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Accretion by a Neutron Star Moving at a High Kick Velocity in the Supernova Ejecta *

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Abstract We suggest a two-dimensional time dependent analytic model to describe the accretion of matter onto a neutron star moving at a high speed across the ejecta left in the aftermath of a supernova explosion. The formation of a strange star resulting from the accretion is also addressed. The newborn neutron star is assumed to move outward at a kick velocity of $v_{\rm ns} \sim 10^3$ km s^{-1}, and the accretion flow is treated as a dust flow. When the neutron star travels across the ejecta with high speed, it sweeps up material, and when the accreted mass has reached a critical value, the neutron star will undergo a phase transition, for instance, to become a strange star. Our results show that the accretion rate decreases in a complicated way in time, not just a power law dependence: it drops much faster than the power law derived by Colpi et al. We also found that the total accreted mass and the phase transition of the neutron star depend sensitively on the velocity of supernova ejecta.

Key words: pulsars: general-pulsars — stars: neutron — X-rays: stars — accretion disk — instability

1 INTRODUCTION

The classical problem of spherical accretion onto a compact object, e.g., a black hole or a neutron star (NS), has been studied by many authors. A hydrodynamical solution was presented by Bondi 1952, who showed that inside a certain capture radius, black holes or NSs accrete material at the Bondi accretion rate $\dot{M}_B$ in a static ambient medium. However, if the compact objects accrete in an expanding ambient mediums, e.g., in a supernova explosion, then the mass accretion rate is significantly different from the Bondi accretion rate. In a supernova explosion, the reverse shock resulting from the collision of the outgoing material with the stationary stellar envelope will lead to the formation of a dense uniformly expanding gas surrounding the compact remnant of the explosion (Chevalier 1989). Therefore fall-back of material onto the newly formed compact remnant is expected to take place. It is interesting to note that this situation has been used to account for the properties of anomalous X-ray pulsars (Lu & Cheng 2002; Lu et al. 2003). The problem of spherical accretion onto a compact object from a medium that is initially uniform but radially outflowing was first explored by Colgate (1971) and Zeldovich et al. 1972. More recently, Chevalier 1989 estimated that, for parameters relevant to SN 1987A, a total mass of about $0.1M_\odot$ is accreted by the compact remnant due to a substantial infall. One-dimensional Lagrangian code that follows a spherically symmetric accretion of fluid for the case of a neutron star sitting at rest in the center of a supernova was discussed in detail in Colpi et al. 1996. The situation in which a NS moving at a high kick velocity and accreting in an expanding ambient medium has not been discussed so far. If the NS gets a kick, it will move away from the initial center

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of symmetry of the explosion: spherical symmetry breaks down completely. The motion of the initially outflowing shells bound to the neutron star also changes completely. The relative velocity between the neutron star and the ejecta then varies with the radius and the azimuthal angle.

Although the exact mechanism by which NSs are given substantial "kick" velocities at birth is not known, observations show that the kick velocity exceeds $500\,\mathrm{km\,s^{-1}}$ in approximately 20% of all NSs (Cordes et al. 1998). It is argued by Bethe 1990 that the core collapse of massive stars of $10-25M_\odot$ produces Type II supernovae accompanying neutron stars whose initial mass is likely to be near the Chandrasekhar limit of $\sim 1.4M_\odot$. The NSs are believed to have proper velocities. Recent observations and analysis on the proper motion of pulsars even give $\sim 450\,\mathrm{km\ s^{-1}}$ as the average 3-dimension velocity of NSs at birth (Lorimer et al. 1997; Hansen & Phinney 1997; Cordes et al. 1998), with possibly a significant fraction having velocities greater than $1000\,\mathrm{km\ s^{-1}}$. Direct evidence for pulsar velocities $\geq 1000\,\mathrm{km\ s^{-1}}$ is provided by observations of the bow shock produced by PSR B2224 + 65 in the interstellar medium (Cordes et al. 1993). Studies of association of NSs with supernova remnants, in many cases, have indicated large proper velocities (Frail et al. 1994). In particular, the association of soft gamma-ray repeaters (SGRs) with supernova remnants implies that SGR $0526-66$ and SGR $1900+14$ have velocities of $\sim 2900(3\,\mathrm{kyr}/t_{\mathrm{SNR}})\,\mathrm{km\ s^{-1}}$ and $\sim 1800(10\,\mathrm{kyr}/t_{\mathrm{SNR}})\,\mathrm{km\ s^{-1}}$, respectively ($t_{\mathrm{SNR}}$ the age of the supernova remnant), although the associations seem problematic. Since many isolated pulsars have such large proper velocities, it seems necessary to invoke "natal kicks" imparted to newborn NSs due to asymmetrical processes during the supernova explosion. Several mechanisms have been suggested for the natal kicks: local hydrodynamical instabilities, neutrino - magnetic field driven asymmetry, local high-order gravity mode instabilities, and electromagnetic radiation of an off-center rotating magnetic dipole (from a review, see Lai et al. 2001). With these mechanisms in mind, we assume that a newborn NS has an initial kick velocity of $v_{\mathrm{ns}} \sim 10^3\ \mathrm{km\ s^{-1}}$. It is interesting to note that the birth of high speed NS may be related to the production of gamma-ray bursts (Huang et al. 2003).

One can infer from the above investigations that the situation of accretion onto a NS with a high velocity in the supernova is completely different from that of spherical accretion in a static NS, as in the case of SN 1987A. Percival 1995 first suggested a scenario for SGRs and anomalous X-ray pulsars (AXPs) that involves high kick velocity NSs (HVNSs) capturing disk material from the co-moving supernova ejecta. They suggested that a $10^{-4}M_\odot$ accretion disk may be acquired by a HVNS as it moves through nearly co-moving supernova ejecta. However, detailed calculations of the time dependent accretion rate for ranges of interstellar medium density, progenitor mass loss parameters, the neutron star magnetic field, initial spin period and initial velocities are required. In this paper, we carry out a full scale, 2-dimensional time dependent hydrodynamical calculation to model the accretion of mass onto a HVNS in a supernova. For simplicity, we focus our calculations on dust-like flows (Colpi et al. 1996).

This paper is organized as follows: in Section 2, we first review the properties of the fall-back process in dust-like regime, then we use a full scale 2-dimensional time dependent model to investigate the accretion of dust onto HVNSs and discuss the phase transition of HVNSs resulting from the accretion and possible astrophysical applications. Conclusions and a discussion are given in Section 3.

2 MODEL DESCRIPTIONS

2.1 Properties of Dust-Like Flows

Motion of supernova outflow is described by its velocity and position (see Fig. 1). Let $\boldsymbol{v}_{\mathrm{sn}}$ and $\boldsymbol{v}_{\mathrm{ns}}$ be, respectively, the velocity vectors of the supernova outflow and of the kicked NS in the center-of-mass (CM) frame. Then the relative velocity vector of the outflow, $\boldsymbol{v}_{\mathrm{rel}}$, is

$$\boldsymbol{v}_{\mathrm{rel}} = \boldsymbol{v}_{\mathrm{sn}} - \boldsymbol{v}_{\mathrm{ns}} \ , \tag{1}$$

and the position vector of the outflow (reckoned from the center of supernova), $\boldsymbol{r}$, satisfies

$$\boldsymbol{r} = \boldsymbol{r'} + \boldsymbol{v}_{\mathrm{ns}} t \ , \tag{2}$$

where $\boldsymbol{r'}$ is the position vector reckoned from the neutron star, and t is the time since the explosion. Usually, for simplicity, the velocity of the outflow behind the shock is assumed to follow a Hubble-type flow when solving the equation of the final free expanding state of the supernova explosion. That is,

$$\boldsymbol{v}_{\mathrm{sn}} = \boldsymbol{r}/t \ . \tag{3}$$

From Equations (1), (2) and (3), we have

$$\boldsymbol{v}_{\rm rel} = \boldsymbol{r'}/t \ . \tag{4}$$

Equation (4) implies that if the outflow is a Hubble flow, then a NS with or without a kick velocity would behave in the same way. However, the real explosion of a supernova is very complex. The velocity profile of the outflow could deviate from a Hubble flow, and it could depend on many factors of the explosion, such as, shells, magnetic fields and rotations, non-spherical core collapse, neutrinos and so on. The widely-performed simulations of supernova-driven ejecta have shown that the velocity of the outflow is a complex function of $\boldsymbol{r}$ and t, and could even be just a constant (Ardeljan et al. 2000; Kifonidis et al. 2003). In this paper, we concentrated on the situation where the outflow has a constant velocity, and that the value of $v_{\rm sn}$ is assumed to follow the value given by Arzoumanian et al. (2002),

$$\begin{aligned} v_{\rm sn} \ &= \ \sqrt{\frac{2E_{\rm sn}}{M_{\rm ej}}} \\ &\sim \ 3\times 10^8 \Big(\frac{E_{\rm sn}}{10^{51}\,{\rm erg}}/\frac{M_{\rm ej}}{10\,M_\odot}\Big)^{1/2} \ {\rm cm\ s^{-1}} \ , \end{aligned} \tag{5}$$

$E_{\rm sn}$ is the explosion energy and $M_{\rm ej}$ is the total mass of the ejecta. The crossing timescale of the ejecta is then comparable to the initial expansion timescale t_1, which is,

$$t_1 = \frac{R_0}{v_{\rm sn}} = \frac{R_0}{\sqrt{2E_{\rm sn}/M_{\rm ej}}} \sim 9\times 10^3 \Big(\frac{R_0}{3\times 10^{12}\,{\rm cm}}\Big)\left[\Big(\frac{M_{\rm ej}}{10M_\odot}\Big)/\Big(\frac{E_{\rm sn}}{10^{51}\,{\rm erg}}\Big)\right]^{1/2} \ {\rm s}\,, \tag{6}$$

with $R_0 \sim 3\times 10^{12}$ cm the initial radius of the supernova ejecta (Shigeyama et al. 1988). Namely, the ejecta expands freely outwards with a uniform velocity $v_{\rm sn}$ at the early evolution stage of $t \le t_1$ (Padmanabhan 2001), which we shall refer to as the free expansion phase. This ejecta can be assumed to be a pressureless fluid (dust-like flows). Within the dust approximation, the density evolution of the outflowing material in the ejecta is assumed to satisfy (Chevalier 1989),

$$\rho = Qt^{-3} \ , \quad t_0 \le t \le t_1 \ , \tag{7}$$

where t_0 is the time since the accretion of the NS began, t_0 will be given later, Q is a constant depending on the detailed properties of the particular supernova under consideration. For the case of SN 1987A, $Q \sim 10^9$ g s^3 cm^{-3}(Chevalier 1989). Equation (7) shows ρ is independent of the radius and decays with time as a power law of index -3.

It should be noted that the actual motion of a dust flow in the ejecta is controlled by the relative velocity $v_{\rm rel}$ and the two angles δ and θ, shown in Figure 1. Because the NS receives a kick velocity, the accretion is no longer spherically symmetric, rather, it is now axisymmetric, with a complicated topology and the accretion of the dust flow onto the neutron star becomes more difficult to calculate. In this paper, we consider an axisymmetric dust flow with density ρ as defined in Equation (7) and velocity $v_{\rm rel}$ with respect to the NS. For simplicity, we use polar coordinates (centered on the NS) and denote the velocity of the dust flow by $v_{\rm rel}(R,\phi)$, R being the polar radius, and ϕ the polar angle from the direction of the initial kick (see Fig. 1). All the involved angles range between 0 and $\pi/2$

$$0 \le \alpha\,, \beta\,, \theta\,, \delta < \frac{\pi}{2} \ , \tag{8}$$

The angles and the velocities are related in classical mechanics by

$$\beta = \delta + \theta \ , \tag{9}$$

$$v_{\rm rel}(\theta,t) = \sqrt{v_{\rm sn}^2 + v_{\rm ns}^2 - 2v_{\rm sn}v_{\rm ns}\cos\theta} \ , \tag{10}$$

$$\frac{v_{\rm sn}}{\sin\beta} = \frac{v_{\rm rel}}{\sin\theta} = \frac{v_{\rm ns}}{\sin\delta} \ , \tag{11}$$

$$\frac{R}{\sin\theta} = \frac{v_{\rm ns}t}{\sin(\alpha+\theta)} = \frac{\sqrt{(v_{\rm ns}t)^2 + R^2 - 2v_{\rm ns}tR\cos\alpha}}{\sin\alpha} \ . \tag{12}$$

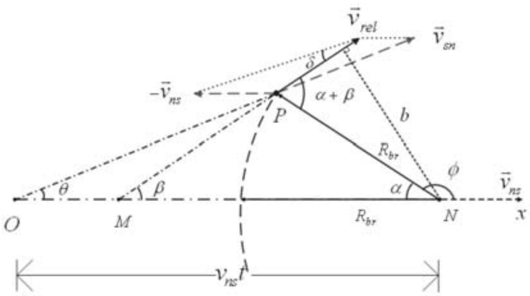

Fig. 1 Sketch showing the motion of the dust flow. Here $v_{\rm sn}$ is the velocity of the supernova ejecta, and $v_{\rm ns}$, that of the kicked NS, $v_{\rm rel}$ is the relative velocity between the two. N is the current location of the kicked NS, O is the center of the supernova explosion.

From Equations (9) and (12), we have

$$\theta = \arcsin \frac{R \sin\alpha}{\sqrt{R^2 - 2Rv_{\rm ns}t\cos\alpha + (v_{\rm ns}t)^2}} \ , \tag{13}$$

$$\beta = \arcsin \frac{v_{\rm sn}\sin\theta}{\sqrt{v_{\rm sn}^2 + v_{\rm ns}^2 - 2v_{\rm sn}v_{\rm ns}\cos\theta}} \ , \tag{14}$$

$$\theta = \arccos \frac{v_{\rm ns}\sin^2\beta + \cos\beta\sqrt{v_{\rm sn}^2 - v_{\rm ns}^2\sin^2\beta}}{v_{\rm sn}} \ . \tag{15}$$

From Equations (13) to (15), we obtain

$$\beta = \arcsin\chi \ , \tag{16}$$

where

$$\chi = \frac{v_{\rm ns}\frac{R\sin\alpha}{\sqrt{R^2-2Rv_{\rm ns}t\cos\alpha+(v_{\rm ns}t)^2}}}{\sqrt{v_{\rm sn}^2 + v_{\rm ns}^2 - 2v_{\rm sn}v_{\rm ns}\sqrt{1-\frac{R^2\sin^2\alpha}{R^2-2Rv_{\rm ns}t\cos\alpha+(v_{\rm ns}t)^2}}}} \ . \tag{17}$$

Substituting Equation (13) into Equation (10), we rewrite the relative velocity $v_{\rm rel}$ as a function of the angle α

$$v_{\rm rel}(\alpha, t) = \sqrt{v_{\rm sn}^2 + v_{\rm ns}^2 - 2v_{\rm sn}v_{\rm ns}\cos\left[\arcsin\frac{R\sin\alpha}{\sqrt{R^2 - 2Rv_{\rm ns}t\cos\alpha + (v_{\rm ns}t)^2}}\right]} \ . \tag{18}$$

The impact parameter b, as shown in Figure 1, satisfies

$$b = R\sin(\alpha + \beta) \ , \tag{19}$$

or

$$b = R\sin(\alpha + \arcsin\chi) \ . \tag{20}$$

2.2 Accretion Radius

We assume that the accretion of bound or returning dust flow onto the NS is restricted to a cone of opening angle α with respect to the initial polar axis (see Fig. 1). It depends on the motion of the dust flow with velocity of $v_{\rm rel}$ and the distributions of β and θ. To illustrate these relations, we introduce two limiting radii, an innermost radius $R_{\rm in}$ and an outermost radius $R_{\rm out}$, as critical distances for the accretion of the dust flow onto the NS. We also need an initial condition, in order to define the initial timescale t_0.

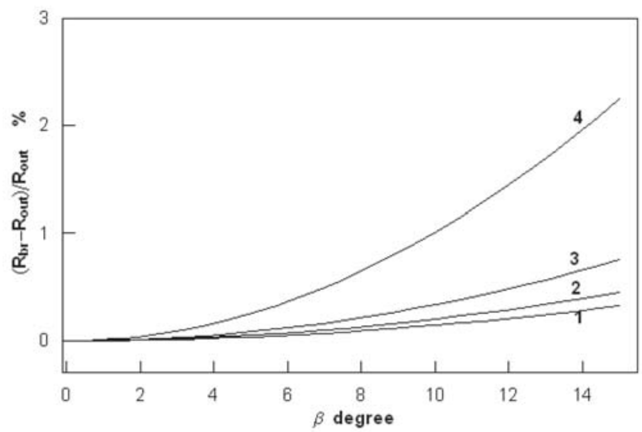

Fig. 2 Ratio between $(R_{\rm br} - R_{\rm out})$ and $R_{\rm out}$ as a function of angle β. The lines 1, 2, 3 and 4 correspond to four different $v_{\rm sn}$ values: 2.1×10^9 cm s^{-1}, 1.5×10^9 cm s^{-1}, 9×10^8 cm s^{-1}, and 3×10^8 cm s^{-1}.

The innermost radius $R_{\rm in}$ for possible accretion by the NS is assumed to be

$$R_{\rm in} \simeq R_{\rm ns} \ , \tag{21}$$

where $R_{\rm ns} \simeq 10^6$ cm is the radius of the neutron star.

Note that far away from the central gravitating mass of the neutron star, pressure gradient has little effect, we therefore assume that the outermost bounding and returning (hereafter 'br') radius for the dust flow is given by the critical distance within which the influence of the gravitation of the NS predominates. Fixing the boundary of the 'br' volume, the distance of the critical 'br' flow assumed in circular orbit can be obtained from the equation of energy conservation of each dust flow,

$$\begin{aligned} \frac{1}{2}v_{\rm rel}^2 &\sim \frac{GM_{\rm ns}}{R_{\rm br}} \ , \\ R_{\rm br} &\sim \frac{2GM_{\rm ns}}{v_{\rm rel}^2} \ , \end{aligned} \tag{22}$$

where $R_{\rm br}$ is the orbit radius or distance for dust flows from the NS, G is the gravitation constant, $M_{\rm ns}$ is the mass of the NS. Replacing R with $R_{\rm br}$ and combining Equations (10), (12), (15) and (22), we have

$$R_{\rm br} = \frac{2GM_{\rm ns}}{v_{\rm sn}^2 + v_{\rm ns}^2 - 2v_{\rm ns}^2 \sin^2\beta - 2v_{\rm ns}\cos\beta\sqrt{v_{\rm sn}^2 - v_{\rm ns}^2 \sin^2\beta}} \ . \tag{23}$$

The outermost radius $R_{\rm out}$ is set to be equal to $R_{\rm br}$ for $\beta = 0$, that is

$$R_{\rm out} \equiv R_{\rm br}|_{\beta=0} \equiv \frac{2GM_{\rm ns}}{(v_{\rm sn} - v_{\rm ns})^2} \ . \tag{24}$$

Giving $v_{\rm ns} \simeq 10^8$ cm s^{-1}, we plot the value $(R_{\rm br} - R_{\rm out})/R_{\rm out}$ as a function of β for various values of $v_{\rm rel}$ in Figure 2. The plot shows that $R_{\rm br}$ changes little with the angle β for values of $v_{\rm sn} \simeq 10^9$ cm s^{-1}. For $v_{\rm sn} = 3\times10^8$ cm s^{-1}, the fractional change is about 0.2%, 0.4%, 1% at $\beta = 4^\circ, 6^\circ, 10^\circ$. The plot shows that the ratio decreases with increasing $v_{\rm sn}$ for a given β. The above analysis indicates that the critical distance $R_{\rm br}$ inside which the motion of the dust flow is dominated by the gravitation of the NS can be replaced by $R_{\rm out}$. Beyond $R_{\rm out}$, we neglect the influence of the gravitation of the NS.

The center of the explosion is outside the gravitational influence of the NS when $v_{\rm ns}t - R_{\rm out} \geq 0$, where t is the time since the NS was kicked away from the center of the explosion of the supernova. The corresponding initial time t_0 for the dust flow to remain bound or returning then satisfies

$$t_0 \geq \frac{R_{\rm out}}{v_{\rm ns}} = \frac{2GM_{\rm ns}}{v_{\rm ns}(v_{\rm sn} - v_{\rm ns})^2} \ . \tag{25}$$

Equation (25) is taken as the initial condition, which implies that the dust flow accreted by the NS happens when $t \geq t_0$. Because $R_{\rm out}$ is independent of the angle β (see Eq. (24)), the problem considered in this paper is greatly simplified.

In some theoretical models for the supernova kick, one expects the kick to be preferentially along the initial spin or magnetic-field axis, for example in the neutrino propulsion mechanism (Duncan & Thompson 1992; Harrison & Tademaru 1975; Schmitt et al. 2005), although this is not always supported by present observations (Anderson & Lyne 1983). For simplicity, we only address the case where the range of kick directions is restricted to a cone of α with respect to the initial kick direction of the exploding star (assumed to be aligned with the polar axis), where all directions within the cone are equally probable. Note that, in this case, 'br' of the dust flow onto the NS is only possible if

$$\alpha \leq \alpha_{\rm max} \ , \tag{26}$$

where $\alpha_{\rm max}$ is the maximum cone angle for 'br' of the dust flow accreted onto the NS at the outermost radius of $R_{\rm out}$. We can estimate $\alpha_{\rm max}$ by the conservations of energy and angular momentum of each 'br' dust flow with the critical boundaries. We adopt the assumptions that the velocity of the dust flow at $R = R_{\rm out}$ is equal to $v_{\rm rel}$ and that the dust flow is entirely radial, so that, the mass and the momentum are conserved in each angular sector; one has

$$\frac{1}{2}v_{\rm in}^2 - \frac{GM_{\rm ns}}{R_{\rm in}} = \frac{1}{2}v_{\rm rel}^2 - \frac{GM_{\rm ns}}{R_{\rm out}} \ , \tag{27}$$

$$R_{\rm in}v_{\rm in} = bv_{\rm rel}\,|_{R_{\rm out}} \ , \tag{28}$$

where $v_{\rm in}$ is the maximum velocity of the dust flow at the innermost radius $R_{\rm in}$. Giving the timescale t, from Equations (18), (27) and (28), we obtain the following equation for $R_{\rm in}$,

$$(R_{\rm out}v_{\rm rel}|_{R_{\rm out}} - 2GM_{\rm ns})R_{\rm in}^2 + 2GM_{\rm ns}R_{\rm out}R_{\rm in} - b^2R_{\rm out}v_{\rm rel}^2|_{R_{\rm out}} = 0 \ . \tag{29}$$

Below, we can derive the root of $R_{\rm in}$ from Equation (29), and with $R_{\rm in} = R_{\rm ns}$, the maximum 'br' angle of $\alpha_{\rm max}$ can be estimated

$$\sin(\alpha_{\rm max} + \arcsin\chi_{\rm max}) = \sqrt{1-\chi_{\rm max}^2}\sin\alpha_{\rm max} + \chi_{\rm max}\cos\alpha_{\rm max} \ . \tag{30}$$

Combining Equations (17) and (30), $\chi_{\rm max}$ can be eliminated in Equation (30), and we can calculate the value of $\alpha_{\rm max}$ numerically.

2.3 Accretion Rate and Evolution

So far, we have addressed the 'br' possibilities of the dust flow onto the NS for the case in which we restrict the accretion by the NS in a spherical cap with radius $R_{\rm out}$ and a critical angle of $\alpha_{\rm max}$. The mass accretion rate of the dust flow onto the NS is

$$\dot{M}(t) = 2t^{-3}\pi Q R_{\rm out}^2 \int_0^{\alpha_{\rm max}} v_{\rm rel}(\alpha, t)\cos[\alpha + \arcsin\chi(\alpha)]\sin\alpha d\alpha \ , \tag{31}$$

where $\chi(\alpha)$ is given in Equation (17). Figure 3 plots the numerically calculated time curve of the accretion rate of the dust flow. We find that a significant increase of the mass accretion rate occurs when $v_{\rm sn} > 10^9$ cm. When $t > t_1$, we have due to $\theta_{\rm max} \to 0$, and the accretion flow can be treated as a flat flow with $v_{\rm rel} = v_{\rm sn} - v_{\rm ns}$. During this phase, the NS is far away from the center of the explosion, the accretion rate is (Lipunov et al. 1992)

$$\dot{M} = \pi b_{\rm max}^2 \rho v_{\rm rel} \ , \ \ t > t_1 \ , \tag{32}$$

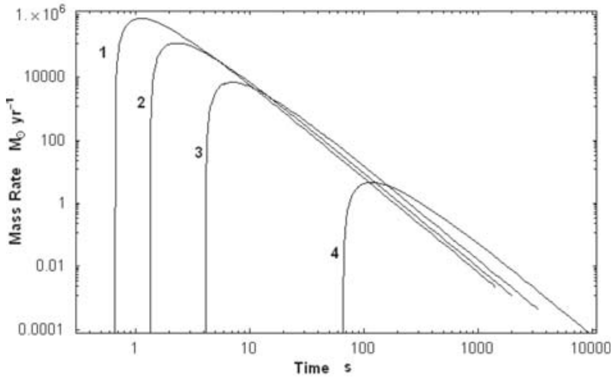

Fig. 3 Accretion rate of the dust flow onto the NS as a function of time t for four values of the velocity of the ejecta. 1, $v_{\rm sn} = 2.1 \times 10^9$ cm s^{-1}; 2, $v_{\rm sn} = 1.5 \times 10^9$ cm s^{-1}; 3, $v_{\rm sn} = 9 \times 10^8$ cm s^{-1}; and 4, $v_{\rm sn} = 3 \times 10^8$ cm s^{-1}.

where $b_{\rm max}$ can be determined from the conservations of the energy and momentum of the accreted flow at the surface of the NS and at the infinity, respectively, that is

$$\frac{1}{2}v_{\rm in}^2 - \frac{GM_{\rm ns}}{R_{\rm in}} = \frac{1}{2}v_{\rm rel}^2 \ , \tag{33}$$

$$R_{\rm ns}v_{\rm in} = b_{\rm max}v_{\rm rel} \ . \tag{34}$$

Solving Equations (33) and (34), we obtain

$$b_{\rm max} = \frac{\sqrt{v_{\rm rel}^2 R_{\rm ns}^2 + 2GM_{\rm ns}R_{\rm ns}}}{v_{\rm rel}} \ . \tag{35}$$

Substituting Equation (35) into Equation (32), we rewrite the accretion rate

$$\dot{M} = 4\pi\lambda\frac{G^2M_{\rm ns}^2\rho}{v_{\rm rel}^3} \ , \quad t > t_1 \ , \tag{36}$$

where $0 \le \lambda = \frac{v_{\rm rel}^2(R_{\rm ns}^2 v_{\rm rel}^2 + 2GM_{\rm ns}R_{\rm ns})}{4G^2M_{\rm ns}^2} \le 1$ for the typical values of $v_{\rm sn}$ and $v_{\rm ns}$. Given $v_{\rm ns} = 1 \times 10^8$ cm, we plot the dependence of λ on $v_{\rm sn}$ in Figure 4.

Comparing with the axisymmetric accretion from a gas with negligible temperature (Hoyle & lyttleton 1939),

$$\dot{M}_{\rm HL} \equiv \frac{4\pi G^2M_{\rm ns}^2\rho}{v_\infty^3} \ , \tag{37}$$

we find that the mass accretion rate of the gas flow when $t > t_1$ is smaller than the Hoyle-Lyttleton limit $\dot{M}_{\rm HL}$ by the factor λ under the assumption $v_\infty = v_{\rm rel}$. The evolution of mass accretion is determined by the properties of $\rho(t)$ of the gas flow. We rewrite the mass accretion of the pressureless gas flow at any time t as

$$\dot{M} = \begin{cases} 2t^{-3}\pi QR_{\rm out}^2\int_0^{\alpha_{\rm max}} v_{\rm rel}(\alpha,t)\cos[\alpha + \arcsin\chi(\alpha)]\sin\alpha d\alpha, & \text{if} \ \ t_0 \le t \le t_1 \ , \\ 4\pi\lambda v_{\rm rel}^{-3}G^2M_{\rm ns}^2\rho(t), & \text{if} \ \ t > t_1. \end{cases} \tag{38}$$

We are interested in the total accretion mass of the dust flow during $t_0 \le t \le t_1$,

$$M_{\rm acc} \equiv \int_{t_0}^{t_1}\dot{M}dt = \begin{cases} 2.8 \times 10^{-5}M_\odot, & \text{for} \ \ v_{\rm sn} \simeq 3 \times 10^8 \, {\rm cm\,s^{-1}} \, , \\ 3.8 \times 10^{-2}M_\odot, & \text{for} \ \ v_{\rm sn} \simeq 2 \times 10^9 \, {\rm cm\,s^{-1}} \, , \\ 5.9 \times 10^{-1}M_\odot, & \text{for} \ \ v_{\rm sn} \simeq 5 \times 10^9 \, {\rm cm\,s^{-1}} \, . \end{cases} \tag{39}$$

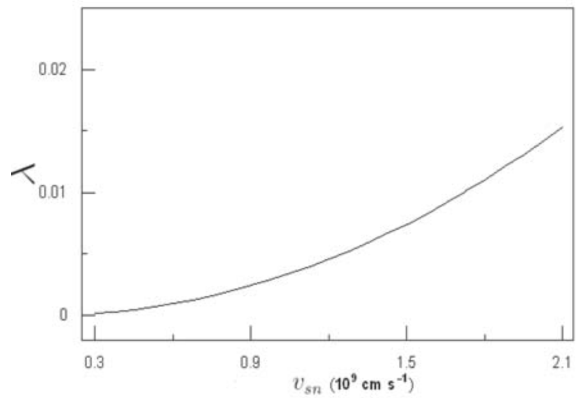

Fig. 4 λ as a function of the velocity of the ejecta of $v_{\rm sn}$.

2.4 Phase Transition of NS

The scenario of an isolated NS undergoing a phase transition to become a black hole was suggested by Brown et al. (1992) for SN 1987A. Such an NS feeds through fallback of material from the progenitor star after the supernova explosion. If the accretion mass onto the NS is over some critical mass (Brandt et al. 1995), then the collapse of the NS leads to a black hole. This may be the reason that there has been no further indication of the existence of a neutron star in the remnant of SN 1987A, in spite of some suggestion that a proto-neutron star was formed in SN 1987A (Percival 1995). The accreted mass may be the key factor to determine whether the remnant in supernova is a NS or a black hole.

The possibility for the NS becoming a strange star has also been discussed by Chevalier (1996) in low-mass X-ray binaries. It has been argued that such conversion requires the formation of a strange matter seed, which is thought to be produced through the de-confinement of neutron matter at a density of $(7-9)\rho_0$ (where ρ_0 is the saturation nuclear matter density) (Baym 1991), much larger than the central density of a $1.4M_\odot$ NS with a moderately stiff to stiff equation of state. To reach this de-confinement density, Chevalier (1996) suggested that, once a $1.4M_\odot$ NS with a moderately stiff to stiff equation of state has accreted a mass $\Delta M_{\rm acc} \geq 0.5\,M_\odot$, a strange matter seed may appear in the core of the neutron star, and subsequently strange matter will begin to swallow its surrounding neutron matter in a hydrodynamically unstable mode. As a result, a NS could convert to a strange star on a timescale of the order of 10 minutes. It should be noted here that the critical de-confinement density for the phase transition from a NS to a strange star is very uncertain currently, which could be lower than the value of $(7-9)\rho_0$. This indicates that the required accreted mass could be less than $0.5M_\odot$, assuming that the phase transition is triggered through the accretion by a NS.

Consequently, as one application of our calculations, we argue that when the total mass of the kicked NS through its accretion reaches about a critical value required for the formation of strange matter seeds, it could make the NS convert to a strange star in $\sim$ 2hr in the supernova. Our calculation shows that the total accreted mass of the dust flow onto the NS could range from 2.8×10^{-5} to $0.6M_\odot$ (see Eq. (39)). We stress here that the accretion rate for the NS's phase transition should obviously be highly super-Eddington. With these mass transferring rates, the gravitational energy at the surface of the NS is dissipated very likely in neutrinos, so the super-Eddington accretion rate could occur (Cooper & Narayan 2005). However, for a black hole, there is no such problem.

3 DISCUSSION AND CONCLUSIONS

We have constructed a 2-dimensional feed-back accretion model for the case of NS with a very high kick velocity. In the earlier stages of supernovae in which the properties of its ejecta can be treated as dust-like corresponding to a non-Hubble flow, we address the accretion rate and the total accreted mass onto HVNS. The results show that the accretion rate of the NS is a sensitive function of the time and the initial condition, particularly the ejecta's velocity, $v_{\rm sn}$. The rare and extreme case with $v_{\rm sn}\approx 5\times10^9$ cm s^{-1} will result in the total accreted mass reaching about $0.6M_{\odot}$. This is larger than the value given by Chevalier (1996) for SN 1987A, in which the accreting NS is assumed to be at rest with respect to the supernova ejecta.

The numerical calculations in Figure 3 show that there exist two effects acting in opposite directions on the mass accretion rate, hence the total accreted mass: an NS with high kick velocity can increase its total accreted mass, in general, with increasing $v_{\rm sn}$; on the other hand, the total accreted mass drops fast with time, unlike the power law dependence as derived in Colpi et al. 1996. In the later stage when $t > t_1$, the evolution of the mass accretion rate onto the kicked NS is very similar to the Hoyle-Lyttleton limit by the factor of λ (see Eqs.(36) and (37)).

We suggest that our results can be used to estimate whether the remnant in supernova ejecta is an NS, a strange star or a black hole. This depends fully on the total accreted mass by the neutron star. When the amount of accreted mass is higher than the critical value, the NS could experience a phase transition and become a strange star.

We should point out that we have investigated a simplified model of accretion onto a kicked NS, which is initialized with a Bondi flow without explicitly considering magnetic field and heating. It is interesting to note that the later scenario refers to as a convection-dominated Bondi flow, which has been examined by Igumenshchev & Narayan 2002. If a gas in a Bondi flow has a frozen-in magnetic field, the field lines are stretched in the radial direction and compressed in the transverse direction, resulting in a dramatic change in the radial structure of the flow from its initial Bondi profile, and the mass accretion rate onto the central compact object, e.g., a black hole, will be completely different from the standard Bondi model, but this is beyond of the present paper.

Secondly, we have evaluated the accuracy of $R_{\rm br}$ by calculating the case of $\beta = 0$, and we found that R_0 is a good approximation at all angles of $\beta > 10^\circ$ from the polar axis and best matches the actual solution between 1° and 10° from the polar axis, and our calculation shows that the degree of approximation increases with increasing $v_{\rm sn}$.

Furthermore, we are interested only in the very earlier stage of a supernova, such as, during the phase of $t_0 < t < t_1$. To make the phase transition of a NS into a strange star via accretion in the ejecta of a supernova, our result requires the ejecta moves with velocity $v_{\rm sn} \simeq 5 \times 10^9$ cm s^{-1} (Mazzali et al. 2000). Because the shock wave triggered in the supernova has not yet reached the surface of the progenitor star, the kinetic energy transferred from the gravitational collapse has not penetrated through the whole ejecta, therefore the velocity of the ejecta in this stage may be larger than $v_{\rm sn} \approx 3 \times 10^8$ cm s^{-1}, a typical velocity of the ejecta observed in the later stages of supernovae. Note that supernovae explode with mean expansion velocities of order 2000 to 10 000 km s^{-1}, as inferred by the Doppler-shifted emission and absorption lines (Arnett 1996). So the assumption of the ejecta velocity $v_{\rm sn} \simeq 5 \times 10^9$ cm s^{-1} may be reasonable in the earlier stage of supernovae. Nevertheless, the real ejecta velocity required by the phase transition of a NS could be lower than the value 5×10^9 cm s^{-1}, because the critical de-confinement density for the phase transition from a NS to a strange star could be lower than that of $(7 - 9)\rho_0$.

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