm$ refer to the primary and the secondary components, (hereafter indicated by subscripts p and s). Thus, Equations (1) and (2) for the two components reduce to

$$\begin{aligned} V_{r,p} &= V_{\mathrm{cm}} + K_p(e\cos\omega_p - \sin 2\pi\phi), \\ V_{r,s} &= V_{\mathrm{cm}} + K_s(e\cos\omega_s + \sin 2\pi\phi). \end{aligned} \tag{8}$$

For a nearly circular orbit (eccentricity much less than unity), neglecting the term $e\cos\omega$ in Equation (8) reduces to two simple sine curves. See equation (5) in Rucinski (2002).

To avoid the mentioned difficulties in obtaining θ in terms of ϕ, we try to remove it in our equations. To do this, first we take the time derivative of Equation (2),

$$\ddot{Z} = -K\sin(\theta+\omega)\dot{\theta}. \tag{9}$$

Then, using Kepler's second law and the relations obtaining for the orbital parameters in the inverse-square field,

$$\dot{\theta} = h/r^2, \tag{10}$$

$$h = \frac{2\pi}{P}a^2\sqrt{1-e^2}, \tag{11}$$

$$r = \frac{a(1-e^2)}{1+e\cos\theta}, \tag{12}$$

one may show that Equation (9) is reduced to

$$\ddot{Z} = \frac{-2\pi K}{P(1-e^2)^{3/2}}\sin(\theta+\omega)(1+e\cos\theta)^2, \tag{13}$$

where r, a and h are the radial polar coordinate, the semi major axis of the orbit and the angular momentum per unit of mass, respectively.

Using Equation (2) one can remove θ from Equation (13), thus,

$$\begin{aligned} P\ddot{Z} \;=\; & \frac{-2\pi K}{(1-e^2)^{3/2}}\sin\Big[\cos^{-1}\Big(\frac{\dot{Z}}{K}-e\cos\omega\Big)\Big] \\ \times\; & \Big\{1+e\cos\Big[-\omega+\cos^{-1}\Big(\frac{\dot{Z}}{K}-e\cos\omega\Big)\Big]\Big\}^2. \end{aligned} \tag{14}$$

To simplify the notation further, we let $Y = P\ddot{Z}$ and $X = \dot{Z}$. Then Equation (14) describes a nonlinear relation, $Y = Y(X, K, e, \omega)$, in terms of the orbital elements K, e and ω. Using the nonlinear regression of Equation (14), one can estimate the parameters $K,\ e$ and ω, simultaneously.

One may show that the adopted spectroscopic elements are related to the orbital parameters. First, according to definition of the center of mass, the mass ratio in the system is obtained as

$$\frac{m_p}{m_s} = \frac{a_s\sin i}{a_p\sin i}. \tag{15}$$

According to the Kepler's third law and Equation (15), the following relation

$$m_p\sin^3 i = a_s\sin i\left(\frac{a_p\sin i + a_s\sin i}{P}\right)^2, \tag{16}$$

is obtained, with a in AU, P in years and m in solar masses. A similar relation is obtained for the secondary component by simply exchanging p and s in Equation (16). Note that in Equations (15) and (16) parameter $a\sin i$ is related to the orbital parameters through Equation (3).

3 NONLINEAR LEAST SQUARES OF ($O-C$)

To obtain the orbital parameters K, e and ω in Equation (14), we use the nonlinear regression method. In this approach, the sum of squares of errors (SSE) for measured data of size N is calculated as

$$\mathrm{SSE} = \sum_{i=1}^{N}(Y_i-\hat{Y}_i)^2 = \sum_{i=1}^{N}[Y_i - Y(X_i, K, e, \omega)]^2, \tag{17}$$

where Y_i and $\hat{Y}_i$ are the real and the predicted values, respectively. To obtain the model parameters, the SSE should be minimized in terms of K, e and ω, with

$$\frac{\partial\mathrm{SSE}}{\partial K} = \frac{\partial\mathrm{SSE}}{\partial e} = \frac{\partial\mathrm{SSE}}{\partial\omega} = 0. \tag{18}$$

To solve Equation (18), we use the SAS (Statistical Analysis System) software. Note that nonlinear models are more difficult to specify and to estimate than linear models. For instance, in contrast to linear models, nonlinear models are very sensitive to the initial guesses for the parameters. Because in practice SSE may have to be minimized at several points in the three dimensional parametric space of K, e and ω. However, the final goal is to find the absolute minimum. Hence we choose the initial parameters that yield the absolute minimum of SSE that is also stationary. This means that if one changes the initial guesses slightly, then the result should reduce to the previous values of the parameters. While if SSE converged at the local minimum, the model would not be stationary, see Sen & Srivastava (1990) and Christensen (1996).

4 NUMERICAL RESULTS

Here we test our new method to derive both the orbital and combined elements for the four double lined spectroscopic systems, AI Phe, GM Dra, HD 93917 and V502 Oph. Using the measured radial velocity data of the two components of the systems obtained by Andersen et al. (1988) for AI Phe, Rucinski et al. (2002) for GM Dra, Rucinski et al. (2003) for HD 93917 and Pych et al. (2004) for V502 Oph, we obtained the fitted velocity curves (as functions of the photometric phase) in Figures 1a, 4a, 4a and 4a.

The radial acceleration values, $\ddot{Z}$ in Equation (14) are obtained by taking the time derivative of the adopted radial velocity curves. Figures 1b, 4b, 4b and 4b show the radial acceleration scaled by the period versus the radial velocity for the four systems. The solid closed curves are the results of the nonlinear regression of Equation (14), and they can be seen to agree closely with the measured data. The figures also show that for the small-eccentricity systems GM Dra, HD93917 and V502 Oph, their radial velocity-acceleration curves display an elliptical shape, while, in contrast, for the eccentric system AI Phe, the acceleration-velocity curve shows some deviation from an ellipse.

The orbital parameters, K, e and ω, resulting from the nonlinear least squares of Equation (14) for AI Phe, GM Dra, HD93917 and V502 Oph, are tabulated in Tables 1, 2, 3 and 4, respectively. The velocity of the center of mass, $V_{\rm cm}$, is obtained by calculating the areas above and below of the radial velocity curve. Where these areas become equal to each other, the velocity of center of mass is obtained. Tables 1, 2, 3 and 4 show that the results are in good agreement with the those obtained by Andersen et al. (1988) for AI Phe, Rucinski et al. (2002) for GM Dra, Rucinski et al. (2003) for HD 93917 and Pych et al. (2004) for V502 Oph.

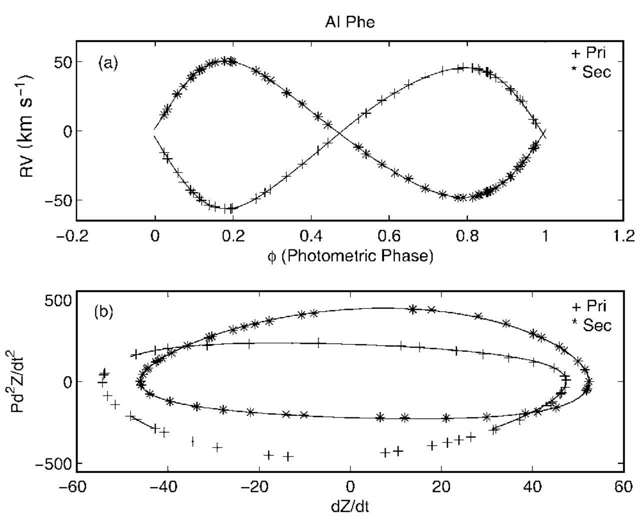

Fig. 1 (a): Radial velocities of the primary and the secondary components of AI Phe plotted against the photometric phase. The observational data are taken from Andersen et al. (1988). (b): Radial acceleration scaled by the period versus the radial velocity of the two components of AI Phe, from the nonlinear regression of Eq. (14). Plus signs for the primary, asterisks for the secondary.

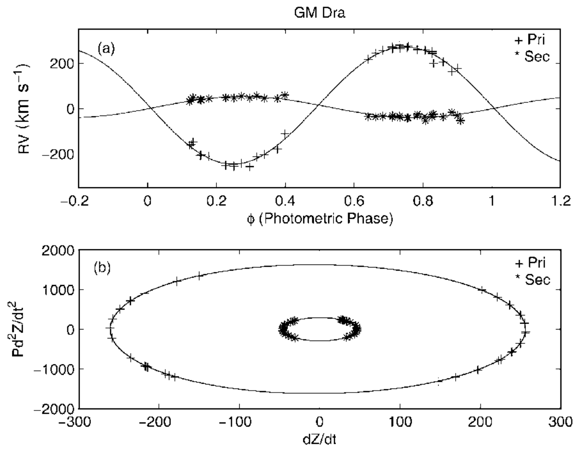

Fig. 2 Same as Fig. 1, for GM Dra. The observational data are from Rucinski et al. (2002).

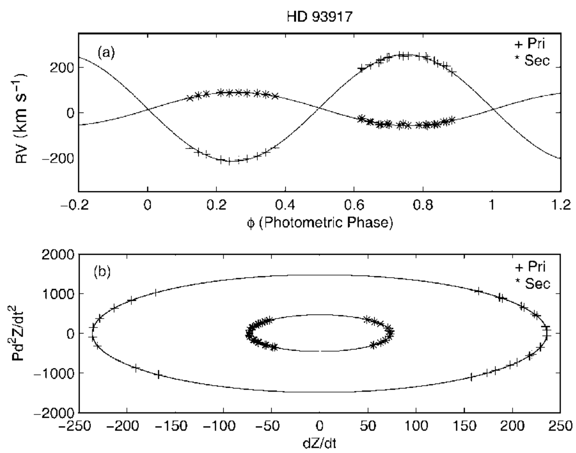

Fig. 3 Same as Fig. 1, for HD 93917. The observational data are from Rucinski et al. (2003).

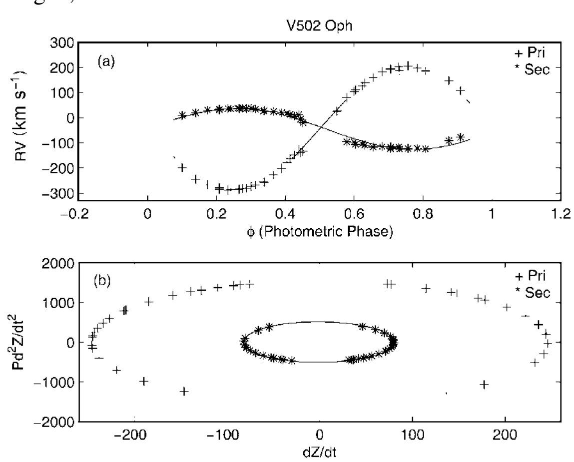

Fig. 4 Same as Fig. 1, for V502 Oph. The observational data are from Pych et al. (2004).

Table 1 Spectroscopic and Combined Orbit of AI Phe

	This Paper	Andersen et al. (1988) e,w (fixed)	Andersen et al. (1988) e,w (free)
Primary			
$V_{\rm cm}$(km s^{-1})	-1.83 ± 0.07	-1.76 ± 0.06	-1.76 ± 0.06
K_p(km s^{-1})	50.94 ± 0.01	50.9 ± 0.08	50.95 ± 0.08
e	0.186 ± 0.003	0.188 ± 0.002	0.1855 ± 0.0016
$\omega(^\circ)$	111.58 ± 0.08	109.9 ± 0.6	111 ± 0.5
Secondary			
$V_{\rm cm}$(km s^{-1})	-1.99 ± 0.08	-1.92 ± 0.06	-1.92 ± 0.06
K_s (km s^{-1})	49.27 ± 0.01	49.24 ± 0.07	49.20 ± 0.08
e	0.183 ± 0.003	$e_p = e_s$	0.1895 ± 0.0015
$\omega(^\circ)$	290.65 ± 0.41	$\omega_s = \omega_p + 180$	289.6 ± 0.4
$m_p \sin^3 i/M_\odot$	1.197 ± 0.004	1.194 ± 0.004	—
$m_s \sin^3 i/M_\odot$	1.237 ± 0.003	1.234 ± 0.005	—
$(a_p + a_s) \sin i/R_\odot$	47.86 ± 0.04	47.78 ± 0.05	—
m_p/m_s	1.033 ± 0.002	1.034 ± 0.002	—

Table 2 Same as Table 1, for GM Dra

	This Paper	Rucinski et al. (2002)
Primary		
$V_{\rm cm}$(km s^{-1})	10.55 ± 0.59	$9.12(1.63)$
K_p(km s^{-1})	258.67 ± 0.59	$258.72(2.66)$
e	0.008 ± 0.004	—
$\omega(^\circ)$	175.14 ± 0.13	—
Secondary		
$V_{\rm cm}$(km s^{-1})	10.55 ± 0.59	$9.12(1.63)$
K_s(km s^{-1})	46.59 ± 0.98	$46.69(1.74)$
e	0.003 ± 0.001	—
$\omega(^\circ)$	338.78 ± 2.33	—
$m_p \sin^3 i/M_\odot$	0.152 ± 0.005	—
$m_s \sin^3 i/M_\odot$	0.846 ± 0.011	—
$(a_p + a_s) \sin i/R_\odot$	2.043 ± 0.011	—
m_p/m_s	0.180 ± 0.004	$0.180(2)$
$(m_p + m_s) \sin^3 i/M_\odot$	0.998 ± 0.016	$1.002(42)$

Table 3 Same as Table 1, for HD 93917

	This Paper	Rucinski et al. (2003)
Primary		
$V_{\rm cm}$(km s^{-1})	17.49 ± 0.94	$17.47(0.79)$
K_p(km s^{-1})	236 ± 0.32	$237.30(1.32)$
e	0.003 ± 0.002	—
$\omega(^\circ)$	266 ± 0.04	—
Secondary		
$V_{\rm cm}$(km s^{-1})	17.49 ± 0.94	$17.47(0.79)$
K_s(km s^{-1})	73.58 ± 0.89	$74.33(0.91)$
e	0.002 ± 0.001	—
$\omega(^\circ)$	85.43 ± 0.01	—
$m_p \sin^3 i/M_\odot$	0.324 ± 0.007	—
$m_s \sin^3 i/M_\odot$	1.0392 ± 0.0095	—
$(a_p + a_s) \sin i/R_\odot$	2.71 ± 0.01	—
m_p/m_s	0.312 ± 0.004	$0.313(5)$
$(m_p + m_s) \sin^3 i/M_\odot$	1.363 ± 0.016	$1.394(30)$

Table 4 Same as Table 1, for V502 Oph

This Paper	Pych et al. (2004)
-44.88 ± 0.98	$-42.56(0.85)$
246.33 ± 0.94	$246.70(1.00)$
0.003 ± 0.002	—
85.25 ± 0.25	—
-44.88 ± 0.98	$-42.56(0.85)$
81.29 ± 0.31	$82.71(1.03)$
0.009 ± 0.004	—
209.84 ± 11.68	—
0.411 ± 0.005	—
1.242 ± 0.014	—
2.935 ± 0.011	—
0.330 ± 0.003	$0.335(9)$
1.652 ± 0.019	$1.679(22)$

The combined spectroscopic elements including $m_p \sin^3 i$, $m_s \sin^3 i$, $(a_p + a_s) \sin i$ and m_p/m_s are calculated by substituting the estimated parameters K, e and ω in Equations (3), (15) and (16). The results obtained for the four systems are tabulated in Tables 1, 2, 3 and 4, and show that our results are in good agreement with the those obtained by Andersen et al. (1988), Rucinski et al. (2002), Rucinski et al. (2003) and Pych et al. (2004) for AI Phe, GM Dra, HD 93917 and V502 Oph, respectively.

5 CONCLUSIONS

A new analytical method to derive the orbital elements of the spectroscopic binary stars is introduced, which is applicable to orbits of all eccentricities and inclination angles. This method takes considerably less time the Lehmann-Filhés method. It should be more accurate as the orbital elements are deduced from all points of the velocity curve, which is not the case with the Lehmann-Filhés method. The present method enables one to vary all three unknown parameters K, e and ω simultaneously instead only one or two at a time. It is possible to make adjustments in the elements before the final presentation. There are some cases, to which the Lehmann-Filhés method is inapplicable, and the present one may be found useful. One such case would be when the observations do not cover all phases. Another case in which this method is useful is when a star is attended by two dark companions with commensurable periods. In this case the resultant velocity curve may have several unequal maxima and the Lehmann-Filhés method will fail altogether.

Using the measured radial velocity data of AI Phe, GM Dra, HD 93917 and V502 Oph obtained by Andersen et al. (1988), Rucinski et al. (2002), Rucinski et al. (2003) and Pych et al. (2004), respectively, we derived the orbital elements of these systems by the above mentioned method. Our numerical calculations show that the results obtained for both the orbital elements and combined spectroscopic parameters are in good agreement with those obtained via the Lehmann-Filhés method. In a subsequent paper we intend to test numerically our method for other different systems.

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