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Time Series Analysis: the "True" Fourier Spectrum Derived by Iterative Process

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Abstract A method to find the "true" Fourier spectrum for unevenly spaced time series is developed. It is found that the "true" Fourier spectrum associates with the conventional Fourier spectrum by a system of linear equations, so the "true" Fourier spectrum can be obtained by any methods of solving the system of linear equations (here the method of iterative process is choosed). It is an effective method for detecting and describing the "true" multiperiodic signals, even in the case where some strong peaks in a conventional Fourier spectrum occur at spurious frequencies. For the "true" Fourier spectrum composed of finite isolated harmonic components, this method gives a better estimation of the frequencies and amplitudes. This method is tested using simulated time series and the published data for servel blazars. Then it is applied to some radio variabilities of a sample of blazars. In some cases, typical timescales of several decades are found, indicating that this method is capable of finding very low frequency signals.

Key words: numerical methods — quasars — radio sources: variable

1 INTRODUCTION

The estimation of the power spectrum of a time series formed by unevenly spaced observations is one of the more difficult problems in time series analysis (Jones et al. 1977). In many branches of science, such as astronomy, geophysics et al., unevenly spaced observations are sometimes unavoidable. Weather, telescope availability, and the seasons are responsible for series of astronomical data where the spacing between consecutive observations may range from hours to weeks, or even months.

In any case, irregular spacing introduces myriad complications into the Fourier transform. First, it can alter the peak frequency (slightly) and amplitude (greatly). Second, when the data have gaps which recur with a period all their own, extremely large false peaks appear, which are ghost images of the real peaks.

Many attempts have been made to deal with these problems; for example, Scargle (1982) introduced the modified periodogram and Ferraz-Mello (1981) introduced the Date Compensated Discrete Fourier Transform (DCDFT). Both these methods improve the estimates of frequency and amplitude for Fourier spectrum.

Based on the DCDFT, Foster (1995) introduced the Cleanest Fourier Spectrum. It is shown to be an effective technique for detecting and describing multiperiodic signals.

For most variable BL Lac objects, long-term observation data exhibit random time spacing which fools the conventional Fourier methods. Motivated by a desire to better analyse long-term BL Lac data, we have developed a new technique that provides an improved measure of the "true" Fourier spectrum for randomly sampled data.

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2 FOURIER ANALYSIS

First, we use frequency ν, as we know, $\omega = 2\pi\nu$.

Most period-search techniques, such as Fourier analysis, are based on modeling the observed data as a linear combination of trial functions,

$$f(t) = \int_{-\infty}^{\infty} e(\nu', t)\, TF(\nu')\, d\nu', \tag{1}$$

where $f(t)$ is a homogeneous stochastic process, $e(\nu', t)$ is the function for a trial frequency ν', and $TF(\nu')$ is the "true" Fourier spectrum. From the Fourier analysis, we have the usual estimator,

$$F(\nu) = \int_{-\infty}^{\infty} e^*(\nu, t) f(t)\, dt, \tag{2}$$

where $F(\nu)$ is the Fourier spectrum, $e^*(\nu, t)$ is the complex conjugate of $e(\nu, t)$.

From Equations (1) and (2), we can see,

$$\begin{aligned} F(\nu) = \int_{-\infty}^{\infty} e^*(\nu, t) f(t)\, dt \; &= \; \int_{-\infty}^{\infty} e^*(\nu, t) \int_{-\infty}^{\infty} e(\nu', t)\, TF(\nu')\, d\nu' dt \\ &= \; \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^*(\nu, t) e(\nu', t)\, dt\, TF(\nu')\, d\nu'. \end{aligned} \tag{3}$$

For the continuous time variation,

$$\int_{-\infty}^{\infty} e^*(\nu, t) e(\nu', t)\, dt = \delta(\nu - \nu'), \tag{4}$$

so we can obtained that,

$$F(\nu) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^*(\nu, t) e(\nu', t)\, dt\, TF(\nu')\, d\nu' = TF(\nu). \tag{5}$$

3 THE "TRUE" FOURIER SPECTRUM AND THE SYSTEM OF LINEAR EQUATIONS

Let $f(t_1)$, $f(t_2)$, ..., $f(t_n)$ be the time series formed by samplings of such periodic process. For the "true" Fourier spectrum composed of finite isolated harmonic components,

$$f(t_i) = \sum_{l=1}^{m} e(\nu_l, t_i)\, TF(\nu_l). \tag{6}$$

The Fourier analysis is substituted by discrete Fourier transform (DFT),

$$F(\nu) = \sum_{i=1}^{n} e^*(\nu, t_i) f(t_i). \tag{7}$$

Then, from Equations (6) and (7), we obtain that,

$$\begin{aligned} F(\nu) = \sum_{i=1}^{n} e^*(\nu, t_i) f(t_i) \; &= \; \sum_{i=1}^{n} e^*(\nu, t_i) \sum_{l=1}^{m} e(\nu_l, t_i)\, TF(\nu_l) \\ &= \; \sum_{l=1}^{m} \sum_{i=1}^{n} e^*(\nu, t_i) e(\nu_l, t_i)\, TF(\nu_l). \end{aligned} \tag{8}$$

We observe that Equation (4) is unavailable,

$$\sum_{i=1}^{n} e^*(\nu, t_i) e(\nu_l, t_i) \neq \delta(\nu - \nu_l). \tag{9}$$

So the Fourier spectrum is not equal to the “true” Fourier spectrum,

$$F(\nu) = \sum_{l=1}^{m}\sum_{i=1}^{n} e^*(\nu, t_i) e(\nu_l, t_i)\, TF(\nu_l) \neq TF(\nu). \tag{10}$$

If we define,

$$\begin{aligned} S_{kl} &= \sum_{i=1}^{n} \bar{e}(\nu_k, t_i) e(\nu_l, t_i), \\ TF_l &= TF(\nu_l), \\ F_k &= F(\nu_k). \end{aligned} \tag{11}$$

From Equation (8), we got a m-unknown system of linear equations,

$$S_{kl} TF_l = F_k. \tag{12}$$

By solving the system of linear equations, $TF(v)$ can be obtained. The method of iterative process is a powerful tool for solving such systems.

4 EXAMPLES

In Figure 1, a simulated time series that result from three isolated harmonic components at the frequencies 0.5, 1, and 2 year^{-1}, with amplitudes 0.75, 5, and 1, respectively, the observed times are same with the observation of QSO 0003–066 in 4.8 GHz. The Date-compensated discrete Fourier transform Ferraz-Mello (1981) are used to get Fourier spectrum. We tested the method described above using both simulated and real data from QSO 0003–066. A simulated time series was constructed with three isolated harmonic components at frequencies 0.5, 1 and 2 year^{-1}. The three components were given relative amplitudes of 0.75, 5 and 1, respectively. We sampled this time series at the same intervals for which observations of QSO 0003–066 were available and used both the DCDFT and our new method to analyze the real and simulated data.

In Figure 2, we see the “true” Fourier spectra of the radio data of QSO 0003–066, with a single dominant frequency of very large amplitude. Note that the other strong features of DCDFT turn out to be false, disappearing from the “true” Fourier spectrum.

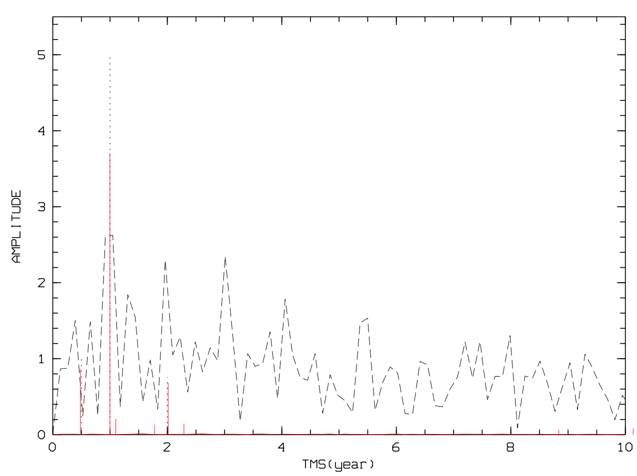

Fig. 1 DCDFT Fourier spectrum of the simulated time series that result from three isolated harmonic components (dotted curve), these three components are marked with dashed lines, and the “true” Fourier spectrum formed using 7 isolated harmonic components and residual spectrum (solid curve).

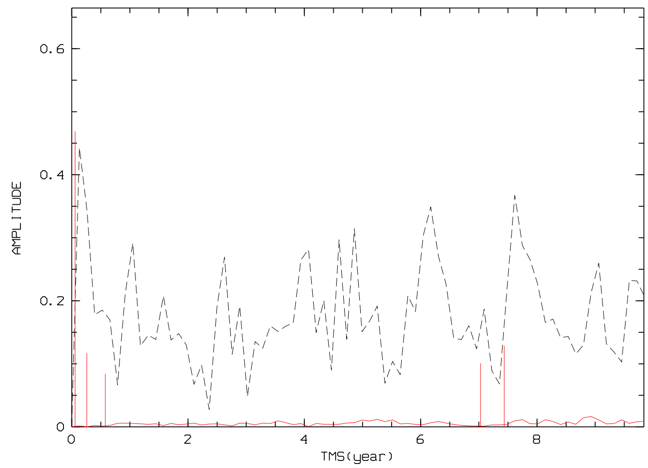

Fig. 2 DCDFT Fourier spectrum of the time series of QSO 0003–066 observed in 4.8 GHz (dotted curve), and the “true” Fourier spectrum which formed by 10 isolated harmonic components and residual spectrum (solid curve).

5 CONCLUSIONS

We have demonstrated that iteratively solving for the “true” Fourier spectrum is a powerful tool for the analysis of randomly sampled data. We plan to further explore the possibilities of this method in the future.

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