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Algorithm of Ensemble Pulsar Time

Alexander E. Rodin[1] ⋆

Pushchino Radio Astronomical Observatory, Pushchino, Moscow region, 142290, Russia

Abstract An algorithm of the ensemble pulsar time based on the Wiener filtration method has been constructed. This algorithm has allowed the separation of the contributions of an atomic clock and a pulsar itself to the post-fit pulsar timing residuals. The method has been applied to the timing data of the millisecond pulsars PSR B1855+09 and PSR B1937+21 and allowed to filter out the atomic scale component from the pulsar phase variations. Direct comparison of the terrestrial time TT(BIPM96) and the ensemble pulsar time $\mathrm{PT_{ens}}$ has displayed that the difference TT(BIPM96) – $\mathrm{PT_{ens}}$ is within $\pm 0.4\,\mu$s range. A new limit of gravitational wave background based on the difference TT(BIPM96) – $\mathrm{PT_{ens}}$ was established to be $\Omega_g \mathrm{h}^2 \sim 10^{-10}$.

Key words: time — pulsars: individual: PSR B1855+09, PSR B1937+21 — methods: data analysis

1 INTRODUCTION

The discovery of pulsars in 1967 (Hewish et al. 1968) and millisecond pulsars in 1982 (Backer et al. 1982) and consequent observations had shown clearly that their rotational stability allowed them to be used as astronomical clocks.

In this paper the author presents a method of forming the ensemble pulsar time scale (PT). The method is based on the optimal Wiener filter. In Section 2 principles of pulsar timing are described with regard to time scales. Section 3 contains a theoretical algorithm of the Wiener filter and construction of the ensemble pulsar time scale. Secttion 4 presents results, Section 5 discusses an application of the algorithm to timing data of pulsars PSR B1855+09 and PSR B1937+21 (Kaspi, Taylor & Ryba 1994).

2 PULSAR TIMING

An observer which is situated on the Earth rotating around its axis and moving around the Sun receives with a radio telescope a pulsar signal during an integration time to obtain sufficient signal-to-noise ratio. Time of arrival (TOA) of the integrated pulses are measured with the observatory frequency standard (e.g. H-maser) by the maximum of the cross-correlation between the integrated pulse and the pulse template. The obtained topocentric TOAs τ_N are in the scale of the local frequency standard and therefore required to be converted to the barycentric time scale via the following expressions (Manchester & Taylor 1977; Doroshenko & Kopeikin 1990):

$$\mathrm{UTC} = \tau_N + \Delta\tau_1,\ \mathrm{TAI} = \mathrm{UTC} + k,\ \mathrm{TT} = \mathrm{TAI} + 32.184\ \mathrm{s}, \quad (1)$$

where $\Delta\tau_1$ is the correction of the local scale to the universal coordinated time UTC. The international atomic time (TAI) differs from UTC by k integer number of seconds introduced to reconcile the lengths of day measured by an atomic clock and the Earth rotation. TAI is related with the terrestrial time TT by the Equation (1), where the constant shift 32.184 s prevents a jump between ephemeris and atomic time. Since a second of TT has various lengths depending on the position and velocity of the Earth in its orbit then a

⋆ E-mail: rodin@prao.ru

transformation from TT to TB scale is required which is performed on the basis of the paper by Fairhead & Bretagnon (1990). Once converted from TT to TB the topocentric TOAs need to be reduced to the barycentre of the Solar system (SSB) according to the following transformation formula (Manchester & Taylor 1977; Doroshenko & Kopeikin 1990):

$$T = t - t_0 + \Delta_R(\alpha, \delta, \mu_\alpha, \mu_\delta, \pi) - \Delta_{\rm orb} - D/f^2 + \Delta_{\rm rel} + \Delta t_{\rm clock}, \tag{2}$$

where t_0 is reference epoch, t is pulsar topocentric TOA in TB scale, T is TOA at the Solar system barycenter in TB scale, $\Delta_R(\alpha, \delta, \mu_\alpha, \mu_\delta, \pi)$ is the Roemer delay along the Earth orbit, $\alpha,\ \delta,\ \mu_\alpha,\ ,\mu_\delta,\ \pi$ are the pulsar right ascension and declination, proper motion in right ascension and declination, and parallax respectively, $\Delta_{\rm orb}$ is the Roemer delay along the pulsar orbit in the case of a binary pulsar, D/f^2 is dispersive delay for propagation at frequency f (corrected for the Doppler shift) through the interstellar medium, $\Delta_{\rm rel}$ is time corrections due to relativistic effects in the Solar system and the pulsar orbit, and $\Delta t_{\rm clock}$ is the offset between the observatory frequency standard and the terrestrial time.

Time of arrivals at the SSB are then used for calculation of the pulsar rotational phase (in cycles)

$$N(T) = N_0 + \nu T + \frac{1}{2}\dot{\nu}T^2 + \varepsilon(T), \tag{3}$$

where N_0 is the initial phase at epoch $T = 0$, ν, $\dot{\nu}$ are the pulsar spin frequency and its derivative respectively at epoch $T = 0$, $\varepsilon(T)$ is phase variations (timing noise). The fitting procedure includes adjustment of the parameters N_0, ν, $\dot{\nu}$, α, δ and so on, to minimise the weighted sum of squared differences between $N(T)$ and the nearest integer. Usually the pulsar rotational phase residuals are expressed in units of time $\delta t = \delta N/\nu$. In this paper we deal with the only part of the residuals that includes the variations of the clock offset $\Delta t_{\rm clock}(T)$.

When comparing different realisations of atomic time scales between each other one can see that they are dominated by flicker frequency noise on intervals of a few months and random walk in frequency on intervals of years (Guinot 1988), i.e. in the frequency domain the clock variations have power spectrum of form $1/\omega^n$, in the time domain clock variations can be expressed in the polynomial form

$$\Delta t_{\rm clock}(\mathcal{T}) = c_0 + c\mathcal{T} + \frac{1}{2}\dot{c}\mathcal{T}^2 + \frac{1}{6}\ddot{c}\mathcal{T}^3 + \ldots. \tag{4}$$

One can see that the appearance $\Delta t_{\rm clock}$ in Equation (3) results in a coupling between pulsar and clock parameters:

$$N(\mathcal{T}) = N_0' + (1+c)f\mathcal{T} + \frac{1}{2}(f\dot{c} + (1+c)^2\dot{f})\mathcal{T}^2, \tag{5}$$

where $\mathcal{T}$ is the ideal time scale, $f,\ \dot{f}$ are the pulsar frequency and its derivative not subjected to the influence of the clock parameters. For this reason one should use TOAs expressed in the best available time scale TT (Guinot & Petit 1991).

3 FILTERING TECHNIQUE

Let us consider n measurements of a random value $\boldsymbol{r} = (r_1, r_2, \ldots, r_n)$ are given. $\boldsymbol{r}$ is a sum of two uncorrelated values $\boldsymbol{r} = \boldsymbol{s} + \varepsilon$, where $\boldsymbol{s}$ is a random signal to be estimated and associated with the clock contribution, ε is random errors associated with the fluctuations of pulsar rotation. Both values $\boldsymbol{s}$ and ε should be related to the *ideal* time scale since a pulsar on the sky "does not know" about the time scales used for their timing. The problem of the Wiener filtration is concluded in estimation of the signal $\boldsymbol{s}$ if the measurements $\boldsymbol{r}$ and the covariances $\langle s_i, s_j\rangle$ and $\langle \varepsilon_i, \varepsilon_j\rangle,\ (i,j = 1,2,\ldots,n)$ are given (Gubanov 1997; Vaseghi 2000). The optimal estimation of the signal $\hat{\boldsymbol{s}}$ is expressed by the formula (Gubanov 1997; Vaseghi 2000)

$$\hat{\boldsymbol{s}} = \boldsymbol{Q}_{sr}\boldsymbol{Q}_{rr}^{-1}\boldsymbol{r} = \boldsymbol{Q}_{ss}\boldsymbol{Q}_{rr}^{-1}\boldsymbol{r} = \boldsymbol{Q}_{ss}(\boldsymbol{Q}_{ss} + \boldsymbol{Q}_{\varepsilon\varepsilon})^{-1}\boldsymbol{r}, \tag{6}$$

where $\boldsymbol{Q}_{rr}$, $\boldsymbol{Q}_{ss}$ are the covariance matrices of the noise data $\boldsymbol{r}$ and signal $\boldsymbol{s}$ respectively, $\boldsymbol{Q}_{sr}$, $\boldsymbol{Q}_{rs}$ are the cross-covariance matrices between $\boldsymbol{r}$ and $\boldsymbol{s}$. The covariance matrix $\boldsymbol{Q}_{ss}$ is calculated as cross-covariances

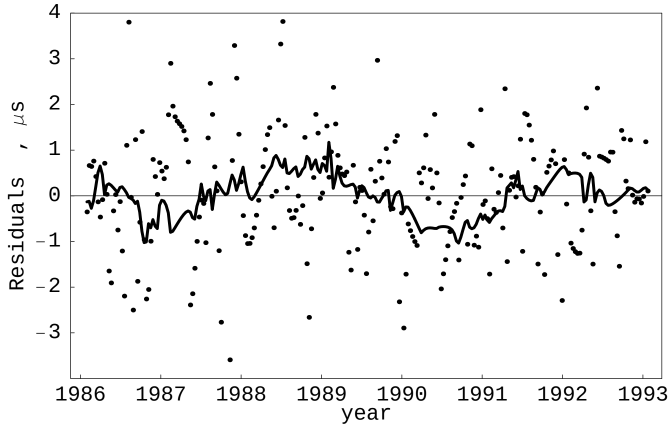

Fig. 1 The barycentric timing residuals of pulsars PSR B1855+09 (dots) and PSR B1937+21 (continuous line).

$\left\langle {}^k r_i, {}^l r_j \right\rangle = \langle s_i, s_j \rangle\,,\ (k,l = 1,2,\ldots,M;\ i,j = 1,2,\ldots,n)$, M is a total number of pulsars. In the formula (6) the matrix $\boldsymbol{Q}_{rr}^{-1}$ serves as the whitening filter. The matrix $\boldsymbol{Q}_{ss}$ forms the signal from the whitened data.

The ensemble signal (time scale) is expressed as following

$$\hat{\boldsymbol{s}}_{\rm ens} = \frac{2}{M(M-1)} \sum_{k=1}^{\frac{M(M-1)}{2}} {}^k\boldsymbol{Q}_{ss} \cdot \sum_{i=1}^{M} w_i {}^i\boldsymbol{Q}_{rr}^{-1} \cdot {}^i\boldsymbol{r}, \tag{7}$$

where w_i is the relative weight of the ith pulsar, $w_i \propto 1/\sigma_i$, σ_i is the root-mean-square of the expression ${}^i\boldsymbol{Q}_{rr}^{-1} \cdot {}^i\boldsymbol{r}$.

4 RESULTS

The method described in the previous section has been applied to the pulsar timing data of PSR B1855+09 and B1937+21 (Kaspi, Taylor & Ryba 1994). Though these data are regular they are unevenly spaced, therefore a cubic spline approximation has been applied to make them uniform with sampling interval 10 days. Such a procedure perturbs a high-frequency component of the data and leaves unchanged a low-frequency component which is of interest.

The common part of the residuals for both pulsars (251 TOAs) has been taken within the interval MJD= $46450 \div 48950$. Since the residuals after the procedure of dropping their ends have the different mean and the slope they have been quadratically refitted for consistency with the classical timing fit. The residuals after all treatments described above are shown in Figure 1.

According to (Kaspi, Taylor & Ryba 1994) the timing data of PSRs B1855+09 and B1937+21 are in UTC time scale, hence the signal to be estimated is the difference UTC – PT. Figure 2 shows the signal estimates based on the residual TOAs of pulsars PSR B1855+09 and PSR B1937+21. The ensemble signal and the difference UTC – TT display similar behaviour.

5 DISCUSSION

For calculation of the fractional instability of a pulsar as a clock a statistic σ_z has been proposed (Taylor 1991). A detailed numerical algorithm for calculation of σ_z has been described in the paper (Matsakis, Taylor & Eubanks 1997).

Figure 3 presents the fractional instability of PSR B1855+09, PSR B1937+21 and TT – PT$_{\rm ens}$. The theoretical lines of σ_z (Kaspi, Taylor & Ryba 1994) in the case when the timing residuals are dominated by the gravitational wave background with $\Omega_g {\rm h}^2 = 10^{-9}$ and 10^{-10} are plotted in the lower right hand corner. One can see that σ_z of TT – PT$_{\rm ens}$ crosses line $\Omega_g {\rm h}^2 = 10^{-9}$ and approaches $\Omega_g {\rm h}^2 = 10^{-10}$.

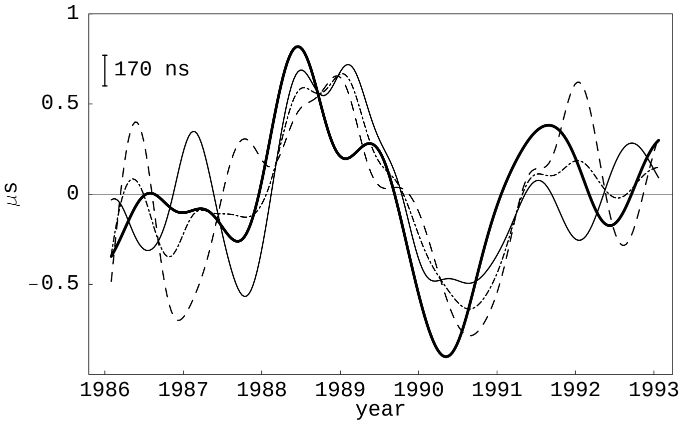

Fig. 2 Combined clock variations of UTC – PT in interval MJD= $46450 \div 48950$ estimated using the optimal filtering method based on the timing residuals of pulsars PSR B1855+09 (thin line), PSR B1937+21 (dashed line), ensemble UTC – PT$_{\rm ens}$ (dot-dashed line) and UTC – TT (solid line).

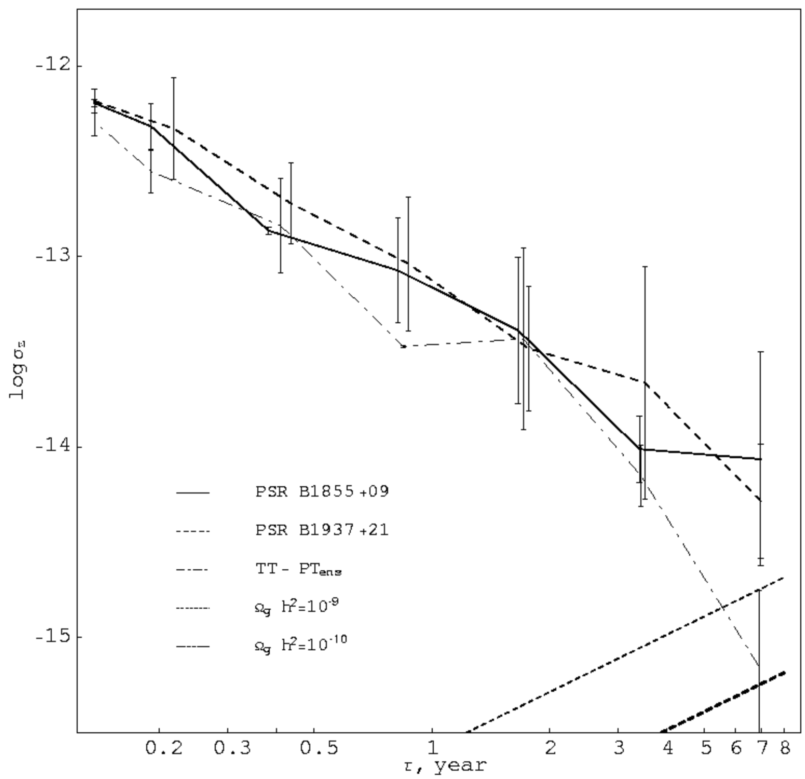

Fig. 3 The fractional instability σ_z for pulsars PSR B1855+09 (solid line), PSR B1937+21 (dashed line) and TT – PT$_{\rm ens}$ (dot-dashed line).

The fractional instability of the TT relative to PT$_{\rm ens}$ is at level of 10^{-15} at 7 years interval and almost one order better than the fractional instability of the pulsars PSR B1855+09 and PSR B1937+21. It is expected that reliability of TT – PT$_{\rm ens}$ estimation will grow up by increasing the number of pulsars participating in PT$_{\rm ens}$ as $M(M-1)/2$ (the number of cross-correlations). Currently the accuracy of the filtering method described above without contribution of the uncertainty of TT algorithm is estimated at level 170 ns. This uncertainty is obtained as root mean square value of the data points taken within the smoothing interval of span m. The span m was calculated from the equivalent bandwidth of the low-pass filter applied to the ensemble data for more easy comparison with UTC – TT line. The uncertainty of PT$_{\rm ens}$ may, in principle, reach the nanosecond level if to use all the observed highly-stable millisecond pulsars.

The method proposed can not distinguish the 2nd order polynomial trends in the reference clock and the pulsar phase due to pulsar period slowing-down. However, this is not a problem if to consider the timing data at more long intervals and process them off-line. Under such processing the long-term details appears as the data span is increased.

The low fractional accuracy of $\dot{P}$ mentioned in the paper (Guinot & Petit 1991) produces no disadvantages when processing off-line since no prediction of the pulsar rotational phase is performed. However if one does need to predict a behaviour of the concrete atomic scale variations, e.g. UTC, then this can be done on the basis of the UTC – PT$_{\rm ens}$ data by using standard forecasting methods for the time series, e.g. the auto-regression method with reservation that only relatively short-term variations without quadratic trend are forecasted. Under such an approach the unsatisfactory fractional accuracy of the spin period derivative does not play significant role since the phase variations are predicted rather than an absolute value.

The proposed filtering method can be applied in "inverse" form: one pulsar and a few reference clocks. In such case it is also possible to separate the pulsar timing noise and the clock variations relative to the ideal time scale rather than to obtain a simple clock difference.

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