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Revisit of Dynamical Mechanisms of Transporting Asteroids in the 3:1 Resonance to the Near-Earth Space *

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Abstract It is well-known that the asteroids in the main belt trapped in the 3:1 Mean Motion Resonance (MMR) with Jupiter (at semi-major axes ~2.5 AU) are few in number, forming one of the so-called Kirkwood Gaps. Wisdom pointed out that chaotic motion of such asteroids can increase their eccentricities and make them approach and cross the orbit of Mars (or even the Earth). We numerically investigated the orbital evolution of the asteroids involved in 3:1 MMR (NEOs) over millions of years and revisited the dynamical mechanisms of transporting such asteroids into the NEO region. The results show that the dynamical evolution of the asteroids around 2.5 AU is mainly dominated by the 3:1 resonance, the ν_5 and ν_6 secular resonances and the Kozai resonance, and these bodies can evolve into NEOs through several of the dynamical mechanisms, so indicating possible dynamical origin of the NEOs.

Key words: celestial mechanics — near-Earth Objects — mean motion resonance — secular resonance

1 INTRODUCTION

Asteroids are sometimes called minor planets. Like the Earth and other major planets, these small objects move around the Sun. According to the modern theory of planetary formation, our Solar System formed about 4.5 billion years ago from a collapsed interstellar gas cloud, the so-called Solar Nebula, the dense protoplanetary disk of dust and gas surround the nascent proto-sun, the μm-sized dust grains collided, coalesced and accreted to form km-sized planetesimals over a few million years, and then planetesimals form larger planetary embryos by gravity focusing. Like comets, asteroids are remnants from the formation of the early Solar System. In this sense, asteroids, which range in size from pebbles or lumps of ice, to rocky or icy worlds close to or more than 1000 km across, were the building blocks of proto-planets, and they delivered building blocks, carbon and water to the Earth biosphere for primal biological evolution like the comets. Millions of asteroids are located in the regions between Mars and Jupiter, known as the main belt asteroids in the Inner Solar System. In 1992, starting from 1992 QB1 (Jewitt & Luu 1993), the finding of the trans-Neptunian Objects (TNOs) or Kuiper Belt Objects (KBOs) has substantiated the conjecture dating back over 40 years by Edgeworth (1943) and Kuiper (1951), and they become new members of the asteroids family, orbiting in the outer solar system. The gravitational forces of the large planets, mostly the giant planet Jupiter, and collisions with other asteroids or comets, slowly alter the orbits of these small bodies. Following cumulative deflections, an asteroid or comet may occasionally become a near-Earth object (NEO), when its orbit intersects that of the Earth. Although countless numbers of asteroids and comets orbit the Sun, only a small fraction of them follow trajectories that bring them near the Earth, which means that

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they may even crash onto our Earth. The NEOs range in size from rocks to mountains, and travel at high speeds in the sky. Such objects have collided with the Earth since its formation [1], and an impactor with diameter ~ 200 m will cause a destruction on national scale. They have also caused widespread variations on the Earth's surface, global climate change, and occasional extinctions of such living organisms as the dinosaurs.

The history of post-formation of the Solar system is dominated by collisions amongst the planetesimals or the collisions into giant planets, terrestrial planets, and natural moons of them, i.e., the craters on the moon, other planets and the Earth as a result of impacts. In particular, one may recall the event of the dramatic collision of pieces of the comet Shoemaker-Levy 9 crashed onto Jupiter in 1994. As for NEO, one of the most eye-striking objects is called 2004 MN4 [now named (99942) Apophis] with diameter ~ 320 m, which was announced to collide into Earth on April 13th, 2029 with a probability of 3%. However, the probability of impact has now been reduced to 0.02% [2] due to the orbital improvement with more follow-up observations. In a word, such a disaster for Earth was true not only in the past, but may well happen someday in future, thus this threat should be taken seriously, and well recognized by humankind.

An object is said to be potentially hazardous when its orbit comes closer to Earth than 0.05 AU (~ 20 times the distance from Earth to Moon) with H brighter than or equal to $V = 22.0$, and a diameter at least 150 meters. As of 2006 Mar. 28, over 760 potentially hazardous objects [3] (PHOs) have been discovered, and this number is increasing all the time as such surveys continue. Given enough accurate measurements of the position of an asteroid, one can predict their paths over centuries, and one of our previous studies (Ji et al. 2001) reported on the computation of the orbits of 160 PHOs, listing their close approaches (e.g., Minimum Orbit Intersection Distance and the Encounter Epoch) with Earth during the next two centuries. However, as mentioned above, in their long-term dynamical evolution history in the Solar System, the NEOs continue to suffer small deflections due to the gravitational perturbations of giant planets, so that their orbits are not wholly predictable far into the future. Follow-up monitoring and ground-based observations [4] are extremely important. At the same time, we must also clarify the process of transport of these bodies.

It is well known that, the distribution of the main belt asteroids is modulated by the commensurabilities with Jupiter: they show concentration at the 3:2, 4:3 and 1:1 resonances, with Jupiter, and rarefaction at the 2:1, 3:1, 5:2 and 7:3 resonances, — the well known Kirkwood Gaps. In a pioneer work of tackling this problem, Wisdom (1983) studied the motion of fictitious planar asteroids near the 3:1 resonance and concluded that chaotic behavior for the small bodies can increase their eccentricities, making them strongly deviate from their initial orbits, and approach or intersect the orbit of Mars. On the other hand, the secular resonances are responsible for the long-term dynamical evolution of the small bodies. There are three governing secular resonances in the asteroidal belt, known as the ν_5, ν_6 and ν_{16} resonances. In general, NEOs are considered to be objects ejected from the main belt through some complicated dynamical process, where mean motion resonances as well as secular resonances play a vital role in their dynamical transportation (Morbidelli & Moons 1993; Moons & Morbidelli 1995; Froeschlé 1997). Moreover, Froeschlé et al. (1995) found that 19 out of 181 NEOs are at present associated with the ν_6 secular resonance, currently believed to be the most important mechanism for pumping up the orbital eccentricity. Morbidelli & Moons (1993) and Moons & Morbidelli (1995) pointed out that the overlapping of mean motion resonances and secular resonances can lead to large chaotic zones. Furthermore, an extensive review on the origin of NEOs by Morbidelli et al. (2002) also suggested other dynamical origins, such as 2:1, 5:2 resonances and chaotic diffusion. In this paper, we mainly focus on the study of the small objects involved in 3:1 resonance and revisit the dynamical mechanisms that transport the asteroids trapped in resonance into NEOs. In addition, in this study, we also found that several NEOs can be temporarily locked a 3:1 orbital resonance and also experience secular resonance ν_5 (or ν_6) with Jupiter (or Saturn), besides, we further show that the Kozai resonance also plays a major role in the evolution of asteroids moving about the near Earth zone.

[1] http://www.nearearthobjects.co.uk/, see NEO reports

[2] http://neo.jpl.nasa.gov/risk/a99942.html

[3] http://cfa-www.harvard.edu/iau/lists/Dangerous.html

[4] As the NEOs will cause catastrophic events to Earth in future, many space-monitoring programs (e.g., LINEAR, SPACEWATCH), have been set up to track down and follow such threats. The Chinese NEO Survey Program using a 1.0 m/1.2 m Schmidt telescope, sponsored by Purple Mountain Observatory, dedicated to the hunting and monitoring of NEOs, will be implemented at Xuyi by the end of 2006.

This paper is organized as follows: Section 2 introduces the dynamical model and the numerical preparations for the study; Section 3 describes the secular resonances for the asteroids; and Section 4 presents several scenarios of the orbital evolution of both the asteroids and test particles in 3:1 resonance. Finally we make a discussion in Section 5.

2 DYNAMICAL MODEL AND NUMERICAL SETUP

In our dynamical model, the large bodies are the Sun, the nine major planets from Mercury to Pluto. The orbits of the major planets are integrated as well as those of the asteroid. In the heliocentric ecliptic coordinate system referred to J2000.0, the equations of motion can be uniformly written as

$$\ddot{\vec{r}}_i = -(1+\mu_i)\frac{\vec{r}_i}{r_i^3} - \sum_{j=1,\, j\neq i}^{10} \mu_j \left(\frac{\vec{\Delta}_{ji}}{\Delta_{ji}^3} + \frac{\vec{r}_j}{r_j^3}\right), \quad i = 1, ..., 10 \tag{1}$$

where $\vec{r}_i$, $\vec{r}_j$ denote position vectors of the major planets ($i = 1 \sim 9$), and the asteroid ($i = 10$), with $\vec{\Delta}_{ji} = \vec{r}_i - \vec{r}_j$, μ_i and μ_j are the masses of these bodies in units of the solar mass. The mass of the asteroid can be neglected, so in the right-side of Equation (1) for the other bodies, there are no gravitational terms from the minor body.

In the simulations, we adopt the symplectic integrator (Feng 1986; Wisdom & Holman 1991; Liu et al. 1999) as a basic tool to numerically explore secular dynamics of the asteroids. The merit of symplectic algorithm is that it preserves the symplectic structure of the Hamiltonian system, and there will be no secular variation in the energy of the system. In addition, the symplectic algorithm has advantage over the traditional algorithms (e.g., Bulirsch-Stoer or RKF integrators) with its higher computing efficiency. At the same time, numerical errors were effectively controlled throughout the integration, and the total energy is generally conserved.

Our numerical setup is as follows: we use the afore-mentioned dynamical model and computational method to investigate long-term orbital evolution of the asteroids over a time span from one to a few 10^6 yr. We take the initial data of all the asteroids from the orbital elements database provided by Bowell [5] for 2000 September 13 (JD 2451800.50). The starting positions and velocities of the major planets, as well as their masses, are adopted from the JPL planetary Ephemerides DE405. Next, we choose the orbital data for a dozen of asteroids (50% are NEOs; the rest are non-NEOs) close to 3:1 MMR from the asteroid database, then numerically investigate the dynamical evolution for these minor planets in time. Here, only the results of the non-NEOs are mainly presented. Additional investigations for test bodies in 3:1 resonance are also carried out in this study.

3 SECULAR SOLUTIONS FOR THE ECCENTRICITIES

For an understanding of the dynamical mechanisms for the evolution of the asteroids, the notion of secular resonances occurring in the main belts is essential.

The secular solutions for the eccentricity vectors $h,\ k$ are (Murray & Dermott 1999)

$$\begin{aligned} h &= e\sin\varpi = e_{\rm free}\sin(At+\beta) + h_0(t), \\ k &= e\cos\varpi = e_{\rm free}\cos(At+\beta) + k_0(t), \end{aligned} \tag{2}$$

where $e_{\rm free}$ is the free eccentricity of the small body, A and β are constants, can be determined from the starting orbital parameters of the asteroid, and

$$\begin{aligned} h_0(t) &= -\sum_{i=1}^{N} \frac{\eta_i}{A-g_i}\sin(g_i t+\beta_i)\,, \\ k_0(t) &= -\sum_{i=1}^{N} \frac{\eta_i}{A-g_i}\cos(g_i t+\beta_i)\,. \end{aligned} \tag{3}$$

Secular resonance happens when the precession rate of the longitude of the periastron (or ascending node) of the small body equals that of Jupiter, $A - g_5 \simeq 0$ (the $\nu_5 (= g_5)$ resonance) or that of Saturn,

[5] ftp://ftp.lowell.edu/pub/elgb/astorb.dat.gz

Table 1 Orbital Parameters for Three Asteroids (JD=2451800.50)

Name	a (AU)	ecc.	Inc. (deg)	Ω (deg)	ω (deg)	M (deg)
1993 FT51	2.50020	0.155474	3.463677	51.3935	183.3266	279.4751
1993 TU19	2.49808	0.218585	2.624119	174.4952	217.1671	263.6700
1993 OE11	2.49888	0.183786	3.482880	109.2782	183.9833	294.7039

$(A - g_6 \simeq 0)$, (the $\nu_6 (= g_6)$ secular resonance). As we mentioned previously, when the asteroid is in a secular resonance, its eccentricity and inclination can be excited greatly. Secular resonance can also occur in the form of mean motion resonances. For example, Moons, Morbidelli & Migliorini (1998) studied the secular dynamics involved in the 2:1 Jovian resonance using a semi-analytical model, but found that this resonance cannot be a dominating source for delivering the asteroids to near the Earth. For the asteroids in 3:1 resonance, however, things could be quite different.

4 THE RESULTS

In the simulations, we found that several instances of asteroids being temporarily locked into the 3:1 resonance and also experiencing the secular ν_5 or ν_6 resonance; in some other cases, the asteroids were linked to the Kozai resonance (see Sect. 4.3), corresponding to the argument of perihelion about librating 90° or 270°. In the following, we will present some examples of the asteroids initially trapped in 3:1 resonance (at $a \simeq 2.50$ AU, see Table 1), then, owing to the above-mentioned mechanisms, transported into the near Earth space. We will also show that the outcomes of the test bodies related to 3:1 resonance, to further examine the dynamical origin of the NEOs.

4.1 1993 FT51, Involved in ν_6

The initial semi-major axes of 1993 FT51 is quite close to 2.50 AU, which means that this asteroid begins its dynamical journey in the neighborhood of the 3:1 resonance, and the starting eccentricity is also small, with $e \simeq 0.155$. Figure 1 shows that the time behavior of the semi-major axis, the eccentricity and $\varpi - \varpi_{\rm S}$. The semi-major axis a librates about 2.5 AU for 1 Myr, indicating that the asteroid is temporarily trapped in the 3:1 orbital resonance with Jupiter. Moreover, the argument $\varpi - \varpi_{\rm S}$ kept librating about 0° from 0.85 Myr to 1.0 Myr. Notice that in the bottom and middle panels, the eccentricity is pumped to above 0.8 due to the secular resonance ν_6 arising from Saturn, at $t \simeq 0.9$ Myr. Therefore, this small object can evolve into an Apollo-type NEO with $q < 1.0$ AU, entering the inner orbit of Earth.

4.2 1993 TU 19, Involved in ν_5

Figure 2 exhibits the dynamical evolution of the semi-major axis, eccentricity and $\varpi - \varpi_{\rm J}$ of 1993 TU 19. Here, a librates about 2.5 AU for 1 Myr with a small amplitudes ∼0.006 AU. In the bottom and middle panels, the eccentricity can be seen to be obviously excited to above 0.3 in a pulsed way in the time spans (0, 0.05 Myr), (0.40 Myr, 0.45 Myr) and (0.70 Myr, 1 Myr), when the asteroid is in ν_5 secular resonance with Jupiter, around $\varpi - \varpi_{\rm J} = 0$. In this way, this asteroid can move quite close to the orbit of Mars and become a Mars-crosser, even approach the Earth. On the other hand, while $\varpi - \varpi_{\rm J}$ is in circulation, the eccentricity remains quite small. The eccentricity goes down when $\varpi - \varpi_{\rm J}$ shifts from libration to circulation. This implies that an NEO can sometimes become an ordinary asteroid through complicated dynamical evolution.

4.3 1993 OE 11, Involved in Kozai Resonance

Kozai (1962) found that the resonance is present in the main belt only at very large inclinations when the perturbation by Jupiter causes the argument of perihelion ω of the asteroid to librate around 90° or 270°. Figure 3 shows the situation for the asteroid 1993 OE 11 locked into the Kozai resonance, when ω moves about 270° for most time on timescales ranging from 0.55 Myr to 0.85 Myr and it can also undergo libration about 90° for some time on the above timescales, the eccentricity can be maintained. The explanations are that for the new Kozai hamiltonian, the transformed system is reduced to one degree freedom and the Delaunay action variable $H = \sqrt{a\,(1-e^2)}\cos i$, remains constant. Additionally the semi-major axis a of the asteroid is unchanged during the secular orbital evolution after eliminating short-period terms of the

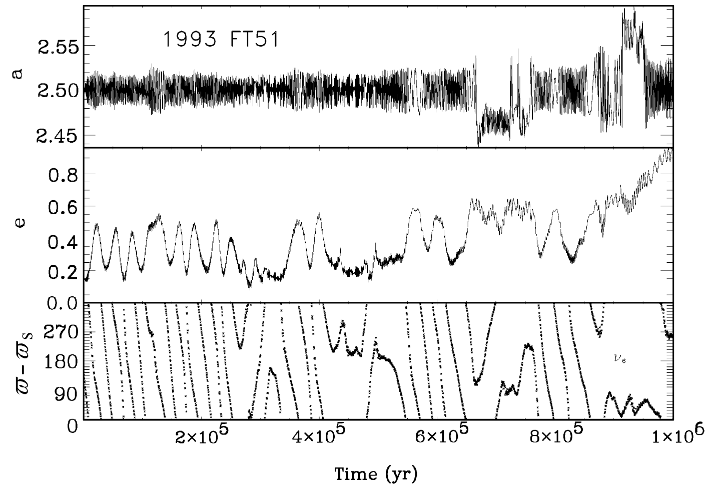

Fig. 1 Semi-major axis, eccentricity and $\varpi - \varpi_S$ plotted against time. The semi-major axis a librates about 2.5 AU for 1 Myr, indicating that the asteroid is temporarily trapped into 3:1 orbital resonance with Jupiter. Notice that in the bottom and middle panels, the eccentricity is pumped to above 0.8 due to the ν_6 secular resonance, at $t \simeq 0.9$ Myr.

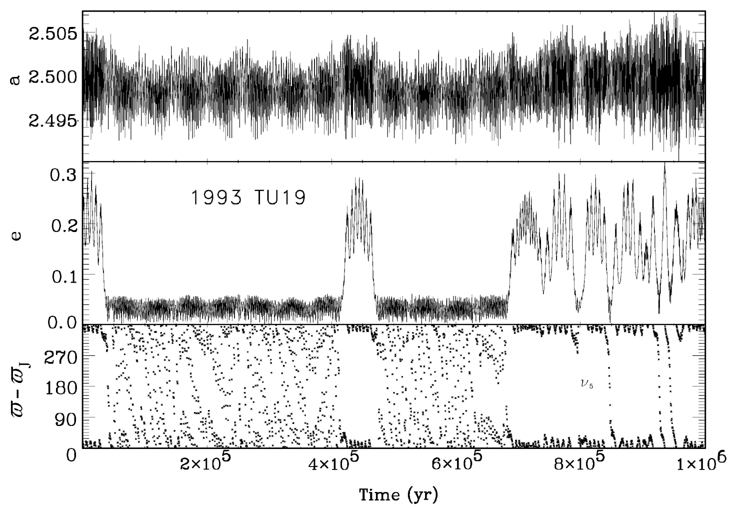

Fig. 2 Time behavior of semi-major axis, eccentricity and $\varpi - \varpi_J$. Here, a librates about 2.5 AU for 1 Myr with small amplitudes. In the bottom and middle panels, the eccentricity is obviously excited to above 0.3 when the asteroid is in the ν_5 resonance, when $\varpi - \varpi_J = 0$. When $\varpi - \varpi_J$ is in circulation, the eccentricity remains small.

perturbation hamiltonian, so one obtains $\bar{H} = \sqrt{(1-e^2)}\cos i$, a constant of motion, so that the inclination i is minimum when the eccentricity e is maximum and *vice versa*. However, the eccentricity balance is broken when the asteroid leaves the Kozai resonance, ω changes into circulation from libration, and finally the eccentricity is excited to above 0.60, indicating that this body becomes an NEO candidate in the end.

4.4 Test Particles Involved in 3:1 Resonance

As the 3:1 commensurability is a major gap in the asteroidal belt, we also make additional computations for the fictitious asteroids near the 3:1 resonance to further study the dynamical origin of the NEOs. We

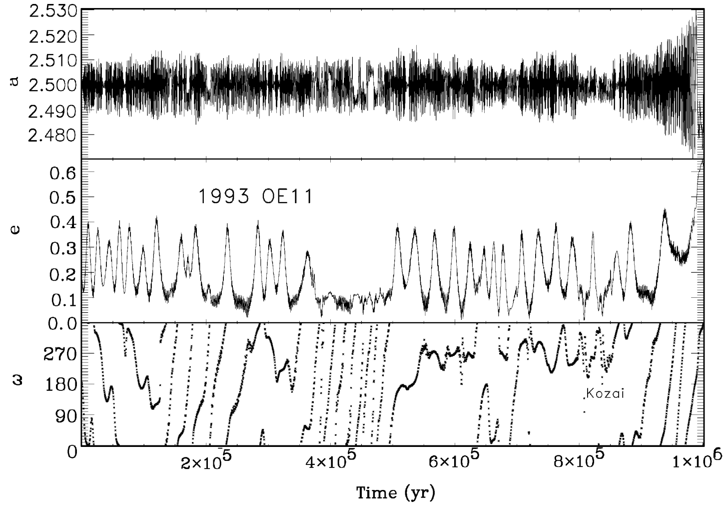

Fig. 3 Time behavior over 1 Myr of the semi-major axis, eccentricity and ω. Here, a librates about 2.5 AU with small amplitudes, ω librates about 270° for most time on timescales ranging from 0.55 Myr to 0.85 Myr, when higher values of eccentricity are maintained.

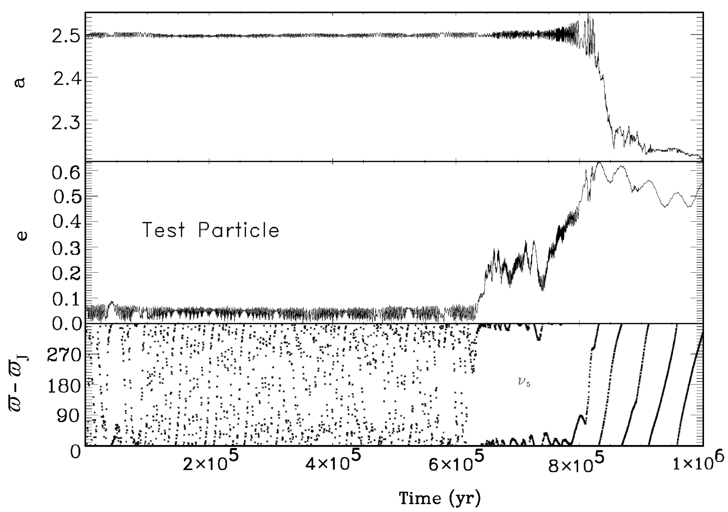

Fig. 4 Time behavior of semi-major axis, eccentricity and $\varpi - \varpi_J$ of a test particle. The semi-major axis a slightly vibrates about 2.50 AU during the first 0.6 Myr. However, in the time intervals of (0.65 Myr, 0.80 Myr), due to the ν_5 resonance (see the bottom and middle panels), the eccentricity e is excited to above 0.60, while a goes down to 2.20 AU, and the test particle finally becomes an NEO candidate.

numerically integrated 100 test particles, each for 1 Myr, with semi-major axes about 2.50 AU, eccentricities in the range $0 < e < 0.3$, inclinations $0° < I < 5°$, and the other angles randomly chosen between $0°$ and $360°$. The numerical results yielded the following statistics: over the timescale of 1 Myr, we found that 82 out of 100 test particles remained close to the 3:1 resonance with semi-major axes ~2.50 AU, 78 test bodies were temporarily involved in the ν_5 resonance with a typical timescale of 0.2–0.4 Myr, 26 orbits were occasionally associated with the ν_6 resonance for 0.1–0.2 Myr, and 65 bodies were sometimes involved with the Kozai resonance with ω librating about 90° or 270° on a timescale of 0.1 Myr. These dynamical mechanisms combine to excite the eccentricities, and eventually 56% of the total population became NEO candidates within the calculation span of 1 Myr, with greatly shrunken semi-major axes and greatly pumped-up eccentricities. For example, Figure 4 illustrates the orbital evolution of a test particle: the semi-major axis a slightly vibrates about 2.50 AU during the first 0.6 Myr, then, during the time (0.65 Myr, 0.80 Myr) due to the ν_5 and 3:1 resonances, the eccentricity is excited to above 0.60 while a goes down to 2.20 AU, becoming an Earth-crossing body. Although the rest of the test particles still remained in the main belt (at the end of 1 Myr), they may well be excited by these same dynamical mechanisms on longer timescales. All the outcomes of the test bodies near the 3:1 resonance confirm the dynamical origin of asteroids previously obtained from observational database.

5 CONCLUSIONS AND DISCUSSION

In this paper, we numerically investigated the orbital evolution of three asteroids (NEOs) and 100 test bodies involved in the 3:1 resonance, and revisited the dynamical mechanism of the asteroid transported into the NEO region. The results show that the dynamical evolution of the asteroids about 2.5 AU is mainly determined by the 3:1 resonance, ν_5 and ν_6 secular resonances and Kozai resonance: the asteroids can evolve into NEOs through one or more of the dynamical mechanisms. Our results are in good agreement with the former studies, e.g., Bottke et al. (2002) also found that 3:1 and ν_6 can be an effective mechanism for the origin of NEOs. In addition, we also pointed out ν_5 plays the same important role of pumping-up the eccentricity of the asteroid as does ν_6. In the planetary systems, the ν_5 resonance is also at work (see Ji et al. 2005; Nagasawa et al. 2006) to excite the eccentricity of planetesimals or terrestrial planets and make them move inward due to angular momentum exchange. Hence, the study on the dynamical mechanisms of how the asteroids evolve into NEOs remains very important and may contribute to useful information when modelling the NEOs population (Morbidelli et al. 2002; Jedicke et al. 2003) to match the observed distribution of the NEOs.

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References

Bottke W. F., Morbidelli A., Jedicke R. et al., 2002, Icarus, 156, 399
Edgeworth K. E., 1943, JBAA, 53, 181
Feng K., 1986, J. Comp. Math., 4, 279
Froeschlé Ch. et al., 1995, Icarus, 117, 45
Froeschlé Ch., 1997, Celest. Mech., 65, 165
Jedicke R. et al., 2003, Icarus, 161, 17
Jewitt D. C., Luu J. X., 1993, Nature, 362, 730
Kuiper G. P., 1951, In: Hynek J. A., ed., On the Origin of the Solar System, in Astrophysics: a Topical Symposium. McGraw-Hill, New York, p.357
Ji J. H., Liu L., 2001, Chin. J. Astron. Astrophys. (ChJAA), 1, 549
Ji J. H., Liu L., Kinoshita H., Li G. Y., 2005, ApJ, 631, 1191
Kozai Y., 1962, AJ, 67, 591
Liu L., Ji J. H., Liao X. H., 1999, Chin. Astron. Astrophy., 23, 108
Morbidelli A., Moons M., 1993, Icarus, 102, 316
Morbidelli A. et al., 2002, In Asteroids III, W. F. Bottke Jr., A. Cellino, P. Paolicchi, R. P. Binzel (eds), Univ. of Arizona Press, Tucson, p.409
Moons M., Morbidelli A., 1995, Icarus, 114, 33
Moons M., Morbidelli A., Migliorini F., 1998, Icarus, 135, 458
Murray C. D., Dermott S. F., 1999, Solar System Dynamics, New York: Cambridge Univ. Press
Nagasawa M., E. W., Thommes S., Kenyon B. et al., 2006, The diverse origins of terrestrial planet systems. in Protostars & Planets V, in press
Wisdom J., 1983, Icarus, 56, 51
Wisdom J., Holman M., 1991, AJ, 102, 1528