Velocity Distance of the Open Cluster M11 *

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Abstract On the reasonable hypothesis that the internal motions of member stars of a cluster are random and isotropic, a method which can be used to estimate the velocity distance of the cluster and its uncertainty is developed. The velocity distance so determined is an absolute distance estimate, and is completely independent of the (widely used) luminosity distance, which is a relative distance estimate. Using the published high-accuracy observational data of radial velocities and proper motions of the stars in the open cluster M11 region, we have determined the distance of M11 to be 1.89 ± 0.52 kpc. This is in quite good agreement with the published luminosity distances of the cluster. We briefly discuss the problems concerned, including the sources of errors in the method and its applicable range.

Key words: open cluster—radial velocity—proper motion —luminosity distance—velocity distance

1 INTRODUCTION

The open cluster M11 (NGC6705) is in the constellation Scutum near the Galactic center ($\alpha(2000) = 18^{\rm h}51^{\rm m}05^{\rm s}, \delta(2000) = -6°16'01''$; $l = 27.30°, b = -2.77°$). It is an intermediate-age open cluster with rich members and a total mass of some $11\,000\ M_\odot$ (Santos Jr. et al. 2005). Because of the important role of M11 in the studies of stellar evolution and dynamical evolution of clusters, quite a number of researches on various respects of the cluster have been done and a large amount of observational data have been accumulated in recent years, and its membership and some fundamental physical parameters, including size, mass, luminosity, age, metallicity, reddening and distance, have been determined (McNamara & Sanders 1977; McNamara et al. 1977; Mathieu 1984; Lee et al. 1989; Kjeldsen & Frandsen 1991; Su et al. 1998; Sung et al. 1999; Hargis, Sandquist & Bradstreet 2005).

The distance of an object is of great astrophysical importance, since many of the basic physical properties cannot be quantitatively determined unless the distance is known. In general, there are two approaches for determining the distances of objects like stars or star clusters: absolute and relative distance measurements. In the absolute distance measurement approach, such as the geometric distance obtained from trigonometric parallax of stars or that given by the moving cluster method (Trumpler & Weaver 1953; Hanson 1975), the object's distance is directly estimated without calibration involving some known distances of other objects. On the other hand, such a calibration is necessary when we attempt to derive absolute distance estimates from the relative distance measurements (such as the luminosity distances) based on the period-luminosity relation of Cepheid variables, or on using SN Ia as standard candles (Jacoby et al. 1992; Branch & Tammann 1992).

A variety of methods can be used for measuring the absolute distances of particular classes of objects to interpret their observed data on a straightforward physical basis provided these objects possess unusually simple geometries. Therefore, the number of objects to which the absolute distance measurement approach

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is applicable is relatively limited. On the other hand, the relative distance measurement approach is based on the fact that many objects do seem to follow some simple empirical relations between their intrinsic properties, one of which does vary with the distance. Once calibrated on to an absolute scale, the relative approach holds a number of advantages over the absolute methods, one of which is that it can be used for many more objects. However, errors in the distance estimates caused by uncertainties in the calibration increases with increasing distance (Binney & Merrifield 1998).

Anyway, because of the importance of distance measurements in the determination of many of the absolute properties of astronomical objects and phenomena, astronomers have developed, and improved on different distance measurement methods both observationally and theoretically. The practice has emerged of measuring the distance of an object by several different techniques so as to reduce the uncertainty in the final adopted value as much as possible (Binney & Merrifield 1998). For examples, there have been the Baade-Wesselink method (Baade 1926; Wesselink 1946; Gautschy 1987), the time delay of supernovae (Panagia et al. 1991; Jakobsen et al. 1991) and water-maser proper motions (Miyoshi et al. 1995) in the absolute distance measurement approach, luminosity functions of globular clusters (Jacobin et al. 1992) and planetary nebulae (Ciardullo et al. 1989), and the Tully-Fisher relation (Tully & Fisher 1977; Pierce & Tully 1992) in the relative distance measurement approach.

Usually, one can use features appearing in the CM diagram to measure the distances of clusters in the Galaxy by means of fitting to the main sequence, to obtain relative distance of a kind called luminosity distance. In this paper we use the purely kinematical observational data (radial velocities and proper motions) of the members of the cluster M11 to estimate what might be called the velocity distance of M11. This approach is completely different from that used for finding the luminosity distance of a cluster.

2 THE PRINCIPLE OF MEASURING THE VELOCITY DISTANCE OF CLUSTER

Let the linear velocity of an object S be V and the corresponding angular velocity be ω. We have

$$V = r\omega, \tag{1}$$

where r is the distance to S. Thus we have

$$r = V/\omega. \tag{2}$$

The distance found from Equation (1) may be called the velocity distance of the object, which is in some respects like the distance of a cluster determined by the moving cluster method.

The velocity data of an object which can be obtained from astronomical observations are the (linear) radial velocity V_r and the (angular) proper motion $\mu \equiv (\mu_\alpha \cos \delta, \mu_\delta)$. Now, in general, the object moves in a random direction in space, so its radial velocity V_r does not correspond to its total proper motion μ or either of its components $\mu_\alpha \cos \delta$ or μ_δ, and so Equation (2) cannot be used to determine the velocity distance of the object. Let V be the speed of the spatial movement of the object and θ the angle between the direction of the spatial movement and that of the line-of-sight of the observer, we have

$$V_r = V \cos \theta, \ \mu = [(\mu_\alpha \cos \delta)^2 + \mu_\delta^2)]^{1/2} = V \sin \theta / kr, \tag{3}$$

and

$$V_r = kr\mu \tan \theta, \tag{4}$$

where $k = 4.74$ is the factor of units conversion. Essentially, Equation (4) is a form of Equation (1) for an object with radial velocity and proper motions available. Except for the particular cases such as moving clusters, one can not find the angle θ observationally, hence cannot find its distance from Equation (4).

Individual members of a cluster must be in random movements in the cluster, in addition to participating in the movement of the cluster as a whole, and hence there must be a dispersion in the observed velocities of the cluster members. We can reasonably suppose (or as a first approximation) that the internal random movements of the members are isotropic, and follow a three-dimension normal distribution with a certain variance (a spherical distribution).

Now let σ_V and $(\sigma_\alpha, \sigma_\delta)$ be the intrinsic velocity dispersions in the radial velocities and the two components of proper motions of the members. Then, from the above hypothesis we have

$$\sigma_V = kr\sigma_\alpha = kr\sigma_\delta. \tag{5}$$

Hence the velocity distance of the cluster is

$$r = \frac{\sigma_V}{k\sqrt{(\sigma_\alpha^2 + \sigma_\delta^2)/2}} = \frac{\sigma_V}{k\sigma_\mu}. \tag{6}$$

where $\sigma_\mu = \sqrt{(\sigma_\alpha^2 + \sigma_\delta^2)/2}$. Actually, the basic principle of measuring the velocity distance of a star cluster mentioned above is the same as what has been used to determine the distance of water-masers from their observed radial velocities and proper motions by the VLBI technique (Miyoshi et al. 1995). We note that the radial velocities and proper motions have different observing errors, which moreover vary from one individual member to the next, so here σ does not mean the observed dispersions of the members, but, should rather mean their intrinsic velocity dispersions. The latter can be found in two ways: (i) The intrinsic dispersions of the proper motions are taken as two distribution parameters and directly found in the membership determination of the cluster that includes an estimation of the dispersions (Zhao & He 1990). (ii) By correcting the observed dispersions for the observing errors in the radial velocities and proper motions (Jones 1970; McNamara & Sekiguchi 1986).

Taking radial velocities as an example, let V_i be the observed radial velocity of the ith member, the observed dispersion of radial velocities of all n members can be calculated as follows:

$$\sigma_V'^2 = \frac{1}{n-1}\sum_{i=1}^{n}(V_i - \bar{V})^2, \tag{7}$$

where $\bar{V} = \sum_1^n V_i/n$, and corresponding intrinsic σ_V is

$$\sigma_V^2 = \sigma_V'^2 - \frac{1}{n}\sum_{i=1}^{n}\varsigma_i^2, \tag{8}$$

where ς_i is the mean error of the observed radial velocity of the ith member. The mean error $\varepsilon(\sigma_V)$ of σ_V can be estimated as follows:

$$\varepsilon^2(\sigma_V) = \frac{1}{4\sigma_V^2}[\varepsilon^2(\sigma_V'^2) + \varepsilon^2(\sigma_m^2)], \tag{9}$$

where

$$\varepsilon^2(\sigma_V'^2) = \frac{2\sigma_V'^4}{n}, \tag{10}$$

and

$$\varepsilon^2(\sigma_m^2) = \frac{2}{n^2}\sum_{i=1}^{n}\varsigma_i^4. \tag{11}$$

As far as the two components of proper motions are concerned, we can obtain the corresponding intrinsic dispersions from their observed ones in a similar way to the above.

3 OBSERVATIONAL DATA

There are few clusters like M11, that have enough member stars with sufficiently accurate radial velocity and proper motion data. This is the main reason that M11 has been chosen as the first target for a determination of the velocity distance of a cluster.

Mathieu et al. (1986) published accurate radial velocities of 39 stars in the M11 region, including their mean errors, as well as their equatorial coordinates, apparent V magnitudes and color indices $B - V$. In

this sample the V magnitudes are all brighter than 12.8 mag and, except for a few stars, the radial velocities all have observed accuracies better than $\pm\,1\,\mathrm{km\,s^{-1}}$. After cross identification, 27 of the stars are found to have proper motion membership probabilities higher than 0.7, and 25, higher than 0.95 (McNamara et al. 1977). The V magnitude range of these member stars is $10.98-12.05$ mag.

Using 10 plate-pairs with time baselines of $16-70$ years, taken by the 40 cm Astrograph at the Shanghai Astronomical Observatory, Su et al. (1998) measured th proper motions of 872 stars in the M11 region. The apparent B magnitude range of these stars is $9.3-16.4$ mag, and the mean accuracy of the observed proper motions is $\pm\,1.1\,\mathrm{mas\,yr^{-1}}$. It is found from the membership determination that there are 541 stars with probabilities higher than 0.7, which can be taken as the sample of member stars. Instead of the observed dispersions of the proper motions, the intrinsic dispersions are taken as two distribution parameters in their membership determination, which can be directly used to estimate the velocity distance of the cluster. Moreover, Su et al. (1998) divided all the stars into different B magnitude groups, and then solved the distribution parameters including the intrinsic proper motion dispersions separately for each group in order to discuss possible velocity mass segregation effect within M11.

4 RESULT AND DISCUSSION

4.1 The Velocity Distance of the Open Cluster M11

In view of the observational data of M11 we can use to estimate the velocity distance of the cluster we should consider well the following three facts: (i) There exists a possible velocity mass segregation effect in the cluster, which means stars of different magnitudes probably have different intrinsic dispersions. (ii) The stars with radial velocities available and those with proper motions available have different magnitudes: the former are mostly brighter ones, with V magnitudes in the range 10.98–12.05 mag (Mathieu et al. 1986). In contrast, the stars with proper motions available are mostly fainter ones, with V magnitudes in the range 10.26–14.80 mag (Su et al. 1998). (iii) Su et al. (1998) have presented the average intrinsic dispersion σ_μ of the two components of proper motions (in right ascension and declination) for stars of different B (rather than V) magnitude groups.

In the sample published by Mathieu et al. (1986), there are 27 members of M11 with radial velocities and V magnitudes available, of which 24 (or some 89%) also have $B-V$ indices given. The average B magnitude of these 24 members is 13.18 ± 0.19 mag, which can be reasonably used as the average B magnitude of all the members with radial velocities available. Using Equations (7)–(11), the intrinsic radial velocity dispersion σ_V and the corresponding error $\varepsilon(\sigma_V)$ for the 27 members are:

$$\sigma_V = 1.25 \pm 0.21\,\mathrm{km\,s^{-1}}. \tag{12}$$

On the other hand, according to the data listed in table 6 of Su et al. (1998), the average B magnitude in the $B = 13.0 - 13.4$ group is 13.22 ± 0.11 mag and the corresponding σ_μ and $\varepsilon(\sigma_\mu)$ are:

$$\sigma_\mu = 0.14 \pm 0.03\,\mathrm{mas\,yr^{-1}}. \tag{13}$$

Because the average magnitudes of stars in the two samples used for deriving Equations (12) and (13) are almost the same, the values of Equations (12) and (13) can be substituted into Equation (6) and the velocity distance of M11 is then estimated to be

$$r = 1.89 \pm 0.52\,\mathrm{kpc}. \tag{14}$$

4.2 Accuracy Analysis

In the result given by Equation (14), the mean error $\varepsilon(r) = \pm 0.52\,\mathrm{kpc}$ of the velocity distance is estimated from the following equation (easily derived from Equation (6)):

$$\varepsilon^2(r) = \frac{\varepsilon^2(\sigma_V)}{(k\sigma_\mu)^2} + \frac{r^2\varepsilon^2(\sigma_\mu)}{\sigma_\mu^2}, \tag{15}$$

in which the first term on the right is due to the mean error $\varepsilon(\sigma_V)$ of the intrinsic radial velocity dispersion,

$$\varepsilon_V(r) = \frac{\varepsilon(\sigma_V)}{k\sigma_\mu}, \tag{16}$$

and the second one is due to the mean error $\varepsilon(\sigma_\mu)$ of the intrinsic proper motion dispersion,

$$\varepsilon_\mu(r) = \frac{r\varepsilon(\sigma_\mu)}{\sigma_\mu}. \tag{17}$$

It can be seen from Equation (15) that the accuracy $\varepsilon(r)$ of the velocity distance is dependent on the following three factors:

(i) The accuracies $\varepsilon(\sigma_V)$ and $\varepsilon(\sigma_\mu)$ of the intrinsic radial velocity dispersion and the intrinsic proper motion dispersion: $\varepsilon(r)$ improve with decreasing $\varepsilon(\sigma_V)$ and/or decreasing $\varepsilon(\sigma_\mu)$.

(ii) The intrinsic proper motion dispersion σ_μ, $\varepsilon(r)$ decrease with increasing σ_μ. This is easy to see from Equation (16), and the accuracy of r is independent of the intrinsic radial velocity dispersion σ_V.

(iii) The accuracy of the velocity distance derived from Equation (6) will get worse as the distance gets larger: this can be seen from Equation (17).

For M11, we have

$$\varepsilon_V(r) = \pm 0.32 \text{ kpc}, \ \varepsilon_\mu(r) = \pm 0.41 \text{ kpc}. \tag{18}$$

Thus, the contributions of $\varepsilon(\sigma_V)$ and $\varepsilon(\sigma_\mu)$ to the mean error $\varepsilon(r)$ of the velocity distance, estimated from the observational data, are not significantly different.

4.3 Comparison

Table 1 lists the main results of distance determination of the cluster M11 given by different authors, where the mean errors of distances with an asterisk are lower limits, and where some authors did not give estimates of errors in their papers.

Table 1 Distances of M11 Obtained by Different Authors

Year	Method	Distance (kpc)	Reference
1956	MS fitting	1.66±0.23	Johnson et al. (1956)
1970		1.70	Hagen (1970)
1971	MS fitting	1.70±0.17*	Backer et al. (1971)
1978		2.09	Harris et al. (1978)
1980	MS fitting	1.90±0.15*	Solomon et al. (1980)
1985	two-color diagram	1.81	Cameron (1985)
1989	MS fitting	2.00	A-Twarog et al. (1989)
1991	MS fitting	2.14±0.20	Kjeldsen et al. (1991)
1996		1.88	Mermilliod (1996)
1999	MS fitting	2.04±0.10	Sung et al. (1999)
2006	velocity distance	1.89±0.52	This paper

All the results listed in Table 1 except ours are luminosity distances derived from features in the CM diagrams and/or two-color diagrams, with different authors using different features as the distance indicator, using different methods to determine the color excess and the extinction, and adopting different values when correcting the apparent distance modulus for interstellar extinction to obtain the intrinsic distance modulus.

It can be seen from Table 1 that the velocity distance of M11 derived by us is in quite good agreement with the luminosity distances given by the other authors, the average of these luminosity distances being 1.89 kpc, with a mean error smaller than ours. Thus, it has been demonstrated that our method can be used to derive the velocity distance from the data of radial velocities and proper motions, to obtain a reasonable result, at least in the case of the cluster M11.

4.4 Discussion

It should be pointed out that, so far as distance determination is concerned, the luminosity distance is a relative distance determination, the fundamental principle of which is that the apparent distance modulus $m - M$ of the object is derived from its observed apparent magnitude m and the absolute magnitude M is found from some features in the CM diagram, and then the intrinsic distance modulus $(m - M)_0$ of the object is derived after an extinction correction. Finally, the luminosity distance r of the object is

estimated from the formula $(m - M)_0 = 5 \log r - 5$. As Binney & Merrifield (1998) pointed out, the relative distance estimators are based on simple empirical relations, which must be calibrated by measuring at least one absolute distance using an absolute distance estimator before the true distance of the object can be determined. Therefore, the distance of the object derived from this approach must contain errors due to uncertainties of the calibration.

On the other hand, the velocity distance of a cluster we developed in this paper is an absolute distance estimation, the basic principle of which is completely different from that of measuring the luminosity distance. The velocity distance of a cluster can be determined without any calibrations, which is a distance estimate of the cluster completely independent of its luminosity distance, and can be used to compare with the latter. However, the absolute distance estimators must be founded on simple and reasonable geometries. If such geometries are not rigorously satisfied in reality, then the absolute distance estimates must suffer from potential systematic errors (Binney & Merrifield 1998). The basic hypothesis, from which Equation (6) is derived and used to estimate the velocity distance of a cluster, is that the internal random movements of the members of the cluster are isotropic and their observed velocities follow the Maxwellian distribution. Obviously, even if the members of the cluster participate in some global motion of the cluster such as a rotation or an expansion (contraction), the above hypothesis can still hold, at least as a first approximation, provided that the members with radial velocities or proper motions available are randomly distributed throughout the cluster, and not concentrated in some small area.

Furthermore, it can be seen from Equation (17) that the contribution of the error $\varepsilon(\sigma_\mu)$ of the intrinsic proper motion dispersion to $\varepsilon_\mu(r)$ increases with increasing cluster distance. Since the observed accuracies of proper motions can not be improved indefinitely and the sample of cluster members is more or less limited, the applicable range of Equation (6) used for the estimation of the velocity distance is also limited. Considering the accuracy of observational data presently attainable, the range seems to be some 2 kpc in general, and the errors of distance determination will become more or less unacceptable at distances $r \geq 2$ kpc.

It can be expected that when more data with higher accuracies of radial velocities and especially proper motions of members of a cluster can be obtained observationally, the method we suggested to estimate the velocity distance of the cluster can be used to provide absolute estimates of distances of clusters in the Galaxy, that are completely independent of their luminosity distances and the accuracies of the velocity distance, especially for clusters with distances $r \leq 1$ kpc, would be satisfactorily high.

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