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Low Dimensional Chaos from the Group Sunspot Numbers *

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Abstract We examine the nonlinear dynamical properties of the monthly smoothed group sunspot number R_g and find that the solar activity underlying the time series of R_g is globally governed by a low-dimensional chaotic attractor. This finding is consistent with the nonlinear study results of the monthly Wolf sunspot numbers. We estimate the maximal Lyaponuv exponent (MLE) for the R_g series to be positive and to equal approximately 0.0187 ± 0.0023 (month^{-1}). Thus, the Lyaponuv time or predictability time of the chaotic motion is obtained to be about 4.46 ± 0.5 years, which is slightly different with the predictability time obtained from R_z. However, they both indicate that solar activity forecast should be done only for a short to medium term due to the intrinsic complexity of the time behavior concerned.

Key words: Sun: activity – Sun: sunspot – chaos – Sun: Wolf sunspot numbers

1 INTRODUCTION

It is well known that the Zürich or Wolf sunspot numbers (R_z) are the single, most important index of solar activity and have served as the primary time series that defines solar activity for more than 100 years, especially as related to terrestrial climate (Eddy 1980; Hoyt & Schatten 1997, 1998a, b; Wilson 1998; Hathaway et al. 2002). On the basis of R_z, many kinds of properties of solar activity, including nonlinear underlying dynamics properties such as chaos and fractal, have been extensively revealed (Kremlevskii et al. 1992; Rozelot 1995; Zhang 1994, 1995; Letellier et al. 2006;). A very important conclusion about chaos is obtained, namely, the behavior of solar activity is governed by a chaotic attractor of low dimension and the attractor's dimension (D_2) is around 3, e.g., $D_2 \approx 2.3$ (Mundt 1991) , $D_2 = 2.4 \pm 0.2$ (Kremliovsky 1994), $D_2 \approx 3$ (Rozelot 1995; Lettllier et al. 2006), and so on. However, the Wolf sunspot numbers may be inaccurate or may have gaps in some periods, which may result in unconvincing analysis.

A new parameter, designed to be an alternative to the Wolf sunspot numbers, was proposed by Hoyt & Schatten (1998a, b): it is defined as $R_g = \frac{12.08}{N} \sum k_i^{'} g_i$, where g_i is the number of sunspot groups recorded by the ith observer, $k_i^{'}$ is ith observer's correction factor, N is the number of observers used to form the daily value, and 12.08 is a normalization number chosen to produce an activity index that closely mimics R_z. Hoyt & Schatten (1998a, b) succeeded in collecting many observations missed by R_z and in improving the quality of the raw data for some observers. They pointed out that the Wolf sunspot numbers contain many inhomogeneities arising from observing noise, and this noise affects the daily, monthly and yearly means. The group sunspot numbers also have observing noise, but it is considerably less than that in the Wolf sunspot numbers. For example, solar activity before 1882 is lower than is generally assumed and consequently solar activity in the last few decades is higher than it has been for several centuries (Hoyt & Schatten 1998a, b). Thus, R_g is better as regards internal self-consistency and noise than R_z, and describes

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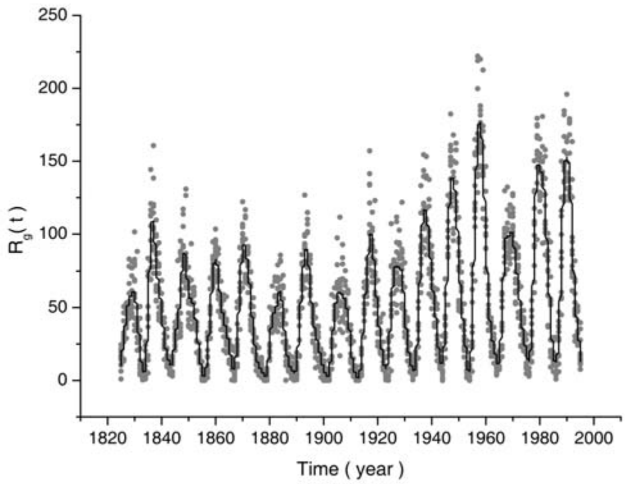

Fig. 1 Monthly averaged group sunspot numbers (grey dots) and the smoothed series (black line) obtained after going through a low-pass filter.

more reliably the solar activity. It will be used in the present study instead of the Wolf number to bring out the nonlinear dynamical features.

The layout of this paper is as follows. Displayed in Section 2 are the data used in this work, as well as their preparation, the nonlinear approach employed and results obtained with the nonlinear technique. A discussion and conclusions are shown in Section 3.

2 PHASE SPACE RECONSTRUCTION

2.1 The Group Sunspot Numbers

The monthly averaged group sunspot numbers (R_g) have no gaps after 1825 (Hoyt & Schatten 1998a, b), we chose the preiod from January 1825 to December 1995 for our analysis [1]. Although R_g are less noisy than R_z, noise is still present in them. Therefore they must be smoothed by a smoothing filter before the phase space is reconstructed from the noisy time series (Kremliovsky 1994). A low-pass filter, which is based on a Fourier transform, and a moving window of size w_s, are employed to remove the high frequency components in R_g. Here we assume that a window size w_s = 13. Plotted in Figure 1 are the original and smoothed time series.

2.2 Estimation of Delay Time and Embedding Dimension

The reconstruction of an m-dimensional phase space is a very important step when studying the underlying dynamics of a scalar time series (Packard et al. 1980 and Takens 1981). From the phase space picture of the system, we can obtain some information on the asymptotic properties of the studied system, such as the positive Lyapunov exponents that reflect how chaotic the system is, and the topological dimension of the attractor, if exists. Here, we will follow Packard et al. (1980) and Takens (1981) to reconstruct the phase space of R_g. For a time series $x(1), x(2), x(3), ..., x(n)$, the vector of the reconstructed phase space is defined by $\boldsymbol{y(i)} = [x(i), x(i+\tau), x(i+2\tau), ..., x(i+(m-1)\tau)]$, where $i = 1, 2, ..., n$, τ is the time delay, and m is the embedding dimension. Thus the two parameters, the time delay τ and the embedding dimension m, need to be selected before reconstructing phase space. The method of mutual information, proposed by Fraser & Swinney (1986), is employed to determine a reasonable value of the delay time τ.

[1] *ftp://ftp.ngdc.noaa.gov/stp/solar_data/sunspot_ numbers/group_sunspot_numbers/monthrg.dat*

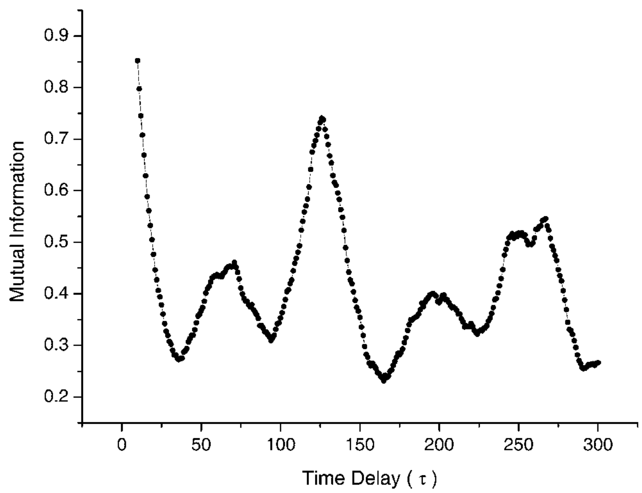

Fig. 2 The mutual information curve of the smoothed data of R_g shows that $\tau = 36$ is a reasonable time delay.

The mutual information between $x(i)$ and $x(i+\tau)$ is defined as

$$I(\tau) = \sum_{x(i),x(i+\tau)} P(x(i), x(i+\tau)) \log_2 \frac{P(x(i), x(i+\tau))}{P(x(i))P(x(i+\tau))}, \tag{1}$$

where $P(x(i))$ is the probability of finding a time series value $x(i)$. Here, with the use of the code provided by TISEAN-2.1 (Hegger et al. 1999) we compute $I(\tau)$ of R_g. The result is shown in Figure 2. As suggested by Fraser & Swinney (1986). If the time delayed mutual information exhibits a marked minimum at a certain value of τ, then that value is a good candidate for reasonable time delay. Hence, as shown in the figure, the reasonable time delay τ is 36 for R_g.

The other parameter, the minimal sufficient embedding dimension m, is determined by using the false nearest neighbor (FNN) method (Kennel et al. 1992). This method finds the nearest neighbor of every point in a given dimension, then checks if these points are still close neighbors in the next higher dimension. If they are not, then they are called FNNs. The percentage of FNNs should drop to zero when the appropriate embedding dimension has been reached. We denote the nearest neighbor of $\boldsymbol{y(i)}$ by $\boldsymbol{y}^{\boldsymbol{NN}}\boldsymbol{(i)}$ in the reconstructed d-dimensional phase space. We compare the $(d+1)$-dimensional coordinates of $\boldsymbol{y(i)}$ and $\boldsymbol{y^{NN}(i)}$, e.g., $x(i+d\tau)$, $x^{NN}(i+d\tau)$. On the basis of definition of FNN, the neighbor is declared false if

$$\frac{|x(i+d\tau) - x^{NN}(i+d\tau)|}{\|\boldsymbol{y(i)} - \boldsymbol{y^{NN}(i)}\|} > R_{\text{tol}}. \tag{2}$$

The parameter R_{tol} is fixed beforehand, and in most studies it has been set to $10-20$. According to Kennel et al. (1992), we can set R_{tol} at 10.0 here. The computation of FNN is also completed with TISEAN-2.1 (Hegger et al. 1999) and then we calculate the percentage of FNNs for different embedding dimension m. See Figure 3. Figure 3 shows that when the embedding dimension m is equal to 3 the percentage of FNNs has dropped to zero. Therefore, the reasonable embedding dimension m of R_g should be 3, suggesting that the group sunspot numbers may display the dynamical properties of a low dimensional chaos.

2.3 Phase Portrait and Maximal Lyapunov Exponent

A phase portrait is reconstructed with the estimated parameters in the two-dimensional plane $(y(t), y(t+\tau))$, as shown in Figure 4. From Figure 4 we can see that the phase space has been totally unfolded and

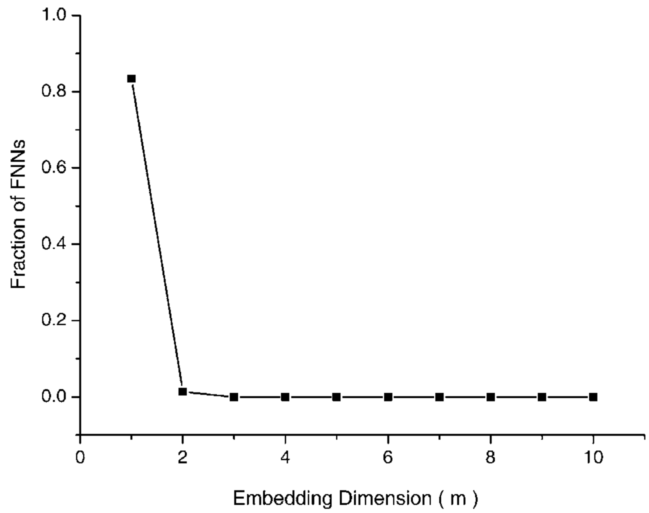

Fig. 3 Fraction of the nearest false neighbors as a function of the embedding dimension m for the smoothed data of R_g.

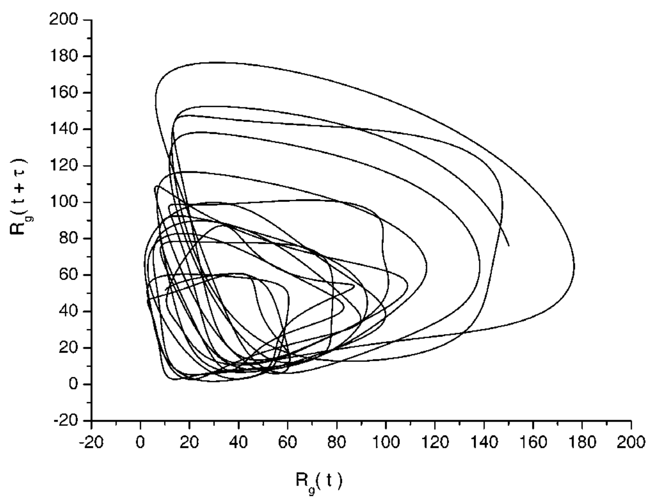

Fig. 4 Reconstructed phase space of the smoothed data of R_g with delay time $\tau = 36$ and embedding dimension $m = 3$.

there seems to be some structure underlying the data of R_g, suggesting that a low-dimensional dynamics could govern the long-term behaviour of solar activity as reflected in R_g, since the embedding dimension of the dynamics is a lowish number, 3.

In order to further study the low-dimensional dynamics underlying the solar activity, we will estimate Lyapunov exponents, which are the average exponential rates of divergence or convergence of the trajectories of the dynamical system in the phase space. If the Lyapunov exponent is less than zero, then the system attracts to a fixed point or a stable periodic orbit, and the absolute value of the exponent indicates the degree of stability. If the Lyapunov exponent is zero, then the system is neutrally stable, and if the Lyapunov exponent is positive, then the system is chaotic and unstable, indicating that nearby trajectories of the system diverge exponentially. The size of the exponent reflects the time scale on which the system dynamics becomes unpredictable. Usually, any system containing at least one positive Lyapunov exponent is defined

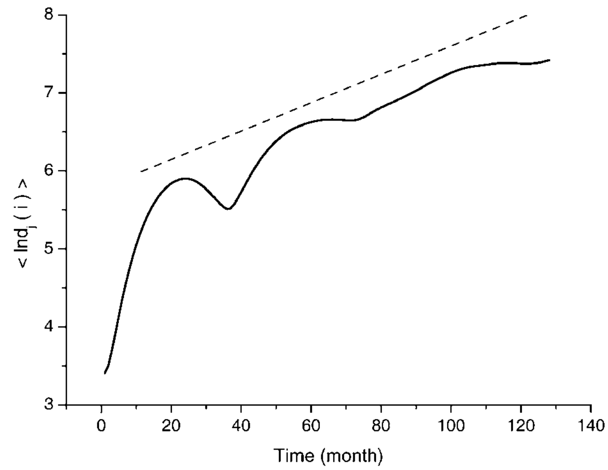

Fig. 5 Divergence $\langle \ln d_j(i) \rangle$ (the solid line) as a function of time, $i\triangle t$ (here, $\triangle t = 1$). The slope of the dashed straight line is the MLE of R_g.

to be chaotic, and therefore the maximal Lyapunov exponent (MLE) is the most important Lyapunov exponent that determines the dynamic properties of system (Wolf et al. 1985). Here the method proposed by Rosenstein et al. (1993) is employed to estimate the MLE of R_g.

According to Rosenstein et al. (1993), the j-th pair of nearest neighbors then diverges at a rate approximately equal to the MLE in the reconstructed phase space: $d_j(i) \approx d_j(0)e^{\lambda_1(i\triangle t)}$, where $\triangle t$ is the sampling period of the time series, $d_j(i)$ is the j^{th} pair of nearest neighbors after i discrete time steps, $d_j(0)$ is the initial separation and λ_1 is the MLE. Then we obtain $\ln d_j(i) \approx \ln d_j(0) + \lambda_1(i \triangle t)$, which represents a set of approximately parallel lines (for $j = 1, 2, ..., (n - (m - 1)\tau)$, each with a slope approximately proportional to λ_1. The variation of $\langle \ln d_j(i) \rangle$ with $i \triangle t$ is shown in Figure 5. We also plot a dashed line indicating approximately the linear increasing segment in the solid curve. Here, $\triangle t = 1$ month. In Figure 5 the dashed line has a slope equal to the theoretical value of λ_1. Thus we obtain a positive MLE, namely, $\lambda_1 \approx 0.0187 \pm 0.0023(\text{month}^{-1})$. This result indicates that the dynamics underlying the data of R_g should be chaotic.

3 DISCUSSION AND CONCLUSIONS

The group sunspot numbers, which are more reliable and more suitable to describe the solar activity than the Wolf sunspot numbers, are used to show the nonlinear dynamical properties of solar activity, and our results show that the dynamical behavior of R_g is that of a low-dimensional chaotic attractor, and this result is consistent with the results from the Wolf sunspot numbers. It is well known that a strange attractor always globally asymptotically tends to one point or a space with boundary condition, i.e., the strange attractor has a "length" (Rosenstein et al. 1993). This "length" can been described by the Lyapunov time or predictability time ($t_p = 1/MLE$), such that for times exceeding the Lyapunov time, the dynamical state of the system ceases to be predictable. We deduce that the Lyapunov time $t_p(R_g)$ is equal to about 4.46 ± 0.5 years. This value is nearly consistent with the result of Zhang (1995)(about 3.6 years), Rozelot (1995) (ranging from 2 to 4 years), and Sello (2001) (4.85 years). Therefore, we conclude that R_g manifests a very similar global phenomenon of the solar activity as does R_z, but is locally slightly different in terms of the maximal Lyapunov exponent. Anyway, the Lyapunov time of R_g and R_z both indicate that the solar activity forecast can be predicted only for a short to medium term, but not for a long term, due to the intrinsic complexity of the related time behavior.

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