Collapse Velocity and Prompt Explosion for the Presupernova Model Ws15 M_{\odot} *

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Abstract For the presupernova model Ws15 M_{\odot} , we re-calculate the electron capture (EC) timescale and hydrodynamical (HD) timescale. We found that the EC timescale can be smaller than the HD timescale in the inner region of the collapse iron core at the moment immediately before the shock wave bounce. The change in these two timescales at the late stage of core collapse is expected to affect the collapse velocity. If the late-time collapse velocity is artificially increased by a small quantity, then prompt explosion of the supernova may happen. Further calculations are still needed to check the plausibility of the acceleration mechanism caused by the faster EC process.

Key words: star: supernova — nuclear reaction — nucleosynthesis

1 INTRODUCTION

Despite the fact that the explosion mechanism of the core-collapsed supernova (SN) has been investigated extensively in the last four decades and significant progress has been achieved by many authors, some of the most fundamental questions are still unanswered. Theoretically, a massive star $(M > 8M_{\odot})$ develops an iron core after a period of nuclear burning. When the mass of the iron core exceeds the appropriate Chandrasekhar mass ($M_{\rm ch} = 5.83 Y_{\rm e}^2 M_{\odot}$, $Y_{\rm e}$ is the electron fraction), electron degeneracy pressure can no longer prevent the star from contracting: the star becomes gravitationally unstable and begins to collapse. In the initial stage of collapse, electron capture (EC) plays an essential role. It not only reduces the number of electrons, but also carries away energy and entropy from the core in the form of neutrinos. Both these effects conspire to accelerate the collapse. As the density at the center reaches a maximum, the falling outer core collides with the stiffened inner core and produces a bounce shock. If the shock can rush out of the iron core with an energy of about 10^{51} erg, prompt explosion happens. However, two effects act to prevent the development of prompt explosion. The first is that too much energy is lost in the photodisintegration of iron nuclei to free nucleons (Wang et al. 1997). The second is neutrino emission from behind the shock, especially as it moves to lower-density regions below 10¹²g cm⁻³ where neutrinos can diffuse out ahead of the shock. Here μ and τ neutrinos participate in the shock cooling, as electron neutrinos do (Woosley et al. 2002). Detailed numerical simulation indicates that the shock is not able to rush out of the iron core and

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stall in the envelopes. The intense neutrino flux from the proto-neutron star (PNS) heats the stalled shock and the revived shock drives off the envelopes, — the so-called "delayed explosion" (Wilson et al. 1993). In order to obtain success of explosion, Wilson et al. (1993, 2006) increased the neutrino energy deposition above the "gain radius" and the neutron-finger convection in the PNS, but the existence of neutron-finger instability depends on the very specific thermodynamical properties of equation of state and on the details of neutrino transport (Woosley et al. 2002). Baron & Cooperstein (1990) changed the gravity constant to be an adjustable variable. Wang et al. (1997) changed the photodisintegration energy of α particle from -7.075 MeV to -3.075 MeV. Up to now, various physical factors, including updated nuclear information and neutrino physics, are taken into account (Buras et al. 2003; Mezzacappa 2005). The weak-interaction process in the sub-nuclear regime is reexamined (Langanke et al. 2000; Luo et al. 1997). Many different models of presupernova and multi-dimension hydrodynamic simulations have been performed (Kane et al. 2000; Heger et al. 2001). More recently, some new mechanisms were proposed (Burrows et al. 2006). Peng (2004) took into account possible effect on the late-time collapse velocity of the difference between the EC timescale t_{ec} and the hydrodynamical (HD) timescale t_{hd} . However, little numerical simulation has been done based on this new proposal. We think such work is necessary and helpful to the exploration of SN. The aim of this paper is to make a preliminary exploration on the validity of Peng's new collapse scenario.

In this work, numerical simulations are performed using a modified version of the program Wlyw89, in which general relativistic effect, neutron finger convection, Schwarzchild convection and Ledoux convection are taken into account. The method used to deal with neutrino transport is the same as that adopted in Xie et al. (1996). The presupernova model we choose is Ws15 M_{\odot} with an iron core of $1.377 M_{\odot}$ (Woosley et al. 1995). It is divided into 96 mass layers with the boundary set at $1.6 M_{\odot}$. Matter in the model follows the "four particles model", consisting of four types of particles, i.e., free protons, free neutrons, α -particles, and heavy nuclei, which can well represent the whole property of presupernova (Lattimer et al. 1991; Wang et al. 2003).



Fig. 1 Calculated average mass numbers of heavy nuclei under different mass densities at the beginning of core collapsing for the model $Ws15M_{\odot}$.

2 EC AND HD TIMESCALES

EC is a key physical process for nucleosynthesis and neutrino production in SN, especially for corecollapsed SN (including type SNII, SNIb and SNIc) (Peng 2001; Langanke et al. 2000). Now, the time step in the numerical simulation is usually decided by $t_{\rm hd}$ and the difference between $t_{\rm ec}$ and $t_{\rm hd}$ is ignored, however, because $t_{\rm ec}$ is usually much longer than $t_{\rm hd}$ when the density is not high enough. As the iron core contracts, the densities of the central region of the core will become higher, the EC rates will increase more rapidly and the EC timescale will become shorter. The relationship between EC timescale $t_{\rm ec}$ and EC rate $\lambda_{\rm ec}$ is

$$t_{\rm ec} = \lambda_{\rm ec}^{-1} \,. \tag{1}$$

In general, the rate of change of electron fraction caused by EC is given by Luo et al. (2001)

$$\dot{Y}_{\rm e} = -\sum_k \frac{X_k}{A_k} \lambda_{\rm ec}^k \,, \tag{2}$$

where, X_k and A_k are the fraction and the mass number of the k-th nucleus, and λ_{ec}^k is its rate of EC. In principle, accurate EC rate should be calculated by the shell model (Fuller et al. 1982; Langanke et al. 2000). However, different nuclei have different EC rates and most nuclei in presupernova star are in unstable excited states. We therefore use "four particles model" to calculate the EC of the iron core. In this case, free electrons can only be captured by the free protons and average heavy nuclei, so the total \dot{Y}_e can be written as

$$\dot{Y}_{\rm e} = \dot{Y}_{\rm eH} + \dot{Y}_{\rm ep} \,, \tag{3}$$

where \dot{Y}_{eH} and \dot{Y}_{ep} are the rates of the electron fraction caused by the protons and heavy nuclei, respectively (Bethe et al. 1979; Wang et al. 2003),

$$\dot{Y}_{\rm eH} = -Y_{\rm e}\rho N_{\rm A}x X_H c\hat{\sigma}_0 \int \frac{3}{\mu_{\rm e}^3} \left(\frac{\varepsilon_{\nu}}{m_{\rm e}c^2}\right)^2 \varepsilon_{\rm e}^2 d\varepsilon_{\rm e} \frac{3m_p}{k_f^2} d\varepsilon_p \,, \tag{4}$$

$$\dot{Y}_{\rm ep} = -X_p \frac{8\pi c\hat{\sigma}_0}{(hc)^3} \int \left(\frac{\varepsilon_\nu}{m_{\rm e}c^2}\right)^2 \varepsilon_{\rm e}^2 d\varepsilon_{\rm e} \,. \tag{5}$$

Here ρ is density, N_A is the Avogardro constant, x = Z/A, X_H , X_p are the fraction of heavy nuclei and free proton, respectively; $\hat{\sigma}_0 = 1.18 \times 10^{-44} \text{ cm}^2$, c is the velocity of light, μ_e the chemistry potential of electron, ε_{ν} , ε_p , ε_e the energy of neutrino, proton and electron, respectively; m_p the mass of proton, and k_f the Fermi momentum. From Equations (2)–(5) the EC rate can be rewritten as

$$\lambda_{\rm ec} = AY_{\rm e}\rho N_{\rm A}xc\hat{\sigma}_0 \int \frac{3}{\mu_{\rm e}^3} \left(\frac{\varepsilon_{\nu}}{m_{\rm e}c^2}\right)^2 \varepsilon_{\rm e}^2 d\varepsilon_{\rm e} \frac{3m_p}{k_f^2} d\varepsilon_p + \frac{8\pi c\hat{\sigma}_0}{(hc)^3} \int \left(\frac{\varepsilon_{\nu}}{m_{\rm e}c^2}\right)^2 \varepsilon_{\rm e}^2 d\varepsilon_{\rm e} \,, \tag{6}$$

where A is the average mass number of the heavy nuclei, which is a function of the temperature, density and electron fraction. Figure 1 shows the average mass number of the heavy nuclei as a function of the mass density, at the beginning of core collapse (different densities correspond to different mass shells). It can be seen that the average mass number increases with increasing mass density and that the maximal mass number at this time is slightly above 60.

Figure 2 shows a comparison between the EC timescale t_{ec} and the HD timescale t_{hd} . In our calculation of t_{ec} we have included the contribution of β decay (the inverse process to EC) to the electron number density. The HD timescale can be expressed as (Woosley et al. 2002)

$$t_{\rm hd} \approx 446 \rho^{-1/2} \,, \tag{7}$$

 ρ being the mean density interior to radiation radii, close to the local density. Figure 2 shows that $t_{\rm ec}$ is much greater than $t_{\rm hd}$ in the initial stage of collapse. The contrary condition $t_{\rm ec} < t_{\rm hd}$ is satisfied only in the central part of the 0.112 s before-bounce curves. As the collapse proceeds, the region which satisfies the condition $t_{\rm ec} < t_{\rm hd}$ will extend to about $1.0 M_{\odot}$. The reason is that EC rate increases rapidly with increasing density of the inner part of the iron core. From the analysis we find that there exits a moment before shock bounce when the EC timescale is comparable to the collapse proceeds.

Now, consider a mass shell at a given time step during the collapse stage. For regions with $t_{ec} < t_{hd}$, more electrons will be captured than in the fiducial case, and consequently the number densities and the pressure of electron gas will decrease quickly. Now, the total pressure, $P = P_e + P_h + P_Y + P_v + P_G$, is the sum of the pressures of electrons and nucleons (P_e and P_h), the gas pressure of neutrons, protons, α particles and the average heavy nuclei (P_Y), and the pressures of neutrinos and photons (P_v , P_G). So the total pressure will also decrease more in this time step. Let a rough estimate of the difference between the two timescales be $\Delta T = t_{\rm hd} - t_{\rm ec}$, so for a time step decided by $t_{\rm hd}$, as is usual in simulations, the electron capture time is $\Delta T_{\rm ec} = (t_{\rm ec}/t_{\rm hd})\Delta T$, and the residual time in this time step is $\Delta T' = \Delta T - \Delta T_{\rm ec}$. The decrease of $Y_{\rm e}$ can also be derived from $Y_{\rm e} \rightarrow Y_{\rm e} + \dot{Y}_{\rm e}\Delta T'$. During $\Delta T'$ the free protons and heavy nuclei can capture more electrons than before, which must result in a decrease of the electron degenerate pressure. The decrease of the total pressure P depends not only on the difference between $t_{\rm ec}$ and $t_{\rm hd}$, but also on the EC rate in that time step. Since the time step is very short in our calculation, the EC rate was assumed to be a constant for each mass layer in a given time step. The calculation results yielded from the above method are shown in Table 1. From it we can see that the total pressure, electron pressure and Y_e all decrease.

 Table 1
 Comparison of Some Parameters at 0.022 s before Bounce in a Time Step

j	$t_{ m ec}$	$t_{ m hd}$	P	P'	$P_{\mathbf{e}}$	$P'_{ m e}$	$Y_{\rm e}$	$Y'_{ m e}$
10	1.5906×10^{-4}	1.3890×10^{-3}	2.0517×10^{11}	2.0506×10^{11}	2.0460×10^{11}	2.0443×10^{11}	0.3950	0.3948
20	3.3659×10^{-4}	1.7789×10^{-3}	1.1309×10^{11}	1.1245×10^{11}	1.1217×10^{11}	1.1103×10^{11}	0.4001	0.3970
30	7.7433×10^{-4}	2.3389×10^{-3}	5.6026×10^{10}	5.5758×10^{10}	5.5658×10^{10}	5.5144×10^{10}	0.4170	0.4140
40	2.0441×10^{-3}	3.1872×10^{-3}	2.5177×10^{10}	2.5068×10^{10}	2.4899×10^{10}	2.4675×10^{10}	0.4313	0.4283

Notes: *j* represents the *j*-th mass layer. (P, P_e, Y_e) and (P', P'_e, Y'_e) are the total pressure, electron pressure and electron fraction before and after considering the difference between the two timescales, respectively.

The collapsing process at mass layers with $t_{ec} < t_{hd}$ will accelerate due to the faster decrease of the electron pressure, which may increase the energy of the bounce shock and hence lead to a successful explosion. In order to investigate by at least how much the velocity should increase in a time step, we use the following velocity (v_i) instead of the conventional collapse velocity (v_{i0}) , defined as

$$v_{i} = \begin{cases} \left(1 + \frac{t_{\rm hd} - t_{\rm ec}}{t_{\rm hd}} \alpha\right) v_{i0} & (t_{\rm ec} < t_{\rm hd}) \\ v_{i0} & (t_{\rm ec} \ge t_{\rm hd}) \end{cases}, \quad (i = 1, 2, \cdots, 96) \tag{8}$$

where α is an adjustable parameter, *i* the *i*-th mass layer.

3 RESULTS OF SIMULATION

Table 2 shows the energy of the bounce shock at different α . It is found that the explosion energy increases with increasing α . When $\alpha = 0.0017$, the shock wave energy at $1 M_{\odot}$ is 4.837 foe (1 foe = 10^{51} erg) and decreases to 0.0393 foe at $1.38 M_{\odot}$. Thus, prompt explosion cannot happen under this condition. When $\alpha = 0.002$, the shock wave rushes out of the iron core with the energy of 0.6927 foe. This induces the SN to explode weakly. When $\alpha = 0.0023$ the SN will explode powerfully. Based on these simulation results, we conclude that prompt explosion will occur if the velocity is increased by some proper parameters at each time step when is satisfied the condition $t_{\rm ec} < t_{\rm hd}$ at the collapse stage.

Table 2 Energy Profile (in units of 0.1 foe) at Three Values of α

α	$0.8M_{\odot}$	$0.9M_{\odot}$	$1.0~M_{\odot}$	$1.1 \ M_{\odot}$	$1.2~M_{\odot}$	$1.28 M_{\odot}$	$1.3 M_{\odot}$	$1.38 M_{\odot}$
0.0017 0.0020	$0.00 \\ 0.00$	15.49 19.84	48.37 60.90	40.59 49.97	22.57 32.53	11.62 20.98	6.915 15.79	0.393 6.927
0.0023	0.00	27.21	73.64	59.98	42.16	29.37	24.21	15.11

Figure 3 compares the velocities before and after the collapse velocity is modified. We find in the fiducial case, the maximal velocity at the junction of the outer and inner cores is about 8×10^9 cm s⁻¹, the shock stalls at the position about $1.2 M_{\odot}$; and when Equation (8) is applied, the maximal velocity becomes more than 10^{10} cm s⁻¹, the shock velocity at the edge of the iron core is about 3×10^9 cm s⁻¹, which is almost the explosion velocity (Wooosley et al. 1995; Lin et al. 1996).



Fig. 2 EC and HD timescales as functions of the mass shell. The numbers 1, 2, 3 refer to, respectively, before-bounce times of 0.112 s, 0.022 s and 0.003 s.



Fig. 3 (a) Velocity profile after modification of the collapse velocity. 1: at the beginning of collapse; 2: when the density of the central core reaches the maximum; $3 \sim 9$: when the shock wave arrives the positions of 0.8, 0.9, 1.0, 1.1, 1.2, 1.3 and 1.38 M_{\odot} , respectively ($\alpha = 0.002$). (b) Velocity profile in the fiducial case at the same moments as indicated in Fig. 3a.

4 SUMMARY

We have found, from numerical simulation, that the EC timescale is shorter than HD timescale in the inner core in the late stage of collapse of the SN but the time is very limited, only 0.112 s before the shock bounce for presupernova model Ws15 M_{\odot} . It is advantageous for the SN explosion if this difference is taken into account in the numerical simulation. Our conclusions also support the new proposal of core collapse SN explosion (Peng 2004), which emphasized the importance of EC and HD timescales. However, the physical origin of the increase in the collapse velocity is not understood thoroughly. We suppose that the problem can possibly be solved by considering the minute difference between the two timescales.

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