# Tail Emission from a Ring-like Jet: Its Application to Shallow Decays of Early Afterglows and GRB 050709 * 

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#### Abstract

Similar to the case of pulsars the magnetic axis and the spin axis of gamma-ray burst sources may not lie on the same line. This may cause the formation of a ring-like jet due to collimation of the precessing magnetic axis. We analyze the tail emission from such a jet, and find that it has a shallow decay phase with a temporal index of $-1 / 2$ if the Lorentz factor of the ejecta is not very high, which is consistent with the shallow decay phase of some early X-ray afterglow detected by Swift. The ring-like jet has a tail cusp with sharp rising and very sharp decay. This effect can provide an explanation for the re-brightening and sharp decay of the X-ray afterglow of GRB 050709.


Key words: gamma rays: bursts - X-rays: general

## 1 INTRODUCTION

It is well known that pulsars originate from the core collapse of massive stars. The average angle between their spinning and magnetic axes is about $27^{\circ}$ (Leahy 1991). Similarly, the spin axis and magnetic axis of the central engine of gamma-ray burst may not lie on one line. Since ejecta may be collimated by a precessing magnetic axis, the ejecta may assume a spiral shape initially (Fargion \& Grossi 2005, and reference therein). According to the standard fireball model of gamma-ray bursts (Piran 2005), the ejecta have diverse velocities, and the spiralling ejecta ejected at different times will collide and merge eventually into one shell, and the collisions will produce internal shocks of gamma-ray bursts. Eventually, the collisions cause the ejecta to merge into a ring-shaped jet. Even if the the ejecta is conical in shape, the baryonloaded region will still be ring-like (Eichler \& Levinson 2003). Granot(2005) and Eichler \& Granot (2005) have analyzed afterglows from ring-like jets which has also been used to interpret the $h \nu_{\text {peak }}-E_{\text {iso }}$ relation (Eichler \& Levinson 2004).

Tail emission plays an important role at the times when shocks disappear. The temporal index is $-(2+$ $\beta$ ) for a cone-shaped jet, where $\beta$ is the spectral index of the emission (Kumar \& Panaitescu 2000; Yamazaki et al. 2005). Considering the zero point effect of time, the light curves can be steeper during a short period (Nousek et al. 2005; Zhang et al. 2005; Wu et al. 2006). Nousek et al. (2005) and Zhang et al. (2005) have also shown that a shallow decay with index of about $-1 / 2$ follows the steep decay for most X-ray afterglows. For the X-ray afterglow of short burst GRB 050709 , there is an unexpected high-flux point followed by a very steep decay (Fox et al. 2005). Both these two observations can be explained naturally by considering the tail emission of ring-like jets. In Section 2 we give the expressions of tail emission from a ring-like jet. Sections 3 and 4 analyze respectively the shallow decay and X-ray afterglow of GRB 050709. Section 5 summarizes our results.

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## 2 MODEL

Consider several ring-like sub-jets emitted from the central engine: they merge into one whole ring-like jet accompanied with internal shocks. This final ring with uniform energy density and sharp edges expands with Lorentz factor $\gamma$, as sketched in Figure 1. Let the radiation from the ring-like jet begin and suddenly cease at radius $R_{c}$ at time $t_{c}$. We calculate the tail emission from high latitudes of the ring. The relation is $R_{c} \simeq 2 \eta^{2} c t$, where $\eta$ is the mean Lorentz factor of the internal shocks.


Fig. 1 Sketch of a ring-like jet at a distance $R_{c}$ from the central engine, where the main emission has just ceased, while an observer begins to receive the tail emission from high latitudes. Here $\theta_{p}$ is the half opening angle of the ring, $\theta_{w}$ is the half width angle, $\theta$ is the latitude of tail emission region corresponding to observed time $t$.

The relation between the latitude angle $\theta$ and the observed time $t$ is

$$
\begin{equation*}
R_{c}(1-\cos \theta)=c\left(t-t_{c}\right) /(1+z) \tag{1}
\end{equation*}
$$

where $z$ is the cosmological redshift. Neglecting the depth of the ejecta and the emission from the equal arrival-time surface, and defining the emissivity $I_{\nu^{\prime}}^{\prime}$ per unit area in the comoving frame, which is uniform in the whole ring, the flux density in the observer's frame is

$$
\begin{equation*}
f_{\nu}\left(t>t_{c}\right)=\frac{I_{\nu^{\prime}}^{\prime}}{4 \pi D_{L}^{2}} D^{2} \frac{\mathrm{~d} S}{\mathrm{~d} t /(1+z)} \tag{2}
\end{equation*}
$$

where $D_{L}$ is the luminosity distance, $D=1 /\left[\gamma\left(1-\sqrt{1-1 / \gamma^{2}} \cos \theta\right)\right]$ is the Doppler factor, and $\mathrm{d} S$ is the emitting area during period $\mathrm{d} t$.

At an early time when $\theta<\theta_{w}$, the tail emission is the same as in the case of an on-axis conical jet, which has been investigated by several authors (Kumar \& Panaitescu 2000; Fan \& Wei 2005). There are two limiting cases: for $\theta \ll 1 / \gamma$,

$$
\begin{equation*}
f_{\nu}\left(t>t_{c}\right) \propto \delta t^{0} \tag{3}
\end{equation*}
$$

and for $1 \gg \theta \gg 1 / \gamma$,

$$
\begin{equation*}
f_{\nu}\left(t>t_{c}\right) \propto \delta t^{-(2+\beta)} \tag{4}
\end{equation*}
$$

where $\delta t \equiv t-t_{c}$. We consider the emission to be a single power-law profile, $I_{\nu^{\prime}}^{\prime} \propto \nu^{\prime-\beta}$, which is valid for the high frequency emission $\nu^{\prime}>\max \left(\nu_{c}^{\prime}, \nu_{m}^{\prime}\right)$, with $\nu_{c}^{\prime}$ the cooling frequency and $\nu_{m}^{\prime}$ typical frequency of synchrotron emission.

In the case $\theta>\theta_{w}$, the width of the ring can be neglected, and the flux density is

$$
\begin{equation*}
f_{\nu}\left(t>t_{c}\right) \propto D^{-(2+\beta)} \frac{\sin \theta_{p} \cos (\theta / 2)}{\sin \theta \sqrt{1-\left(\frac{\sin (\theta / 2)}{\sin \theta_{p}}\right)^{2}}} \tag{5}
\end{equation*}
$$

There are two limiting cases in which Equation (5) can be simplified. For $\theta \ll 1 / \gamma$,

$$
\begin{equation*}
f_{\nu}\left(t>t_{c}\right) \propto \delta t^{-1 / 2} \tag{6}
\end{equation*}
$$

and for $1 \gg \theta \gg 1 / \gamma$,

$$
\begin{equation*}
f_{\nu}\left(t>t_{c}\right) \propto \delta t^{-(5 / 2+\beta)} \tag{7}
\end{equation*}
$$

## 3 SHALLOW DECAY OF EARLY X-RAY AFTERGLOW

Statistics of early X-ray afterglows has shown that there is a shallow decay phase with temporal index about $-1 / 2$ (Nousek et al. 2005; Zhang et al. 2005). This corresponds to the case: $1 / \gamma>\theta>\theta_{w}$, and can be described by Equation (6), where the temporal index is just $-1 / 2$. As a shallow decay generally lasts from $10^{2}-10^{3}$ s to $10^{3}-10^{4} \mathrm{~s}$ (fig. 1 in Zhang 2005), we have the limits: the Lorentz factor of the emitting shell $\gamma<7.3(1+z)^{1 / 2} R_{c, 16}^{1 / 2} \delta t_{3.5}^{-1 / 2}$, and the width of the ring-like jet $\theta_{w}<1.4 \times 10^{-2}(1+z)^{-1 / 2} R_{c, 16}^{-1 / 2} \delta t_{2.5}^{1 / 2}$ (the conventional notation $Q=Q_{k} \times 10^{k}$ is used throughout this paper). This implies that the shallow decay component originates from shocked shells with low Lorentz factors, while the shocks may have been formed from ejected sub-shells with different Lorentz factors.

This model can answer the following questions:
First, why is there a steep decay before the shallow decay in the general case? In Zhang et al. (2005), the temporal index of this steep decay is generally less than -3 . The answer is that the two power-law decays originate from two different emitting shells with different Lorentz factors: the steep decay corresponds to the one with the larger Lorentz factor, which satisfies $1 / \gamma<\theta$, and the temporal indices are $-(2+\beta)$ or $-(5 / 2+\beta)$ corresponding to Equations (4) and (7), respectively. The steep decay may become even steeper because of the zero time selection effect (Wu et al. 2006).

Secondly, why is there no spectral evolution before and after the break time from the shallow decay phase to the steep decay phase? This feature was mentioned for spectral index $\sim-1$ in Zhang et al. (2005). It is believed that after the break the afterglow becomes a "normal" afterglow. It is possible that the tail emission phase and the "normal" afterglow emission phase both belong to in the case $\nu_{X}>\left\{\nu_{m}, \nu_{c}\right\}$ (corresponding to the spectral index $-p / 2$ ) and so their spectra are the same.

Thirdly, since the shallow phase and the steep phase originate from different sources, how are we to understand the conjunction at the break time (also can be seen in fig. 1 of Zhang et al. 2005)? As time goes on, the case of $1 / \gamma>\theta>\theta_{w}$ changes to the case of $\theta>1 / \gamma>\theta_{w}$, and then the light curve of the tail emission decay has a temporal index $-(5 / 2+\beta)$. This is steeper than the "normal" afterglow with temporal index $\sim-1.2$. Some time later, the "normal" afterglow will exceed the tail emission definitely, as in the case GRB 050525 a (fig. 1 of Nousek 2005) (at about 3000 s, there is a steep decay). However, GRB 050315 (Vaughan et al. 2005) can be classified into the case where the "normal" afterglow exceeds the shallow tail emission before the latter breaks to its steep phase.

## 4 X-RAY AFTERGLOW OF GRB 050709

GRB 050709 is a short burst with duration 0.3 s , and five points of X-ray emission after the burst were obtained by Swift and Chandra (Fox et al. 2005). Figure 2 shows the fit by assuming that the latter four points are the tail emissions from the first point, with parameters $R_{c}=7.7 \times 10^{16} \mathrm{~cm}, \gamma=15.5$, $\beta=1.1, \theta_{p}=0.5$ and $\theta_{w}=0.005$. As the first X-ray point occurs at time about 100 s , the radius, $r \simeq 2 \eta^{2} c t /(1+z) \simeq 5.2 \times 10^{16} \eta_{2}^{2} t_{2} \mathrm{~cm}$, is consistent with the value of the parameter $R_{c}$. As a short burst has less total energy than a long burst, the ejected shell can be decelerated quickly. The Lorentz factor at $R_{c}$ is $\gamma \simeq E_{\text {iso }} /\left(\pi R_{c}^{3} n m_{p} c^{2}\right) \simeq 26 E_{\text {iso } 50}^{1 / 2} n_{1}^{-1 / 2} R_{c, 16.5}^{-3 / 2}$, where the external medium density $n$ is chosen to be $1 \mathrm{~cm}^{-3}$ because the host is a star-forming galaxy. Therefore, the parameters chosen to fit the X-ray data are reasonable for this short burst.

We can see four stages for this tail light curve: a horizontal phase corresponding to the regime $\theta<$ $\theta_{w}<1 / \gamma$, a shallow decay with temporal index $-1 / 2$ corresponding to the regime $\theta_{w}<\theta<1 / \gamma$, a sharper decay with temporal index $-(2.5+\beta)$ corresponding to the regime $\theta_{w}<1 / \gamma<\theta$, and finally a tail cusp with sharp rising and very sharp decay, which comes from the end of the ring.


Fig. 2 Tail emission fit to the X-ray emission from GRB 050709. The five observed values are taken from (Fox et al. 2005).

We should note that the solid line does not fit the data very well, especially that the tail cusp of the model cannot reach the observed value. This shortcoming may be overcome by considering a non-uniform ring-like jet or some other mechanism. However, its unique feature, that the emission after the tail cusp decays very sharply, is consistent with the last two observed points. On the other hand, it is possible that the second, third and fifth points in Figure 2 belong to the "normal" afterglow from an external shock.

## 5 CONCLUSIONS

With insights gained from pulsars, we suggest that the magnetic axis and the spin axis of a gamma-ray burst source point to different directions. The ejecta along the magnetic axis will eventually form a ring. Gamma-ray emission will be observed if the observer is located inside the solid angle of the ring. We have investigated the tail emission from a ring-like jet. We find that the early shallow decay phase and the late re-brightening of the X-ray emission of GRB 050709 can be explained by this scenario.

Note that the shallow decay phase exists only in the low Lorentz factor cases. For the case $1 / \gamma<\theta_{w}$, only the steep one appears. As the tail emission from the shells with high Lorentz factors decays very quickly, the main emissions will be dominated by the slower shells at some later time.

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## References

Eichler D., Granot J., 2006, ApJ, 641, L5
Eichler D., Levinson A., 2003, ApJ, 596, L147
Eichler D., Levinson A., 2004, ApJ, 614, L13
Fan Y. Z., Wei D. M., 2005, MNRAS, 364, L42
Fargion D., Grossi M., 2005, MNRAS submitted (astro-ph/0504638)
Fox D. B., Frail D. A., Price P. A., et al., 2005, Nature, 437, 845
Granot J., 2005, ApJ, 631, 1022
Kumar P., Panaitescu A, 2000, ApJ, 541, L51
Nousek J. A., Kouveliotou C., Grupe D. et al., 2006, ApJ, 642, 389
Leahy D. A., 1991, MNRAS, 251, 203
Piran T., 2005, Rev. Mod. Phys., 76, 1143
Vaughan S., Goad M. R., Beardmore A. P., et al., 2006, ApJ, 638, 920
Wu X. F., Dai Z. G., Wang X. Y., et al., 2006, ApJ, submitted (astro-ph/0512555)
Yamazaki R., Toma K., Ioka K., Nakamura T., 2006, MNRAS, 369, 311
Zhang B., Fan Y. Z., Dyks J. et al., 2006, ApJ, 642, 354


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