# Mira Symbiotic Stars * 

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#### Abstract

We have carried out a detailed study of Mira symbiotic stars by means of a population synthesis code. We estimate the number of Mira symbiotic stars in the Galaxy as 1700 - 3100 and the Galactic occurrence rate of Mira symbiotic novae as from $\sim 0.9$ to $6.0 \mathrm{yr}^{-1}$, depending on the model assumptions. The distributions of the orbital periods, the masses of the components, mass-loss rates of cool components, mass-accretion rates of hot components and Mira pulsation periods in Mira symbiotic stars are simulated. By a comparison of the number ratio of Mira symbiotic stars to all symbiotic stars, we find the model with the stellar wind model of Winters et al. to be reasonable.


Key words: binaries: symbiotic — Galaxy: stellar content — stars: Mira variables

## 1 INTRODUCTION

Symbiotic stars (SSs) are interacting binaries consisting of a cool giant, a hot compact companion and an H II region (Berman 1932; Boyarchuk 1967; Boyarchuk 1968). The cool component is a red giant (RG) that belongs either to the first giant branch (FGB) or to the asymptotic giant branch (AGB). In the majority of SSs the hot component is, most probably, a white dwarf (WD) or subdwarf or an accreting low-mass main-sequence (MS) star (Kenyon \& Webbink 1984; Mürset et al. 1991). The SSs are variable, and the variability may be due to either thermonuclear runaway on the surface of an accreting WD (Tutukov \& Yungelson 1976; Yungelson et al. 1995; Iben \& Tutukov 1996) or to variations in the accretion rate onto the hot component (Duschl 1986; Bisikzlo et al. 2002; Mitsumoto et al. 2005). Recent reviews of the properties of SSs can be found in Mürset \& Schmid (1999) and Mikołajewska (2003).

Mira variables are late M-type giant stars, pulsating with periods of 80 to 1000 days or more and with visual amplitudes greater than 2.5 magnitudes (Percy 1997). They are the coolest and most luminous AGB stars; they have a carbon core, surrounded by a helium-rich layer which in turn is surrounded by a hydrogen-rich envelope. Detailed studies of Mira variables have mainly focused on two aspects (Whitelock et al. 2000): i) the connection between mass loss and stellar parameters, such as radius and luminosity; ii) a period-luminosity relation between the extreme luminosities and the pulsation period, which makes Mira variables to have great potential as standard candles among old populations in galactic structure studies.

Mira symbiotic stars (MSSs) are special SSs in which the cool components are Mira variables. Observationally, all MSSs are D-SSs in Webster \& Allen's (1975) classification, that is, MSSs have thick dust shells. The study of MSSs is very helpful for the understanding of Mira variables and D-SSs.

Some theoretical studies on the formation and evolution of SSs have been published, e. g., by Yungelson et al. (1995), Han et al. (1995), Iben \& Tutukov (1996), Hurley et al. (2002) and Lü et al. (2006a, b). Their investigations reproduced successfully many observed properties of SSs. However, there is still not a theoretical review of MSSs. In this paper, according to the theoretical model of SSs in Lui et al. (2006b) and the definition of Mira variables in Zhu \& Zha (2005), we calculate, by population synthesis, their birthrate

[^0]and number in the Galaxy, and some potentially observable parameters such as orbital periods, masses of the components, pulsation period and mass loss rate.

In Section 2 we present our assumptions and describe some details of the modeling algorithm. In Section 3 we discuss the main results. In Section 4 the main conclusions are given.

## 2 MODEL

In MSSs, the binary systems are SSs and the cool components are Mira variables. In the following two subsections, we give the definitions of Mira variables and SSs, respectively.

### 2.1 Mira Variables

As stars ascend the asymptotic giant branch (AGB), it appears that they begin to pulsate when their effective temperature drops below a certain level (usually taken 3800 K (Percy 1997)) and their luminosity increases to a certain value. The star then becomes a Mira variable if it exhibits large amplitude pulsations. Such stars appear in a limited region in the HR-diagram which has been discussed in Wood \& Zarro (1981), Groenewegen \& de Jong (1994), and Gautschy (1999). Gautschy (1999) made a define attempt to determine the region in the $L-T_{\text {eff }}$-plane where the star becomes a Mira variable. According to the position of Mira variables in the HR-diagram (Zhu \& Zha 2005; Ferrarotti \& Gail 2006), we assume that a star is a Mira variable if its effective temperature $T_{\text {eff }}$ is lower than $10^{3.49} \mathrm{~K}$ and its luminosity is higher than $10^{3.20} L_{\odot}$. Based on Vassiliadis \& Wood (1993), Mira pulsation period $P$ (days) is given by

$$
\begin{equation*}
\log P=-2.07+1.94 \log \left(R / R_{\odot}\right)-0.90 \log \left(M / M_{\odot}\right) \tag{1}
\end{equation*}
$$

### 2.2 Symbiotic Stars

In SSs, the cool component loses matter at a high rate by stellar wind and the hot component moves in the wind and accretes enough matter to produce the symbiotic phenomenon. In this paper, a binary is considered as SS if it satisfies the following conditions: (i) The system is a detached system; (ii) The luminosity of the hot component is greater than $10 L_{\odot}$ which is the "threshold" luminosity for the hot component of SSs as inferred by Mikołajewska \& Kenyon (1992), Mürset et al. (1991) and Yungelson et al. (1995), which may be due to thermonuclear burning (including nova outbursts, stationary burning and post-eruption burning) or liberation of gravitational energy by accreted matter; (iii) The hot component is a WD and the cool component is in the FGB or AGB stage.

Lü et al. (2006b) gave a detailed description of the physical models of SSs. Simple physical process of producing symbiotic phenomenon can be shown as:

$$
\binom{\text { mass loss of }}{\text { cool component }} \rightarrow\binom{\text { mass accretion of }}{\text { hot component }} \rightarrow \begin{cases}\dot{M}_{\mathrm{acc}}>\dot{M}_{\mathrm{st}} & \rightarrow \text { stable hydrogen burning } \\ \dot{M}_{\mathrm{acc}}<\dot{M}_{\mathrm{st}} & \rightarrow \text { accumulating accreted mass } \\ & \rightarrow \text { thermonuclear runaways }\end{cases}
$$

where $\dot{M}_{\text {st }}$ is a critical accretion rate of WD. If the accretion rate is higher than $\dot{M}_{\text {st }}$, the accereted hydrogen will burn steadily on the surface of the WD accretor; if the accretion rate is lower than $\dot{M}_{\text {st }}$, then the WD accretor may undergo outbursts. By using the approximation to the results of Iben \& Tutukov (1989), $M_{\text {st }}$ is given by

$$
\begin{equation*}
\log \dot{M}_{\mathrm{st}}=-9.31+4.12 M_{\mathrm{WD}}-1.42\left(M_{\mathrm{WD}}\right)^{2} M_{\odot} \mathrm{yr}^{-1} \tag{2}
\end{equation*}
$$

with $M_{\mathrm{WD}}$ in solar units. In this process, the crucial physical parameters are the outcome of the common envelope evolution, the stellar wind velocity $v_{\mathrm{w}}$ and the mass of hydrogen layer, $M_{\text {crit }}^{\mathrm{WD}}$, at which the white dwarf can accumulate prior to hydrogen ignition (Lü et al. 2006b). However, common envelope evolution has only effects on close binary systems while MSSs usually have long orbital periods in order to contain the Mira variables. So, the common envelope evolution is not important for the formation of MSSs. We only consider the influence of $v_{\mathrm{w}}$ and $M_{\text {crit }}^{\mathrm{WD}}$.

Prior to the thermonuclear runaway, a certain amount of hydrogen, $M_{\text {crit }}^{\mathrm{WD}}$, has to be accumulated. Following Yungelson et al. (1995), we use the "constant pressure" expression for $\Delta M_{\text {crit }}$ which implies
that ignition occurs when the pressure at the base of accreted layer rises to a certain limit

$$
\begin{equation*}
\frac{\Delta M_{\mathrm{crit}}^{\mathrm{WD}}}{M_{\odot}}=2 \times 10^{-6}\left(\frac{M_{\mathrm{WD}}}{R_{\mathrm{WD}}^{4}}\right)^{-0.8} \tag{3}
\end{equation*}
$$

where $R_{\mathrm{WD}}$ is the radius of zero-temperature degenerate objects (Nauenberg 1972):

$$
\begin{equation*}
R_{\mathrm{WD}}=0.0112 R_{\odot}\left[\left(M_{\mathrm{WD}} / M_{\mathrm{ch}}\right)^{-2 / 3}-\left(M_{\mathrm{WD}} / M_{\mathrm{ch}}\right)^{2 / 3}\right]^{1 / 2} \tag{4}
\end{equation*}
$$

with $M_{\mathrm{ch}}=1.433 M_{\odot}$ and $R_{\odot}=7 \times 10^{10} \mathrm{~cm}$. Because of the complicated dependence of $\Delta M_{\text {crit }}$ on the input parameters (Yungelson et al. 1995; Lü et al. 2006b), we ran several simulations at various values of $\Delta M_{\text {crit }}$, see Table 1.

The thermonuclear runaways in SSs are called "symbiotic novae" (SyNe). SyNe are usually in the "plateau" stage with a high luminosity given by Iben \& Tutukov (1996),

$$
\begin{equation*}
L / L_{\odot} \approx 4.6 \times 10^{4}\left(M_{\text {core }} / M_{\odot}-0.26\right) \tag{5}
\end{equation*}
$$

The duration of the "plateau" stage is

$$
\begin{equation*}
t_{\mathrm{on}}=6.9 \times 10^{10} \frac{\alpha_{\mathrm{H}} \Delta M_{\mathrm{crit}}^{\mathrm{WD}}}{L} \mathrm{yr}, \tag{6}
\end{equation*}
$$

where $\alpha_{\mathrm{H}}$ is defined as the ratio of the mass of burnt hydrogen to the mass of matter accreted by WD and it was approximated by Lü et al. (2006b) according to figure 2 in Yungelson et al. (1995) (also fig. 16 of Iben \& Tutukov 1996)

$$
\alpha_{\mathrm{H}}=\left\{\begin{array}{l}
-4.39-1.48 \log \dot{M}_{\mathrm{acc}}-0.10\left(\log \dot{M}_{\mathrm{acc}}\right)^{2}, \text { for } \log \dot{M}_{\mathrm{acc}}<-6.36  \tag{7}\\
11.66+4.56 \log \dot{M}_{\mathrm{acc}}+0.45\left(\log \dot{M}_{\mathrm{acc}}\right)^{2}, \text { for } \log \dot{M}_{\mathrm{acc}} \geq-6.36
\end{array}\right.
$$

After SyNe, the system remains observable as an SS for a time span $t_{\text {cool }}$ until the WD cools to the temperature at which its luminosity falls below $10 L_{\odot}$. According to Prialnik (1986), Lü et al. (2006b) assumed that after SyNe its luminosity decreases as

$$
\begin{equation*}
L(t)=L(0) t^{-1.14} \tag{8}
\end{equation*}
$$

where $L(0)$ is given by Equation (5) and $t$ is in years. When $L(t)=10 L_{\odot}$, the SSs stage terminates, giving the cooling time $t_{\text {cool }}$. The lifetimes of an SS is the sum of $t_{\text {on }}$ and $t_{\text {cool }}$.

For stellar mass loss, we accept the prescription of Hurley et al. (2000). Stellar wind accretion rate is given by the classical Bondi \& Hoyle (1944) accretion formula:

$$
\begin{equation*}
\dot{M}_{\mathrm{hot}}=\frac{-1}{\sqrt{1-e^{2}}}\left(\frac{G M_{\mathrm{hot}}}{v_{\mathrm{w}}^{2}}\right)^{2} \frac{\xi_{\mathrm{w}}}{2 a^{2}} \frac{1}{\left(1+v^{2}\right)^{3 / 2}} \dot{M}_{\mathrm{cool}} \tag{9}
\end{equation*}
$$

where $1 \leq \xi_{\mathrm{w}} \leq 2$ is a parameter ( $\xi_{\mathrm{w}}=\frac{3}{2}$ in this work), $v_{\mathrm{w}}$ is the wind velocity and

$$
\begin{equation*}
v^{2}=\frac{v_{\mathrm{orb}}^{2}}{v_{\mathrm{w}}^{2}}, \quad v_{\mathrm{orb}}^{2}=\frac{G M_{\mathrm{t}}}{a} \tag{10}
\end{equation*}
$$

where $a$ is the semi-major axis of the elliptical orbit, $v_{\text {orb }}$ the orbital velocity and $M_{\mathrm{t}}=M_{\mathrm{hot}}+M_{\text {cool }}$, the total mass.

The accretion rate of the stellar wind (Eq. (9)) strongly depends on the wind velocity $v_{\mathrm{w}}$, which is not readily determined. Following Yungelson et al. (1995), we assume

$$
\begin{equation*}
v_{\mathrm{w}}=\alpha_{\mathrm{w}} v_{\infty} \tag{11}
\end{equation*}
$$

where $v_{\infty}$ is the terminal wind velocity and $\alpha_{\mathrm{w}}$ is approximated by Yungelson et al. (1995):

$$
\begin{equation*}
\alpha_{\mathrm{w}}=\frac{0.04\left(r / R_{\mathrm{d}}\right)^{2}}{1+0.04\left(r / R_{\mathrm{d}}\right)^{2}} \tag{12}
\end{equation*}
$$

where $r$ is the distance from the donor and $R_{\mathrm{d}}$ is the radius of the donor. For the definition of terminal velocity, we consider three cases:
(a) $v_{\infty}=\frac{1}{2} v_{\text {esc }}$ (Lü et al. 2006b), where $v_{\text {esc }}$ is surface escape velocity.
(b) $v_{\infty}$ is determined by the relation between mass-loss rates and terminal wind velocities fitted by Winters et al. (2003)

$$
\begin{equation*}
\log \left(\dot{M}\left[M_{\odot} \mathrm{yr}^{-1}\right]\right)=-7.40+\frac{4}{3} \log \left(v_{\infty}\left[\mathrm{km} \mathrm{~s}^{-1}\right]\right) \tag{13}
\end{equation*}
$$

However, Equation (13) is valid for $\sim 10^{-6} M_{\odot} \mathrm{yr}^{-1}$. For higher mass loss rate, Equation (13) gives a too high $v_{\infty}$. Based on models of Winters et al. (2000), we assume $v_{\infty}=\min \left(30 \mathrm{~km} \mathrm{~s}^{-1}, v_{\infty}\right)$. The wind velocity is given by Equation (11), where $\alpha_{\mathrm{w}}$ is defined by Equation (12).

For $\dot{M} \leq 3.0 \times 10^{-7} M_{\odot} \mathrm{yr}^{-1}$, we assume that the wind velocity decelerates from $v_{\text {esc }}$ at the stellar surface to $5 \mathrm{~km} \mathrm{~s}^{-1}$ at $r / R_{\mathrm{d}}=10$, using an $a d$ hoc function

$$
v_{\mathrm{w}}= \begin{cases}\frac{5-v_{\mathrm{esc}}}{9}\left(r / R_{\mathrm{d}}\right)+\frac{10 v_{\mathrm{esc}}-5}{9}, & r \leq 10 R_{\mathrm{d}}  \tag{14}\\ 5 \mathrm{~km} \mathrm{~s}^{-1}, & r>10 R_{\mathrm{d}}\end{cases}
$$

(c) In addition, a model with "standard" terminal wind velocity of AGB stars equal to $15 \mathrm{~km} \mathrm{~s}^{-1}$ is calculated.

### 2.3 Basic Parameters of the Monte Carlo Simulation

For population synthesis of binary stars, the main input model parameters are: (i) the initial mass function (IMF) of the primaries; (ii) the mass-ratio distribution of the binaries; (iii) the distribution of orbital separation; (iv) the eccentricity distribution; (v) the metallicity $Z$ of the binary systems.

A simple approximation to the IMF of Miller \& Scalo (1979) is used. The primary mass is generated using the formula suggested by Eggleton et al. (1989),

$$
\begin{equation*}
M_{1}=\frac{0.19 X}{(1-X)^{0.75}+0.032(1-X)^{0.25}} \tag{15}
\end{equation*}
$$

where $X$ is a random variable uniformly distributed in the range [0,1], and $M_{1}$ is the primary mass from $0.8 M_{\odot}$ to $8 M_{\odot}$.

The mass-ratio ( $q=M_{2} / M_{1}$ ) distribution is quite controversial. We consider only the uniform massratio distribution (Mazeh et al. 1992; Goldberg \& Mazeh 1994),

$$
\begin{equation*}
n(q)=1,0<q \leq 1 \tag{16}
\end{equation*}
$$

The distribution of orbital separations is given by

$$
\begin{equation*}
\log a=5 X+1 \tag{17}
\end{equation*}
$$

where $X$ is a random variable uniformly distributed in the range $[0,1]$ and $a$ is in $R_{\odot}$.
We assume that all binaries have initially circular orbits. Kenyon (1986) showed that the Galactic SSs are strongly concentrated toward the plane with metallicity about 0.02 . The metallicity in this paper is taken as 0.02 . We follow the evolution of both components using the rapid binary evolution code, including the effect of tides on the binary evolution (Hurley et al. 2002). We take $2 \times 10^{5}$ initial binary systems in each simulation. Since we present for every simulation results of one run of the code, the given numbers are subject to Poisson noise. To calculate the birthrate of SSs, we assume that one binary with $M_{1} \geq 0.8 M_{\odot}$ is formed annually in the Galaxy (Yungelson et al. 1993; Han et al. 1995; Yungelson et al. 1995).

## 3 RESULTS

We construct a set of models in which we vary different input parameters relevant to symbiotic phenomenon produced by hydrogen burning at the surface of WD accretors and select MSSs from SSs by the position of Mira variables in HR-diagram. Table 1 lists all cases considered in the present work and Case $1^{\star}$ is considered as the standard model. In addition to nuclear-burning powered model, we consider the "accretion model" that contains systems in which under assumptions of Case 1 the liberation of gravitational potential energy produces symbiotic phenomenon ( $L_{\text {grav }} \geq 10 L_{\odot}$ ), before the first outburst of nuclear burning occurs, or in the time intervals between consecutive nuclear outburst plus decline "quasi-cycles".

Table 1 Parameters of the models of MSS populations. Case $1^{\star}$ is the standard model. In Case 2 the wind velocity $v_{\mathrm{w}}$ is treated as described under item (b) in Section 2.2.

| Cases | $v_{\infty}$ | $\Delta M_{\text {crit }}^{\mathrm{WD}}$ |
| :--- | :--- | :--- |
| Case $1^{\star}$ | $\frac{1}{2} v_{\infty}$ | $\Delta M_{\text {crit }}^{\mathrm{WD}}$ |
| Case 2 | Eq. (13) | $\Delta M_{\text {crit }}^{\mathrm{WD}}$ |
| Case 3 | $15 \mathrm{~km} \mathrm{~s}^{-1}$ | $\Delta M_{\text {crit }}^{\mathrm{WD}}$ |
| Case 4 | $\frac{1}{2} v_{\infty}$ | $3 \Delta M_{\text {crit }}^{\mathrm{WD}}$ |
| Case 5 | $\frac{1}{2} v_{\infty}$ | $\frac{1}{3} \Delta M_{\text {crit }}^{\mathrm{WD}}$ |

Table 2 Different models of symbiotic stars population. The first column gives the serial number of the model according to Table 1. Column 2 gives the Galactic number of MSSs and Column 3 gives their birthrate. The fourth column shows the occurrence rate of SyNe in MSSs. The number ratio of MSSs to all SSs (see Lü et al. 2006b) is given in column 5.

| Cases | Number <br> of MSSs | Birthrate of <br> MSSs $\left(\mathrm{yr}^{-1}\right)$ | Occurrence Rate of <br> SyNe in MSSs $\left(\mathrm{yr}^{-1}\right)$ | $\frac{N_{\text {MSSs }}}{N_{\text {total }}}$ <br> Standard 2100 |
| :--- | :---: | :---: | :---: | :---: |

### 3.1 Birthrate and Number of MSSs

As Table 2 shows, the Galactic birthrate of MSSs may range from $\sim 0.012 \mathrm{yr}^{-1}$ (Case 2) to $\sim 0.060 \mathrm{yr}^{-1}$ (Case 3). The total number of MSSs ranges from ~ 1700 (Case 4) to $\sim 3100$ (Case 3). Lü et al. (2006b) carried out a large-size numerical simulation of SSs which included all the cases in this paper. The number ratio of MSSs to all SSs is from $\sim 17 \%$ (Case 2) to $84 \%$ (Case 3). In the standard model and the combined nuclear and accretion models, the ratio is $\sim 32 \%$. Observationally, there are 174 SSs in the Galaxy (Belczyński et al. 2000), of which $\sim 30$ SSs are D-SSs showing thick dust shells, and 27 of which contain a Mira variable as the cool component. The observational ratio of the number of MSSs to all SSs in the Galaxy is $\leq 15 \%$. The result of Case 2 agrees reasonably well with the observational estimate. Based on the observational ratio of the number of MSSs to all SSs, the model of MSSs will require a rather low velocity of stellar wind in the low mass loss rate phase, and a high wind velocity in the high mass loss rate phase, and this is consistent with the stellar wind model in Winters et al. (2000). However, we should note that the position of Mira variables in HR-diagram is uncertain and is very sensitive to the effect temperature (Zhu \& Zha 2005), this will lead to some uncertainty.

### 3.2 Properties of MSSs

In this section, we describe the potentially observable physical quantities of MSSs. The standard model Case $2\left(v_{\infty}=\right.$ Eq.(13)) and the accretion model will be compared.

Figure 1 shows the distributions of the progenitors of MSSs, in the "initial primary mass - initial orbital period" plane. There is a large difference between the standard model and Case 2. The main reason is the stellar wind velocity. For long orbital periods, symbiotic phenomenon can be produced until the mass-loss rate reaches a very high value. In Case 2, however, the higher the mass-loss rate is, the higher the wind velocity is. So, the accretion rate of stellar wind decreases, which is not favorable to producing the symbiotic phenomenon. It can be found there is lack of MSSs' progenitors with long orbital period in Figure 1(b).

Orbital Periods: Figure 2 shows the distributions of MSSs in orbital periods. The orbital periods of MSSs are longer than 1000 days. In Case 2, the peak is $\sim 40$ years. MSSs have usually long orbital


Fig. 1 Gray-scale maps of initial primary mass $M_{\mathrm{i}}$ vs. initial orbital period $P_{\mathrm{i}}$ distribution for the progenitors of MSSs. The gradations of gray-scale correspond to regions where the number density of systems is, successively, within $1-1 / 2,1 / 2-1 / 4,1 / 4-1 / 8$ and $1 / 8-0$ of the maximum of $\frac{\partial^{2} N}{\partial \log a_{i} \partial \log M_{i}}$, and that blank regions are regions that do not contain any stars. The cases shown in the individual panels are indicated in the low-right corner.


Fig. 2 Number distribution of orbital periods in three MSS models.
periods and are very hard to detect. Mikołajewska (2003) estimated that the orbital periods of D-SSs are longer than 50 years by assuming a typical dust formation radius of $\geq 5 R_{\text {Mira }}$ ( $R_{\text {Mira }}$ is the radius of Mira variables and $\sim 1-3 \mathrm{AU}$ ). If the estimates are correct, there should be some MSSs which are not D-SSs based on Figure 2. However, we know that all MSSs are D-SSs observationally. Considering a variable dust obscuration, Manari (1988) estimated a $\sim 6$ yr orbital period for the D-SS V1016. The main reason is that Mikołajewska (1999) assumed that the binary separations in MSSs should be longer than the dust formation radius; Manari (1988) considered that the hot components of MSSs could destroy the dust grains by strong and energetic radiation field.

Using the dust formation model in Gail \& Sedlmayr (1999) and Ferrarotti \& Gail (2006), our rough analysis is the following: For a Mira variable, its initial stellar wind velocity is $\sim 2-5 \mathrm{~km} \mathrm{~s}^{-1}$ and the stellar wind forms dust shells in $\sim 4-5 R_{\text {Mira }}$ ( $R_{\text {Mira }}$ is the stellar radius and $\sim 200-600 R_{\odot}$ ). Then, the time span of the stellar wind from leaving the stellar surface to forming the dust shell is longer than $\sim 4 \mathrm{yr}$ and shorter than $\sim 25$ yr. We suggest that D-SSs or MSSs can be classified into two types: 1) those with shorter orbital
periods (from $\sim 4$ to 20 yr ), in which the hot components have great effects on the dust formation and dust shells, and induce variable dust obscuration; 2) those with orbital periods longer than $\sim 20 \mathrm{yr}$, in which the binary separations are greater than the dust formation radius and the hot components have small effects on the dust formation and dust shells.


Fig. 3 Number distribution of hot component mass in three MSS models.


Fig. 4 Number distribution of cool component mass in three MSS models.


Fig. 5 Number distribution of component mass ratio in three MSS models.


Fig. 6 Number distributions of model MSSs in the mass-loss rate of the cool component. (a) shows our model simulation results, (b) shows the observational number distribution from Seaquist et al. (1993).


Fig. 7 Number distributions of the mass-accretion rate of the hot component in three MSS models.


Fig. 8 Number distribution of the Mira pulsation period of the cool component in our three model simulations (a). The observational distribution from Whitelock (1987) is shown in (b).

Components' Masses: Not all the component masses in MSSs have been measured. Here we give only our results. Figures 3 and 4 show the distributions of the masses of the hot and cool components. In Figure 3, the peak is at $\sim 0.6 M_{\odot}$. All the hot component masses are greater than $0.5 M_{\odot}$, that is, not a single hot component in MSSs is an He WD. He WDs form through a common envelope or stable Roche lobe overflow from the stars in the first giant branch. These binary systems have generally short orbital periods so that they can not contain Mira variables with large stellar radii. In Figure 4, the range of the cool components in MSSs is from $0.6 M_{\odot}$ to $6.0 M_{\odot}$ (or greater) and the peak is between $\sim 0.6 M_{\odot}$ and $2.0 M_{\odot}$. Figure 5 shows the distributions of the mass ratios of cool to hot components of the MSSs. Their peaks range from $\sim 1.2$ to 2.4 .

Mass loss and accretion: Figure 6(a) shows the number distribution of MSSs in the mass-loss rate of the cool component. The peaks are $\sim 10^{-5} M_{\odot} \mathrm{yr}^{-1}$. The observational number distribution after Seaquist et al. (1993) is shown in Figure 6(b). Although only 10 MSSs have measured mass-loss rates, a comparison between Figures 6(a) and 6(b) shows that our results agree reasonably well with the observations. According to Winters et al. (2000) and Ferrarotti \& Gail (2006), an AGB star can form dust shells when its mass-loss rate is higher than $3 \times 10^{-7} M_{\odot} \mathrm{yr}^{-1}$. From Figure 6(a), it is obvious that most MSSs can form dust shells, that is, MSSs are D-SSs. In Figure 7, the distributions of model MSSs as a function of the mass-accretion rate of the hot component are shown. The peaks of $\dot{M}_{\text {acc }}$ are around $\sim 2 \times 10^{-7} M_{\odot} \mathrm{yr}^{-1}$ for all the nuclear models and $\sim 10^{-8} M_{\odot} \mathrm{yr}^{-1}$ for the accretion model.

Mira Pulsation Periods: Kafatos et al. (1977) found that the pulsation periods of GCVS Mira variables (Kukarkin et al. 1969) have a median value of 250-300 days. In Figure 8(b), the 12 Mira variables in MSSs whose pulsation periods have been determined showing a mean value of $424 \pm 25$ days with a range from 280 to 580 days (Whitelock 1987). In Figure 8(a), all pulsation periods in our simulations are longer than $\sim 300$ days and their peaks are $\sim 400,450$ and 600 days, in the standard model, Case 2 and accretion model, respectively. A comparison between Figure 8(a) and 8(b) indicates that our results are in reasonable agreement with the observations. Both the simulations and observations show that the pulsation periods of MSSs are longer than the average value of general Mira variables. A possible explanation for the lack of short pulsation period MSSs is that Mira variables with shorter pulsation periods have lower mass losses, which is unfavorable to the formation of SSs. Mira variables with longer pulsation periods can provide high enough mass loss rates for the accretion of WD accretors, hence the symbiotic phenomenon.

## 4 CONCLUSIONS

We performed a detailed study of the formation of MSSs, employing the population synthesis approach to the evolution of the binaries. Several conclusions can be drawn:

1. The number of MSSs in the Galaxy is 1700-3100 and the theoretical estimate of the Galactic occurrence rate of symbiotic novae in MSSs is from $\sim 0.9$ to $6.0 \mathrm{yr}^{-1}$, depending on the model assumptions. Judging by the number ratio of MSSs to all SSs, the results in model 2 with the stellar wind model in Winters et al. (2000) are reasonable.
2. The orbital periods of MSSs are longer than 1000 days. According to the time scale of dust formation in AGB stellar wind, we suggest that D-SSs or MSSs can be classified into two types: 1) those with shorter orbital periods (from $\sim 4$ to 20 yr ), in which the hot components have important effect on the dust formation and dust shells and induce variable dust obscuration; 2)those with orbital periods longer than $\sim 20 \mathrm{yr}$, in which the binary separations are larger than the dust formation radius and the hot components have little effect on the dust formation and dust shells.
3. The Mira pulsation periods in MSSs are longer than $\sim 300$ days and their average value ( $\sim 450$ days) is longer than the average value of general Mira variables; this agrees reasonably well with the observations.
4. Based on the mass loss rates of the cool components in MSSs, most MSSs may form dust shells, that is, most MSSs are D-SSs, in reasonable agreement with the observations.

There are two main areas of uncertainty in our models: the position of Mira variables in the HR-diagram (Zhu \& Zha 2005) and many of the physical parameters in the SSs models (Lü et al. 2006b). Future work in these areas is necessary to reduce the uncertainties.

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