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The 3-D Trajectories of Pulsars in the Galaxy

Ying-Chun Wei[1] ⋆, Xin-Ji Wu[1,2], Qiu-He Peng[3] and Na Wang[1]

[1] National Astronomical Observatories / Urumqi Observatory, Chinese Academy of Sciences, Urumqi 830011
[2] Department of Astronomy, Peking University, Beijing 100871
[3] Department of Astronomy, Nanjing University, Nanjing 210093

Abstract Based on the undisturbed finitely thick disk gravitational potential, 3-D trajectories of pulsars are followed with initial locations and velocities randomly selected from a model distribution. Three typical instances are followed for some 10^{11} yr, and their Poincaré sections are used as diagnostics of their motion. We find that the vertical-to-parallel range ratio (relative to the Galactic plane) to be an important parameter: as this ratio increases, the pulsar's motion changes from being regular to being irregular/chaotic.

Key words: pulsars: general — stars: kinematics — stars: rotation

1 INTRODUCTION

It is generally believed that pulsars are born in the Galactic disk with rather high velocities. Lyne & Lorimer (1994) suggested that the pulsars' mean birth velocity is ∼450 km s^{-1}. Hansen & Phinney (1997) got a smaller value of about 250−300 km s^{-1}. Cordes & Chernoff (1998) suggested a two-component Maxwellian distribution of birth velocities with two 1-D velocity dispersions, σ_v ∼175 km s^{-1} and 700 km s^{-1} and Arzoumanian et al. (2002) suggested a similar one with σ_v ∼90 km s^{-1} and 500 km s^{-1} by.

The undisturbed finitely thick disk gravitational potential obtained by Peng Qiu-He et al. (1978) is adopted here, it is

$$\phi(r,z) = -\pi G\alpha \int_0^\infty J_0(\beta r)S(\beta)\Theta(\alpha,\beta;z)d\beta. \qquad (1)$$

where

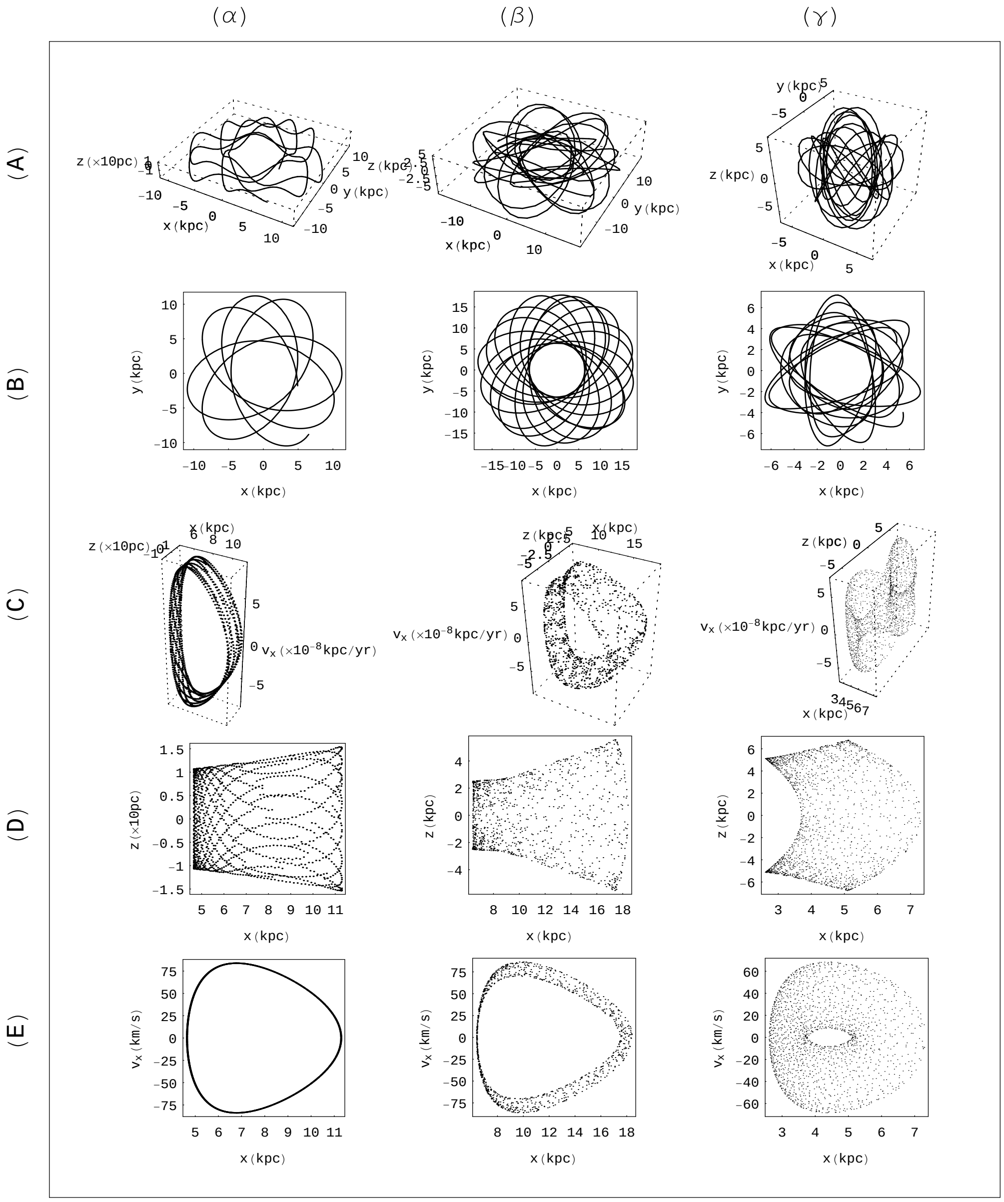
$$\Theta(\alpha,\beta;z) = \int_{-\infty}^{\infty} e^{-\beta|z-z'|-\alpha|z'|}dz' = \frac{2}{\beta^2-\alpha^2}(\beta e^{-\alpha|z|} - \alpha e^{-\beta|z|}). \qquad (2)$$

For our simulation, we adopt $h_d = 4.7\,\mathrm{kpc}$, $\sigma(R_\odot) = 52.1 M_\odot\ \mathrm{pc}^{-2}$ (Hartman et al. 1997; Sun & Han 2004), $R_\odot = 8.0\,\mathrm{kpc}$ following Sun & Han (2004). $h_z = 0.325\,\mathrm{kpc}$ is the scale height of the Galactic disk.

Pulsar's initial condition, i.e. initial location and velocity is selected randomly from some distribution model, then we calculate its 3-D trajectory with a fourth-order Runge-Kutta method as did Paczyński (1990). This calculation is done again and again until finally we think we have got enough instances to say something about the pulsars' 3-D trajectories.

We adopt a rectangular coordinate system with origin at the Galactic center, and the z-axis vertical to the Galactic plane. The pulsars' random initial location is taken from an exponential distribution in z: $\lambda e^{-\lambda|z|}$, $\lambda = 1/0.07\,\mathrm{kpc}^{-1}$, and a Gamma distribution (Paczyński 1990) in $R = \sqrt{x^2+y^2}$. The initial velocity of each pulsar is the vector sum of the circular rotation velocity at the birth position and a random velocity from the supernova explosion from an isotropic Maxwellian distribution of σ_v=200 km s^{-1}. The circular rotation velocity follows Paczyński (1990).

⋆ E-mail: `wuxj@vega.bac.pku.edu.cn`



x=4.932 kpc	vx=−5.5 km/s	x=−14.044 kpc	vx=62.65 km/s	x=4.871 kpc	vx=12.1 km/s
y=−1.794 kpc	vy=172.2 km/s	y=0.1915 kpc	vy=77.44 km/s	y=2.544 kpc	vy=62.6 km/s
z=0.00012 kpc	vz=1.07 km/s	z=0.08448 kpc	vz=58.44 km/s	z=0.236 kpc	vz=−154.4 km/s
T=2×10^9 yr	Tp=7.0×10^11 yr	T=10^10 yr	Tp=6.1×10^11 yr	T=5×10^9 yr	Tp=7.7×10^11 yr

Fig. 1 The three columns (α, β, γ) refer to the three instances of initial locations and velocities listed in the bottom. The various rows are: (A) is the 3-D trajectory; (B) is the projection of the 3-D trajectory on the Galactic plane; (C) is the Poincaré section $x > 0, y = 0$ of the 3-D trajectory; (D) is the projection of the Poincaré section on the $x - z$ plane; (E) is the projection of the Poincaré section on the $x - v_x$ plane.

2 THE CALCULATED PULSAR 3-D TRAJECTORIES IN THE GALAXY

The pulsar's total energy is $E_t = E_p + E_k$, where $E_k = \frac{1}{2}m(v_x^2 + v_y^2 + v_z^2)(> 0)$ is the kinetic energy, and $E_p(< 0)$ is the potential energy in the Galactic gravitational field. In the case of "escaping" pulsars with $E_t > 0$, the 3-D trajectory is a conic hyperbola.

In the cases of $E_t < 0$ and $x/y = v_x/v_y$, the projections of their trajectories on the galactic plane degenerate into straight lines through the galactic center and an oscillation in the z direction.

For the general case, the Poincaré section is utilized as s an effective diagnostic tool of the motion (Henon & Heiles 1964). For a given n-dimensional phase space Poincaré selected a $(n-1)$-dimension hypersurface with one of the coordinates fixed; each time the object passes the section it leaves a point in the section, the resulting map of reflecting the object's continual motion in the phase space. Henon & Heiles (1964) studied a system of two linear oscillators coupled by a power-3 term. The Hamiltonian of the system is

$$H = \frac{1}{2}(p_1^2 + q_1^2 + p_2^2 + q_2^2) + \left(q_1^2 q_2 - \frac{1}{3}q_2^3\right). \qquad (3)$$

The phase space is 4-dimensional (p_1, q_1, p_2, q_2), but the trajectory in the phase space is confined to a 3-dimensional hypersurface, because the total energy is conservative. The Poincaré section is 2-dimensional.

To start with, the pulsar has 6 dimensions (x, y, z, v_x, v_y, v_z) in the phase space, but the total energy and the angular momentum in the z dimension are conserved, reducing the phase space to 4-dimensions (x, y, z, v_x, say) and the Poincaré sections are then 3-dimensional.

We select the Poincaré section of $x > 0, y = 0$. The projections of the Poincaré sections on the $x - z$ and $x - v_x$ planes are also shown in order to bring out their differing structures. T_p is the time span covered in Figure 1(C). When the orbital eccentricity is small, the projection of the pulsar's 3-D trajectory on the galactic plane is approximately a circle, but when the eccentricity is not small, the projection is a precessing rosette around the Galactic center (see Fig. 1(B)).

The three cases (α, β, γ) in Figure 1 are in order of increasing ratio of the ranges perpendicular and parallel to the galactic plane. We can see from the Poincaré sections, especially their projections on the $x - v_x$ plane (E), that as we move from case α, to β, to γ, the pulsars' motion becomes increasingly irregular. In α, the projection of the Poincaré section on $x - v_x$ plane is essentially a closed curve: the motion is quasi-periodic. In β, the closed curve is dispersed and broadens: the pulsar's motion begins to show a certain amount of irregularity. Finally in γ, the motion becomes completely irregular and chaotic and the representative points almost fill an entire region apart from a small part in the middle.

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