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# A New Feature Vector Using Selected Line Spectra for Pulsar Signal Bispectrum Characteristic Analysis and Recognition *

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**Abstract** Average pulse profiles of pulsar signals are analyzed using the bispectrum technique. The result shows that there are nonlinear phase couplings between the two frequency axes of the bispectrum charts, which indicate nonlinear factors in the generation and propagation of pulsar signals. Bispectra can be used as feature vectors of pulsar signals because of their being translation invariant. A one-dimension selected line spectrum algorithm for extracting pulsar signal characteristic is proposed. Compared with selected bispectra, the proposed selected line spectra have the maximum interclass separability measurements from the point of view of the whole one-dimension feature vector. Recognition experiments on several pulsar signals received at several frequency bands are carried out. The result shows that the selected line spectrum algorithm is suitable for extracting pulsar signal characteristics and has a good classification performance.

**Key words:** pulsars: general—stars: magnetic fields—stars: fundamental parameters (classification)

## 1 INTRODUCTION

Pulsars are a kind of neutron stars with rapid rotation and strong magnetic field, and are regarded as one of the four greatest discoveries in the 1960s (Ni et al. 2000). The integrated pulse profiles of pulsar signals are stable, so we can extract average emission characteristics of the emission area from them and hence probe into the physics of the interiors of pulsars. There are many theories to interpret the electromagnetic emission features of pulsars, but none of them can perfectly interpret all the observational phenomena so far. Wu et al. (1998) decomposed the pulsar integrated profiles with the method of Gaussian fit separation, while Van Ommen, Kramer and other researchers used this method in a few cases (Xu et al. 2002). This method assumes that all the components are Gaussian distributed and the signal is a linear sum of these components; it takes no account of non-Gaussian elements and nonlinear phase coupling, hence much of the useful information may have remained hidden.

The integrated pulsar profiles are stable, meaning, for a given pulsar, the profile is approximately the same over a long time, with stable second-order and high-order cumulants. Now bispectra are blind to Gaussian processes, the Gaussian elements of the integrated profiles of pulsar signals are restrained and then the study on non-Gaussian elements can be made. Thus, by using the bispectrum technique, we can analyze the nonlinear phase coupling characteristics of the integrated profiles, which can be used to identify the system's nonlinearity. Additionally, bispectrum can restrain Gaussian noise as well as multiplicative noise and additive noise.

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By extracting the non-Gaussian and non-linear characteristics from integrated profiles of pulsar signals, we have acquired more information for pulsar recognition. Different from power spectra, bispectra have shift-invariant characteristic values and retain the amplitude and phase information except the linear phase information which is related to time shift (Nikias et al. 1993). There are many research papers on pattern recognition using the bispectrum. The leading methods are direct cross-correlation matched bispectra, integrated bispectra (Tugnait 1994; Chandran et al. 1993; Liao et al. 1998) and selected bispectra (Zhang et al. 2001). The method of direct cross-correlation matched bispectra require such huge amount of computation and storage that it is unsuitable for real-time systems. Computationally, the integrated bispectrum method is to sum up the power along a certain path, but some of the bispectrum points belong to ordinary bispectrum ones, which have little effect on target recognition. The integration can also lead to some loss or re-usage of bispectrum points. Additionally, any form of integrated bispectrum has its own disadvantages: e.g., radially integrated bispectra lose amplitude information, and axially integrated bispectra while retaining amplitude information, lose most of the phase information (Zhang 2002). In order to avoid such problems, Zhang et al. (2001) presented a method of selected bispectra. This method chooses the bispectrum points with maximum Fisher interclass separability measures when constructing the feature vectors, which avoids the ordinary bispectrum points and their cross terms. However, the selected bispectrum method only makes every single bispectrum point have the maximum Fisher interclass separability measurements, which means it does not take on the maximum cross-product distance in the whole one-dimension feature vector. In this paper we propose a one-dimension selected line spectrum method for feature extraction of pulsar signals based on the idea of the selected bispectrum technique. Compared with selected bispectra, one-dimension selected line spectra have the maximum interclass separability measurements from the point of view of the whole one-dimension feature vector. Identification experiments on several pulsar signals at several frequency bands have demonstrated the validity of this method.

## 2 BISPECTRUM AND ITS PROPERTY

Let $\{x(t)\}$ be a zero-mean random process, its spectrum is defined as

$$B_x(w_1, w_2) = \sum_{\tau_1=-\infty}^{\infty} \sum_{\tau_2=-\infty}^{\infty} C_{3x}(\tau_1, \tau_2) e^{-j(w_1\tau_1 + w_2\tau_2)}\,, \tag{1}$$

or

$$B_x(w_1, w_2) = \langle X(w_1)X(w_2)X^*(w_1 + w_2)\rangle\,, \tag{2}$$

in which

$$\mathrm{C}_{3\mathrm{x}}(m, n) = E\{x^*(k)x(k + m)x(k + n)\} \tag{3}$$

is the third-order cumulant of the signal$\{x(t)\}$, $X(w)$ denotes the Fourier transform of $x(k)$ and $^*$ represents complex conjugate. In Equation (2) if at least two of the frequency responses of the three signals $X(w_1)$, $X(w_2)$ and $X(w_1 + w_2)$ are correlated, then, $B_x(w_1, w_2) \neq 0$, which means that parts (or all) of the components at the frequency $w_1 + w_2$ are couplings of the components at frequencies $w_1$ and $w_2$ (Ji et al. 1999).

## 3 BISPECTRUM FEATURE ANALYSIS OF PULSAR SIGNALS

We have analyzed power spectra and bispectra of the average pulse profiles of pulsars B1451–68 and B2111+46 at several frequencies. Figures 1 and 2 show the power spectra and bispectra of PSR B1451–68's average pulse profiles at 450 MHz and 674 MHz respectively. Figures 3 and 4 show those of PSR B2111+46's average pulse profiles at 400 MHz and 800 MHz. The data of pulsar signals are from EPN (the European Pulsar Network Data Archive, see *w.jb.man.ac.uk/research/pulsar/resources/epn/browser.html*).

From these figures, we can see that the bispectra of signals at different frequency bands of a given pulsar are similar, while there are obvious differences in the bispectra of different pulsars. The finite zero-point offset between the two frequency axes of the bispectrum charts indicates phase couplings between the two frequency components and the stronger ones denote the main coupled components. These coupled components indicate that there are nonlinear factors in the generation and propagation of pulsar signals. Bispectra can be used as feature vectors of pulsar signals, which are shift invariant and retain the signals' scale and phase information. In contrast, we cannot tell whether some components are intrinsic or coupled from the power spectra which suppress the phase information of the signals.

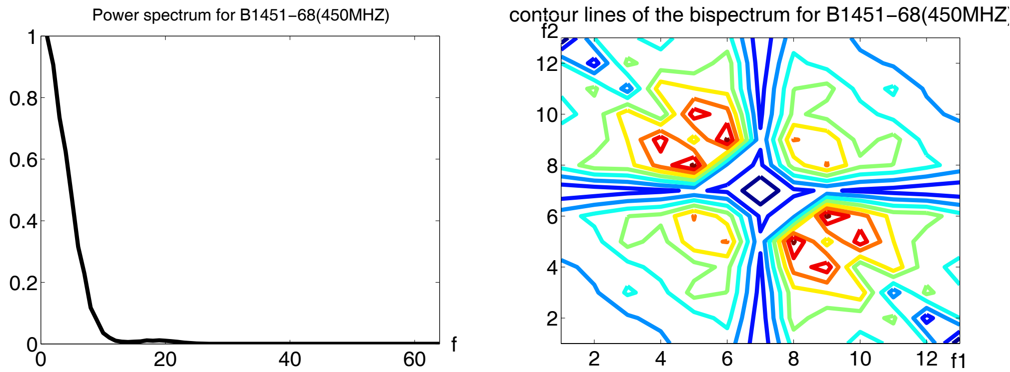


**Fig. 1** Power spectrum (left) and contour lines (right) of the bispectrum of PSR B1451–68 ’s average pulse profile (450 MHz).

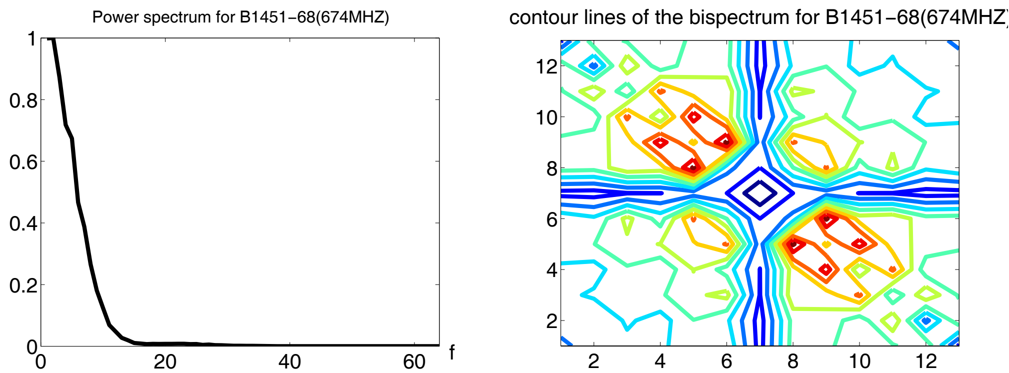


**Fig. 2** Power spectrum (left) and contour lines (right) of the bispectrum of PSR B1451–68 ’s average pulse profile (674 MHz).

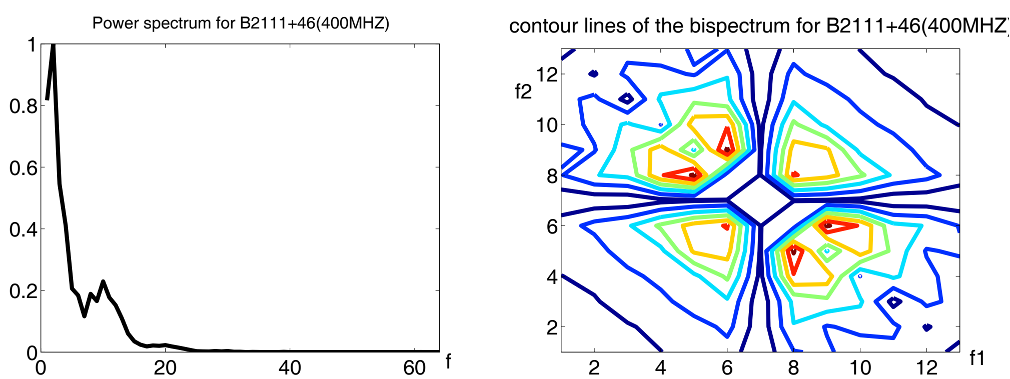


**Fig. 3** Power spectrum (left) and contour lines (right) of the bispectrum of PSR B2111+46’s average pulse profile (400 MHz).

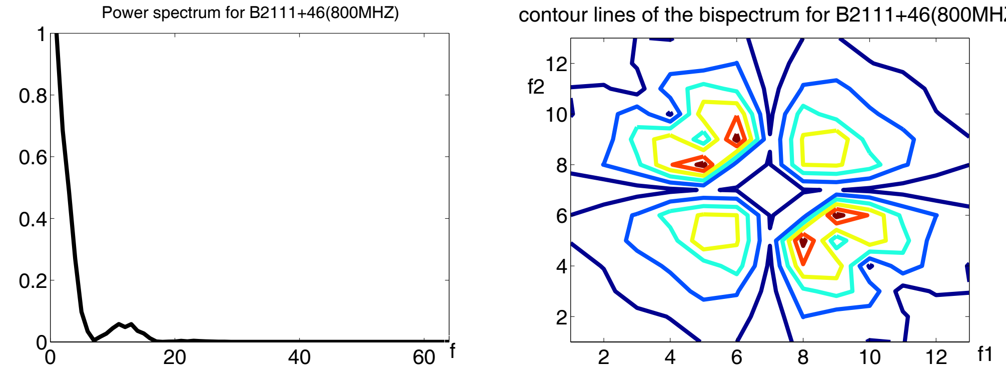


**Fig. 4** Power spectrum (left) and contour lines (right) of the bispectrum of PSR B2111+46's average pulse profile (800 MHz).

## 4 ONE-DIMENSION SELECTED LINE SPECTRUM FEATURE EXTRACTION OF PULSAR SIGNALS

In order to identify pulsar signal, we need to store the characteristic parameters as its template. In the process of classification, it is necessary to extract the features of tested signals, then cross-correlate them with features in the template. Finally, we choose the class with the largest similarity to be our judged result. There are so many pulsars and so many waveform changes induced by frequency changes that it is difficult to extract enough pulsar signal features by employing the integrated bispectrum technique. However, correlative match using bispectra directly involves complex computation, which restricts its application in real-time target recognition. In order to extract enough features of pulsar signals and decrease the computational complexity, we made use of feature core and domain to describe commonness and individuality of bispectra, based on which we define the variance of signal feature vector and the distance between two signal feature vectors. Based on these concepts we redefine the interclass separability measurement and extract the one-dimensional selected line spectrum feature vector with the maximum interclass separability measurement to compose the target template.

### 4.1 Discriminant Measurement

We write $w = (w_1, w_2)$ and $B(w) = B(w_1, w_2)$. Let $\{B_k^{(i)}(w)\}_{k=1,\cdots,N_i}$ and $\{B_k^{(j)}(w)\}_{k=1,\cdots,N_i}$ be the aggregates of bispectra obtained during the training session, here, the subscript $k$ stands for the number of observed datum from which the $k$th bispectra are worked out, the superscript $i$ stands for the number of the pulsar and $N_i$ stands for the signal length of the $i$th pulsar. The Fisher interclass separability measurement is defined as

$$m^{(i,j)}(w) = \frac{\sum\limits_{l=i,j} p^{(l)}[E_k(B_k^{(l)}(w)) - E_l(E_k(B_k^{(l)}(w)))]^2}{\sum\limits_{l=i,j} p^{(l)}\mathrm{var}_k(B_k^{(l)}(w))} \quad , i \neq j, \tag{4}$$

where $p^{(l)}$ is the prior probability of the random variable $B(l) = B_k^{(l)}(w)$, $E_k(B_k^{(l)}(w))$ and $\mathrm{var}_k(B_k^{(l)}(w))$ respectively stand for the mean and variance of the sampled bispectra of the $l$th pulsar at the frequency $w$, while $E_l(E_k(B_k^{(l)}(w)))$ stands for the centroid of the sampled bispectra of all the pulsars at the frequency $w$ (Zhang 2002).

In this paper, the distance between the $i$th pulsar and $j$th pulsar is defined through

$$d^{(i,j)} = \sqrt{\sum_{k=1}^{K} C_i(k) \times C_j^*(k)}\,. \tag{5}$$

The smaller $d^{(i,j)}$ is, the larger the distance between the $i$th and $j$th pulsar is (Ji et al. 1999). The variance of the $i$th pulsar is defined as

$$\mathrm{var}_i = \sum_{k=1}^{K} D_i(k) \times C_i^*(k). \quad (6)$$

The smaller $\mathrm{var}_i$ is, the larger the distance between the distribution $D_i(k)$ and the core $C_i(k)$ is. Suppose that the prior probabilities of all pulsars are equal, the interclass separability measurement can then be redefined as

$$m^{(i,j)} = \frac{\sum_{l=i,j} \mathrm{var}_l}{\sum_{k=1}^{K} C_i(k) \times C_i^*(k)} \;, i \neq j \;. \quad (7)$$

$$R^{(i,j)} = \frac{1}{m^{(i,j)}} \;. \quad (8)$$

Equation (7) indicates that the smaller $d^{(i,j)}$ is, the larger $\mathrm{var}_i$ and $m^{(i,j)}$ are, i.e., the interclass separability measurement between the $i$th pulsar and the $j$th one gets larger as $d^{(i,j)}$ gets smaller. $R^{(i,j)}$ is defined as the cross-correlation between the $i$th and $j$th pulsar. The larger $R^{(i,j)}$ is, the smaller the interclass separability measurement between the $i$th and $j$th pulsar is.

### 4.2 One-dimension Selected Line Spectra

Suppose the length of the one-dimension selected line spectra is $N$, and there are a total of $M$ pulsars. The selected line spectra of the $i$th pulsar are computed in the following steps:

(1) Calculate the bispectra of the $M$ pulsars. Based on certain prior knowledge, choose $K$ points from the bispectra to construct the test series.
(2) Choose $N$ points from the test series as the initial values of the one-dimension line spectra.
(3) Calculate the interclass separability measurements of the $N$ point line spectra between the $i$th pulsar and the others. Calculate the mean and variance of the interclass separability measurement.
(4) Renew the line spectra with the new points selected from the test series. Then repeat step (3).
(5) Determine the final line spectrum vector of the $i$th pulsar based on the principle of maximum-mean and minimum-variance.
(6) Repeat (2), (3), (4), (5) to calculate the one-dimension selected line spectrum vectors of other pulsars.

## 5 IDENTIFICATION EXPERIMENT

We carried out experiments for the identification of PSR B0355+54, B0531+21 and eight other pulsars using the method proposed above.

In Figure 5, the two left panels show the one-dimension selected line spectra of the average pulse profile of PSR B0531+21, the two right panels, the same of PSR B0613–0200. It can be seen from Figure 5 that the one-dimension selected line spectra perfectly reflect the difference between the two pulsars.

Table 1 shows the interclass separability measurements among the feature template vectors of PSR B0531+21, PSR B0613–0200 and the others. It can be seen from Table 1 that the integrated pulse profiles of the same pulsar have the minimum interclass separability measurement, while the interclass separability measurements between the profiles of different ones are larger, according to which we can identify pulsars.

In Figure 6, the cross-correlation measurements of the interclass feature vectors are compared, which are worked out respectively by using the technology of selected bispectra and that of selected line spectra proposed in this paper. In Figure 6, $x$-coordinate stands for the number of pulsars and $y$-coordinate stands for the cross-correlation measurement defined in Equation (8). The solid line represents the result obtained by employing the selected line spectrum technology, while the dotted line stands for the result obtained by using the selected bispectrum technology. For example, the left panel of Figure 6 shows the cross-correlation measurements of the feature vectors between PSR B0531+21 and the others. In this experiment, the number of PSR B0531+21 is 2, so in the left panel of Figure 6 the maximum appears where $x$-coordinate is 2. In the right panel of Figure 6 the maximum appears where $x$-coordinate is 3 which is the number of PSR B0613–0200. It can be seen from Figure 6 that on the compare of feature vectors of different pulsars, the smaller cross-correlation measurement can be obtained by using the selected line spectrum method put forward in this paper compared to the one obtained by using the selected bispectrum method.

**Table 1** Interclass Separability Measurements among One-dimension Line Spectrum Feature Template Vectors of PSR B0531+21, PSR B0613–0200 and Other Eight Pulsars

| Pulsars | B0531+21 (812 MHz) | B0613–0200 (610 MHz) | Pulsars | B0531+21 (812 MHz) | B0613–0200 (610 MHz) |
|---|---|---|---|---|---|
| (1) B0355+54(800 MHz) | 220.0955 | 6.6313 | (6) B1821–24(610 MHz) | 77.2216 | 36.1746 |
| (2) B0531+21(812 MHz) | 2.0000 | 175.5 | (7) B1919+10(1418 MHz) | 21.9840 | 8.3522 |
| (3) B0613–0200(610 MHz) | 38.2522 | 2.0000 | (8) B1929+10(610 MHz) | 17.6088 | 4.7065 |
| (4) J1751–3323(1374 MHz) | 10.2088 | 76.0164 | (9) B1937+21(1410 MHz) | 20.8110 | Infinite |
| (5) B1237+25(800 MHz) | 21.1135 | 7.7528 | (10) B2145–0750(610 MHz) | 13.0151 | 14.6344 |
| (11) J1713+0747(610 MHz) | 38.2980 | 17.8915 | | | |

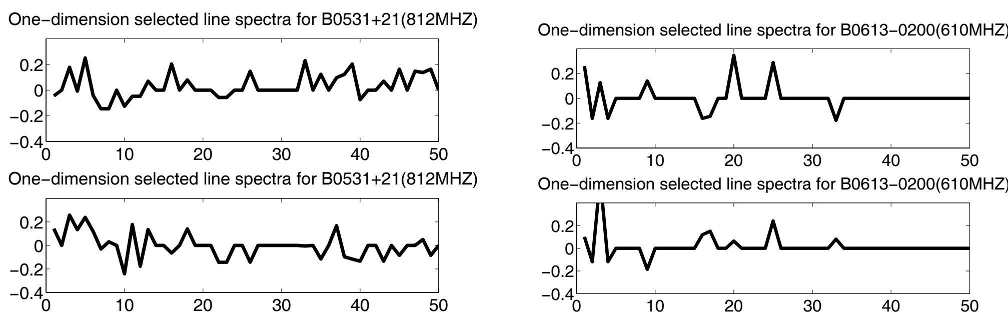


**Fig. 5** One-dimension selected line spectra for pulsar signals. Left: One-dimension selected line spectra for PSR B0531+21. The top is the real component, while the nether is the imaginary component. Right: One-dimension selected line spectra for PSR B0613–0200. The top is the real component, while the nether is the imaginary component.

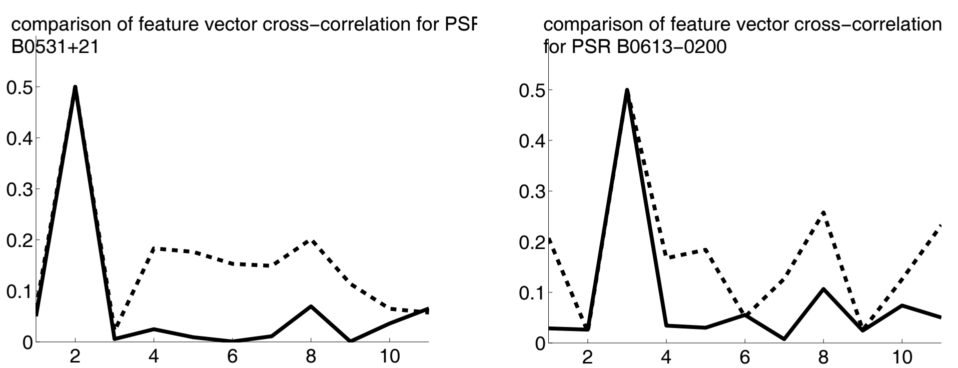


**Fig. 6** Left: comparison of feature vector cross-correlation for PSR B0531+21. Right: comparison of feature vector cross-correlation for PSR B0613–0200 (the solid lines stand for the results obtained with the method of selected line spectra; the dotted lines, those obtained with the method of selected bispectra).

## 6 CONCLUSIONS

In this paper average pulse profiles of pulsar signals are analyzed with the bispectrum technique. From the study on the integrated profiles of several pulsars of several frequency bands, it can be seen that there are nonlinear phase couplings between the two frequency axes of the bispectra, so there are nonlinear factors in the generation and propagation of the pulsar signals. A one-dimension selected line spectrum method for bispectrum characteristic extraction of pulsar signals is put forward. The results of the experiments show that one-dimension feature vectors so extracted have a good classification performance. Furthermore, from the point of view of the whole one-dimension feature vector, a smaller cross-correlation is obtained with the selected line spectrum method proposed in this paper than with the selected bispectrum method.

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