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The Relation between the Critical Accretion Rate of Progenitors of SNe Ia and Metallicity *

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Abstract A carbon-oxygen white dwarf may explode in a Type Ia supernova by accreting matter from its companion via either Roche lobe overflow or from winds, but there exists a critical accretion rate of the progenitor system for the explosion. We study the relation between the critical accretion rate and the metallicity via an AGB star approach. The result indicates that the critical accretion rate depends not only on the hydrogen mass fraction and the white dwarf mass, but also on the metallicity. The effect of the metallicity is smaller than that of the white dwarf mass. We show that it is reasonable to use the model with stellar mass 1.6 $M_{\odot}$ for real white dwarfs.

Key words: stars: evolution — star: AGB — star: supernova: general

1 INTRODUCTION

As the best cosmological distance indicator, Type Ia supernovae (SNe Ia) have been successfully used to determine the cosmological parameters, e.g., $\Omega_{\rm M}$ and Ω_{Λ} (Reiss et al. 1998; Pernutter et al. 1999), but there still exist many problems to be resolved on the progenitors (Umeda et al. 1999). It is believed that SNe Ia are thermonuclear explosions of mass-accreting white dwarfs (WDs) (see the review by Nomoto, Iwamoto & Kishimoto 1997). However, the immediate progenitor binary systems have not been well identified (Branch et al. 1995). Hachisu & Kato (2003a, b) argued that supersoft X-ray source may be a good candidate for the progenitors of SNe Ia.

The widely accepted model of SNe Ia is a Chandrasekhar mass model: a carbon-oxygen white dwarf (CO WD) increases its mass by accreting hydrogen- or helium-rich matter from its companion and explodes when its mass reaches the Chandrasekhar mass limit (Hillebrandt & Niemeyer 2000; Leibundgut 2000). The companion may be a main-sequence star (WD+MS) or a symbiotic star (WD+RG) (Yungelson et al. 1995; Li et al. 1997; Hachisu et al. 1999a, b; Nomoto et al. 1999; Langer et al. 2000). In both of these cases, hydrogen is burned into helium and helium is burned into carbon and oxygen below the accreted layer, thus the mass of the CO WD increases until it approaches the Chandrasekhar mass limit, when carbon ignites at the degenerate center and the star explodes in a thermonuclear supernova. Whether a WD can reach the

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mass limit or not depends crucially on the accretion rate. If the accretion rate exceeds a critical value $\dot{M}_{\rm cr}$, the WD grows into a red giant-like star and the system enters a common envelope phase. On the other hand, if the accretion rate is lower than about $\frac{1}{8}\dot{M}_{\rm cr}$, hydrogen burning becomes unstable (Nomoto 1982a; Han & Podsiadlowski 2004). The unstable hydrogen burning eventually results in nova explosions, in which all the accreted matter may be ejected so that the mass of the WD never reaches the Chandrasekhar limit (Nomoto 1982a; Kovetz & Prialnik 1994). Unfortunately, the parameter space for stable hydrogen burning is so narrow that the derived birth rate of SNe Ia is much less than is observed (Nomoto 1982a, Nomoto et al. 2003). Some mechanism to control the accretion rate is required to overcome this. Hachisu et al. (1996) postulated an optically thick wind solution. Including this optical thick wind, Han et al. (2004) studied the birth rate of SNe Ia in binary population synthesis (BPS) and obtained a birth rate comparable to the observed rate. In their study, the critical accretion rate is a function of the hydrogen abundance and the WD mass, and the effect of metallicity is ignored. Kobayashi et al. (1998) argued that a WD cannot be considered as an SNe Ia progenitor if it has a metallicity lower than 0.002, because the optically thick wind in such low-metallicity environments is too weak for SNe Ia explosion. They predicted a significant decrease of the SNe Ia rate in low-metallicity environments, and so on at high redshifts ($z \geq 1.4$). However, this prediction does not seem to be borne out in redshift surveys (Gilliand et al. 1999; Strolger et al. 2004). Our purpose here is to explore the effect of metallicity on the critical accretion rate $\dot{M}_{\rm cr}$. In Section 2, we describe our physical inputs and model. In Section 3, we give our calculated result. A simple discussion and conclusions then follow in Section 4.

2 PHYSICAL INPUTS AND MODEL

We use the stellar evolution code of Eggleton (1971, 1972, 1973), updated with the latest input physics over the last three decades (Han et al. 1994; Pols et al. 1995, 1998).

We set the ratio of typical mixing length to the local pressure scale height, $\alpha = \ell/H_{\rm P}$, to 2.0, and set the convective overshooting parameter, $\delta_{\rm OV}$, to 0.12 (Pols et al. 1997; Schröder et al. 1997). This roughly corresponds to an overshooting length of 0.25 $H_{\rm P}$. Wind mass loss was not included in our calculation because the effect of wind on the relation of luminosity-core mass can be neglected during the AGB phase as seen in the following paragraph.

Paczyński (1970) studied the evolution of stars of 3.0, 5.0 and 7.0 $M_\odot$ and found that there is a linear relation between the luminosity L and the core mass $M_{\rm c}$ during the thermal pulsing AGB (TPAGB) phase defined by Iben (1983) and this relation is independent of the stellar mass. The core mass $M_{\rm c}$ is the mass within the hydrogen burning shell in their study and we also apply this definition. Strictly $M_{\rm c}$ should be the mass within the helium burning shell, but the helium burning shell is very thin (about 10^{-4} $M_\odot$) during the TPAGB phase and its mass can be neglected.

We assume

$$L = c\dot{M}_{\rm c}, \tag{1}$$

where c is a coefficient determined by the hydrogen abundance and metallicity and $\dot{M}_{\rm c}$ is the core growth rate because the hydrogen shell burning dominates the luminosity (Nomoto 1982a). Using the relation $L(M_{\rm c})$ found by Paczyński's(1970), we have $\dot{M}_{\rm c}$=$\dot{M}_{\rm c}(M_{\rm c})$. *We assume $\dot{M}_{\rm c}$ is the critical accretion rate $\dot{M}_{\rm cr}$ of the WD* (see also Nomoto 1982a), i.e., the maximum rate at which it can burn hydrogen.

Here we use AGB stars with two burning shells to mimic WD accretion for SNe Ia. We justify this as follows: (1) during the TPAGB phase the star has a degenerate CO core covered with double thin burning shells, similar to the stable shell burning of hydrogen and helium on the WD surface; (2) if a star has a given composition, Equation (1) is a unique relation during the TPAGB phase. If some matter is burned, the corresponding energy must be released. So this is a lower limit to L, or L is an upper limit to $\dot{M}_{\rm c}$. This process is the same, whether it occurs in an AGB star or on a WD surface. We calculate the relation $L(\dot{M}_{\rm c})$ for AGB stars with various initial masses and a fixed Z=0.02. The results are shown in Figure 1 by the solid lines. The dashed lines are deduced from Einstein's mass-energy equation. We assume a hydrogen mass fraction of $X = 0.70$ and a helium mass fraction of $Y = 0.28$ and that the mass loss by thermonuclear reactions $\Delta M/M = 0.007$ and 0.0007 for hydrogen and helium burning, respectively (Kippenhahn & Weigert 1990). The remaining mass $(1 - 0.7 \times 0.007 - 0.98 \times 0.0007)M$=$0.994414M$ is added to the degenerate CO core. The increase of core and the depletion of the envelop are synchronous during the

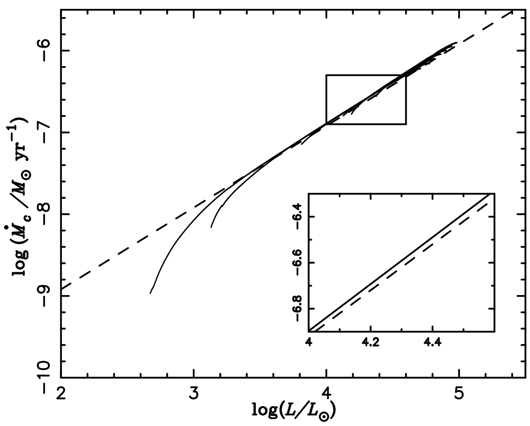

Fig. 1 Relation between total luminosity L and critical accretion rate $\dot{M}_{\rm c}$. The dashed line is derived from the Einstein's mass-energy equation (see the text) and the solid lines combine the calculated results for nine cases with initial masses from 2.0 $M_\odot$ to 6.0 $M_\odot$ in steps of 0.5 $M_\odot$.

TPAGB phase because the helium burning shell is thin. However, before the TPAGB phase when the helium shell is thick, the helium burning shell moves out faster than the hydrogen shell, so the increase of the core and the depletion of the envelope are not synchronous. This makes the relation $L(\dot{M}_{\rm c})$ not unique. In Figure 1, we see obvious differences between the numerical results and the theoretical line in the early evolutionary phases. There exists difference between the two sets of lines even in TPAGB phase, as we have not considered the neutrino energy loss L_ν and the thermal energy loss $L_{\rm th}$ for the theoretical line. From the difference between the two sets of lines, we obtain the ratio (L_ν+$L_{\rm th}$)/(L_ν+$L_{\rm th}$+L)≈5%, which well fits the result of numerical calculation during the TPAGB phase (about 5.7%). In the inset panel of Figure 1, the solid line has a larger gradient than the dashed one, which means that (L_ν+$L_{\rm th}$) becomes slightly larger as the star evolves. In our calculation, the difference between the two sets of lines for the gradient can be neglected, because it has hardly any effect on the final result. (3) When a CO WD accretes matter from its companion, the matter releases gravitational energy to heat itself (Regev & Shara 1989; Prialnik & Kovetz 1995). We equate the accretion heating to the accretion luminosity (Regev & Shara 1989)

$$L_{\rm heat} \simeq \frac{0.2GM_{\rm WD}\dot{M}}{R_{\rm WD}} \simeq \frac{3}{2}NkT \simeq 3\frac{\dot{M}}{\mu_{\rm H}}kT, \tag{2}$$

where G is the gravitational constant, k the Boltzmann constant, $\mu_{\rm H}$ the mass of a hydrogen atom and $M_{\rm WD}$ and $R_{\rm WD}$ the mass and radius of WD, respectively. If we assume, as approximation, that the accreting matter consists only of hydrogen and that the hydrogen is completely ionized, then for a 1 $M_\odot$ WD with a radius of $10^{-2}R_\odot$, we obtain $T \simeq 4.6\times10^8$ K, which is comparable to the temperature of hydrogen-burning shells of AGB stars ($\log(T_{\rm shell}$/K)≃8.3).

3 RESULTS OF CALCULATION

We calculated the evolution of different stars to examine the effect of metallicity on the critical accretion rate. We took stars of initial masses 1.6, 1.8, 2.0, 2.5, 3.0 and 3.5 $M_\odot$ with various Z, $Z = 0.0001$, 0.0003, 0.001, 0.004, 0.01 and 0.02. For a given Z, the initial hydrogen mass fraction is determined by

$$X = 0.76 - 3.0Z \tag{3}$$

(Pols et al. 1998), and then helium mass fraction is $Y = 1 - X - Z$.

The stars were evolved to the AGB phase. It should be noted that in the course of the evolution, there are several dredge-ups, which lead to significant decreases in the hydrogen abundance of the envelope

(Iben 1983; Busso et al. 1999). Though the effect of the first dredge-up may be eliminated by changing the hydrogen abundance in the envelope, it is almost impossible to completely eliminate the influence of the other two dredge-ups for the convergence of the code. Table 1 shows the minimum value of actual changed hydrogen abundance when the star lies in the TPAGB phase. Since the hydrogen abundance varies by only 10^{-5} during the TPAGB phase, we may consider the hydrogen abundance is constant during the whole TPAGB phase and the abundance is shown in Table 1.

During the TPAGB phase, we find a linear relation between the luminosity L and the core mass $M_{\rm c}$,

$$L = L_0(M_{\rm c} - M_0), \tag{4}$$

where L_0 and M_0 are coefficients dependent on the metallicity Z and the initial mass $M_{\rm i}$ (Paczyński 1970). For different Z, Table 2 shows the coefficients for the models with 1.6 $M_\odot$ and 1.8 $M_\odot$. For every $M_{\rm i}$, we use Equations (1) and (4) to calculate the critical accretion rate for various metallicities Z and for core masses $M_{\rm c}$ ranging from 0.6 $M_\odot$ to 1.4 $M_\odot$ in steps of 0.05 $M_\odot$. According to previous studies, SNe Ia can not occur if the initial WD mass is outside this range (Nomoto et al. 1991; Han & Podsiadlowski 2004; Starrfield et al. 2004). The coefficient in Equation (1) should be determined before we calculate $\dot{M}_{\rm c}$. The metallicity and hydrogen abundance may affect this coefficient. Given the hydrogen abundance shown in Table 1, the coefficients for different Z are calculated by the method of Nomoto (1982a) and the results for the 1.6 $M_\odot$ and 1.8 $M_\odot$ models are shown in Table 3.

Table 1 The hydrogen abundance during the TPAGB for various values of the initial mass (shown in Column 1) and the metallicity (Row 1).

Mass	0.0001	0.0003	0.001	0.004	0.01	0.02
1.6	0.7557	0.7567	0.7565	0.7480	0.7300	0.7000
1.8	0.7551	0.7562	0.7562	0.7479	0.7300	0.7000
2.0	0.7543	0.7555	0.7557	0.7479	0.7300	0.7000
2.5	0.7407	0.7436	0.7501	0.7471	0.7298	0.6999
3.0	0.7064	0.7070	0.7204	0.7403	0.7288	0.6994
3.5	0.6843	0.6830	0.6918	0.7117	0.7204	0.6967

Table 2 Coefficients of Equation (4), L_0 (in $L_\odot M_\odot^{-1}$) and M_0 (in $M_\odot$) for various values of metallicity (Row 1) and for two values of $M_{\rm i}$, 1.6 $M_\odot$ and 1.8 $M_\odot$.

Mass	0.0001		0.0003		0.001		0.004		0.01		0.02	
	L_0	M_0	L_0	M_0	L_0	M_0	L_0	M_0	L_0	M_0	L_0	M_0
1.6	61616	0.569	60640	0.555	59077	0.537	57736	0.520	56402	0.507	58844	0.509
1.8	67543	0.589	65808	0.573	61355	0.546	58741	0.524	58282	0.514	59570	0.510

Table 3 Constants of proportionality of Equation (1) (in $10^{10} L_\odot M_\odot^{-1}{\rm yr}$) for different metallicities and for $M_{\rm i}$=1.6 $M_\odot$ and 1.8 $M_\odot$.

Mass	0.0001	0.0003	0.001	0.004	0.01	0.02
1.6	7.525	7.536	7.533	7.448	7.269	6.971
1.8	7.519	7.531	7.530	7.448	7.269	6.971

The relation $\dot{M}_{\rm c}(Z)$ is plotted in Figure 2 for the 1.6 $M_\odot$ and 1.8 $M_\odot$ models, for $M_{\rm c}$=0.65, 0.75, 1.0, 1.4 $M_\odot$. We see that, for a given $M_{\rm c}$, there is a nearly linear relation between $\log(\dot{M}_{\rm c})$ and $\log(Z)$. Considering the effect of $M_{\rm c}$, we may fit this with

$$\log(\dot{M}_{\rm c}) = \log(7.95M_{\rm c} - 3.97) - 7.0 + \log\left(1.03 + \frac{2.40\times10^{-2}}{M_{\rm c}^6}\right)\times\log(Z), \tag{5}$$

and

$$\log(\dot{M}_{\rm c}) = \log(7.09M_{\rm c} - 2.72) - 7.0 + \log\left(1.01 + \frac{2.82\times10^{-2}}{M_{\rm c}^6}\right)\times\log(Z), \tag{6}$$

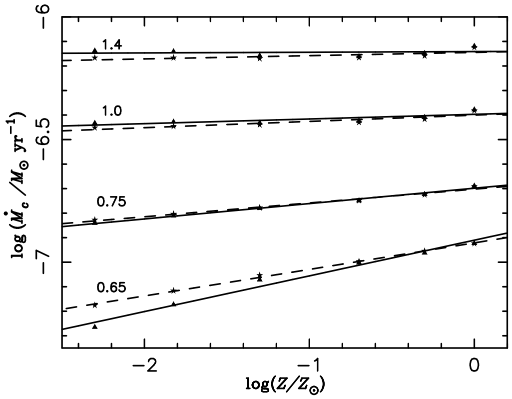

Fig. 2 Relation $\dot{M}_{\rm c}(Z)$ with various core masses for 1.6 and 1.8 $M_\odot$ models. The astral points and dashed lines are for 1.6 $M_\odot$ model and the triangular points and solid ones are for 1.8 $M_\odot$ model. The number above each line denotes the core mass (in solar unit).

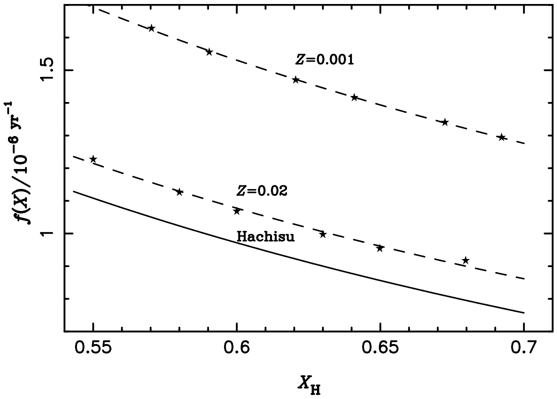

Fig. 3 The function $f(X)$. The solid line is the model of Hachisu (1999a) and the dashed ones are the fitted lines of the star points for different metallicity. $X_{\rm H}$ is the hydrogen mass fraction.

for $M_{\rm i}$ equal to 1.6 and 1.8 $M_\odot$, respectively. The error of $\log(\dot{M}_{\rm c})$ is less than 0.05 for $M_{\rm c} \geq 0.7\ M_\odot$, 0.43 at $M_{\rm c} = 0.6\ M_\odot$ and 0.20 at $M_{\rm c} = 0.65\ M_\odot$. Note, SNe Ia cannot occur if its progenitor has an initial $M_{\rm c}$ lower than about 0.7 $M_\odot$ (see also Han & Podsiadlowski 2004). We have not shown the relation for $M_{\rm i} \geq 2.0\ M_\odot$ because, as Table 1 shows, the effect of dredge-up is then so great that the hydrogen abundance deviates significantly from Equation (3), especially for low Z or large $M_{\rm i}$. Comparing Equations (5) and (6) with previous studies (Hachisu et al. 1999a, b), we find that Z just gives a small correction on the $\dot{M}_{\rm c}(M_{\rm c})$ relation. The effect of Z decreases as $M_{\rm c}$ increases and it may almost be neglected when $M_{\rm c}$ is close to the Chandrasekhar mass limit. This is because, as the core mass approaches the Chandrasekhar limit, the degeneracy increases so that the temperature is neglectable compared to the Fermi temperature and the core radius is only a function of mass. Given the core temperature, a same surface area of core results in a same $\dot{M}_{\rm c}$.

The hydrogen abundance of the accreted matter may also affect $\dot{M}_{\rm c}$. Hachisu et al. (1999a) showed that $\dot{M}_{\rm c}$ is inversely proportional to hydrogen abundance. For convenience, we examined the dependence of $\dot{M}_{\rm c}$ on X with a 2.5 $M_\odot$ model, for Z=0.02 (Pop I) and 0.001 (Pop II). After the central helium is exhausted, we artificially change the hydrogen abundance in the envelope. Defining $f(X)$ by

$$\dot{M}_{\rm c} = f(X)(M_{\rm c} - M_0), \tag{7}$$

we find $f(X) = 2.5 \times 10^{-7}\frac{(4.3-X)}{X}$ for M_0=0.60 and Z=0.001 and $f(X) = 4.3 \times 10^{-7}\frac{(2.1-X)}{X}$ for M_0=0.50 and Z=0.02. The minimum hydrogen abundance during the TPAGB phase is used as we fit the function $f(X)$, for X from 0.5 to 0.7. Figure 3 shows the function $f(X)$. The relation given by Hachisu (1999a) is also shown in Figure 3. We see that our $f(X)$ behaves similarly to Hachisu's but is generally higher. However, $\dot{M}_{\rm c}$ for Z=0.02 in our study is roughly equal to that of Hachisu's (Note, M_0=0.40 in his equation). Figure 3 also shows that $f(X)$ decreases with X. To maintain energy conversation, lower X means more matter masses burning in the shell to release enough energy to support a given luminosity. If the nuclear reaction rate is a constant, then $\dot{M}_{\rm c}$ increases. For the same reason, we may understand the result that $\dot{M}_{\rm c}$ increases with Z, a larger Z results in a lower X.

We calculated a large number of models, including 1.0, 1.2 and 1.4 $M_\odot$ models at each metallicity. We find that as the star evolves along the AGB, after the central helium is exhausted, the hydrogen shell burning experiences an extinction phase if $M_{\rm i} \geq 1.0\ M_\odot$. During this phase, the hydrogen luminosity $L_{\rm H}$ decreases significantly, as shown in Figures 4 and 5. However, it re-ignites soon after: as seen in Figures 4 and 5, $L_{\rm H}$ quickly recovers its average level. Hereinafter, we shall refer to these phenomena as “extinction

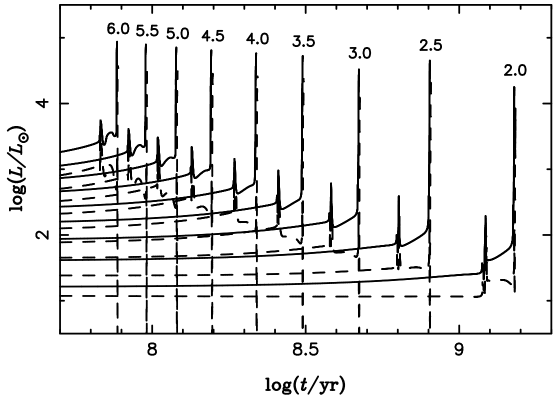

Fig. 4 Time evolutions of the hydrogen luminosity (dashed lines) and the total luminosity (solid lines) for various stellar masses with $Z = 0.02$.

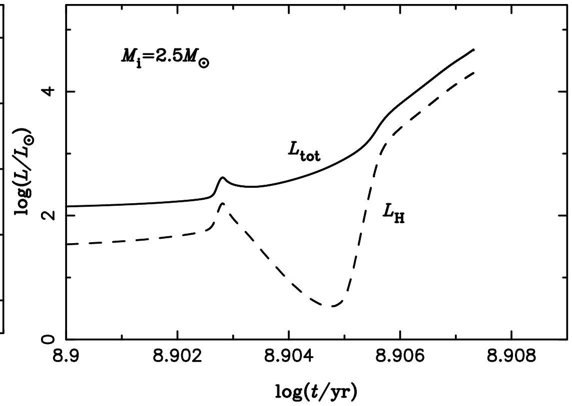

Fig. 5 Blow-up of part of the $2.5\,M_{\odot}$ curves in Figure 4.

and re-ignition". For numerical reasons, Figure 4 does not show the results for the models below $2.0\,M_{\odot}$. The existence of extinction and re-ignition during the AGB phase is easily understood as follows: after the exhaustion of the central helium, the CO core contracts while the helium burning shell expands quickly. At the same time, the hydrogen burning shell also expands and its temperature, pressure and density decrease. When the temperature of the hydrogen shell is too low to continue hydrogen burning, hydrogen burning is extinguished. However, the helium shell continues its burning and moves outwards. As it catches up with the bottom of the hydrogen-rich envelope, the hydrogen is ignited again.

4 DISCUSSION AND CONCLUSIONS

Paczyński (1970) studied the evolution of AGB stars of 3.0, 5.0 and 7.0 $M_{\odot}$, and found a linear relation between the luminosity L and the core mass $M_{\rm c}$, which is independent of the star mass. Based on homology argument, Kippenhahn (1981) found that L is proportional to $M_{\rm c}$ if radiation pressure is dominant in the burning shell (see also Jeffery 1988). Figure 6 shows the $L(M_{\rm c})$ relation (for Z=0.02) found in our study. We see that our relation $L(M_{\rm c})$ is not only a function of $M_{\rm c}$: it also depends on the stellar mass $M_{\rm i}$. Consistent with our result, Iben (1977) actually found that the luminosity is proportional to $M_{\rm i}^{0.4}$.

In appendix A, we find that the relation $L(M_{\rm c})$ is unique only in the extreme relativistic case where the pressure of the core is dominated by that of a completely degenerate electron gas, i.e., $P \propto \rho^{4/3}$. However, the core of an AGB star cannot satisfy this limiting condition. For a given core mass $M_{\rm c}$, a larger stellar mass results in a less degenerate core, so the relation $L(M_{\rm c})$ moves away from the extreme relativistic case of completely degenerate electron gas. Moreover, since metallicity has a mass-like effect on the stellar evolution (Umeda & Nomoto 1999), the relation $L(M_{\rm c})$ is also a function of the metallicity.

As seen in Figure 2, the metallicity contributes to the dispersion of the accretion rate and we obtain different relations between the accretion rate and the metallicity for different masses. The dispersion of $L(M_{\rm c})$ derived from different masses also leads to dispersion of $\dot{M}_{\rm c}(M_{\rm c})$ (Fig. 7). We shall now simply discuss which mass is closer to the real WD. At the onset of accretion, the CO WD has a much lower surface temperature than the core of an AGB star and so is more degenerate. When the accretion rate reaches $\dot{M}_{\rm c}$, the time scale of heat conduction is about 10^6 yr (Nomoto et al. 1984), hence the WD has a lower temperature than that of the core in an AGB star during this prolonged time interval. Even when the WD mass reaches the Chandrasekhar mass limit, it still has a larger density and a lower temperature comparing with the core of AGB stars, and is more degenerate (Nomoto et al. 1982b, 1984; Kippenhahn & Weigert 1990). Therefore, the model with lower stellar mass in our study, i.e., $M_{\rm i}$=1.6 $M_{\odot}$, is closer to the real WD. In addition, the effect of dredge-ups will cause the hydrogen abundance to deviate from

Equation (3), and the effect is least on the 1.6 $M_\odot$ model amongst our models. We did not consider stars with masses below 1.6 $M_\odot$ because the core in such cases can not increase to the WD mass required.

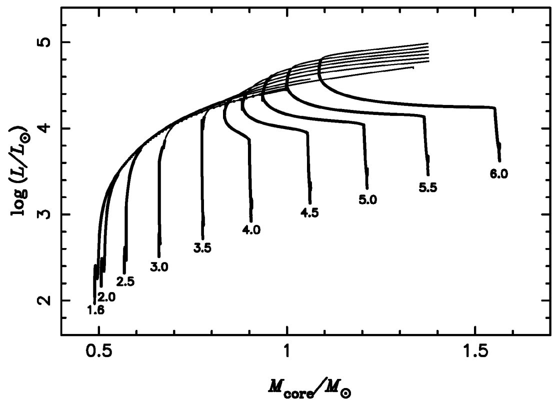

Fig. 6 Relation $L(M_{\rm c})$ for various masses with $Z = 0.02$. Thick lines represent the EAGB phase and thin lines represent the TPAGB phase. The core mass is the mass within the shell where the hydrogen abundance is equal to 0.1.

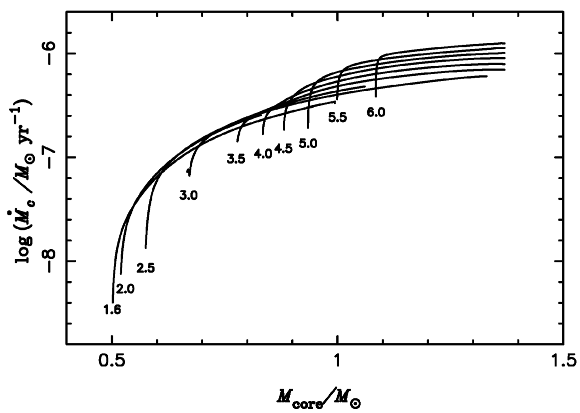

Fig. 7 Relation $\dot{M}_{\rm c}(M_{\rm c})$ for various masses with $Z = 0.02$.

In summary we have obtained the following results:

1. A new critical accretion rate of WD for the explosion of SNe Ia, i.e., $\dot{M}_{\rm c}$ dependent on the core mass $M_{\rm c}$ and metallicity Z is given. The influence of Z on $\dot{M}_{\rm c}$ is small and amounts to a correction on the relation of $\dot{M}_{\rm c}(M_{\rm c})$.
2. We have shown that the dependence of $\dot{M}_{\rm c}$ on $X_{\rm H}$, described by Hachisu (1999a) is valid for X ranging from 0.5 to 0.7.
3. The relation $L(M_{\rm c})$ in AGB stars is not unique. The luminosity L is affected not only by the core mass, but also by the stellar mass and metallicity.

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APPENDIX A: DEDUCTION OF THE RELATION $L(M_{\rm c})$

We assume the equation of state of the core

$$\rho \propto P^{\alpha} T^{-\delta}, \tag{A1}$$

where ρ, P and T are the local density, pressure and temperature of the core, respectively, α and δ are constants. Using a polytropic model (Kippenhahn & Weigert 1990), we find

$$\ln T = \frac{\frac{4}{3}\alpha - 1}{\delta} \ln \rho + c = \eta \ln \rho + c, \tag{A2}$$

where c is a constant of integration and η is a function of the degeneracy ψ. Then we have

$$T \propto \rho^{\eta}. \tag{A3}$$

We make the following five assumptions.

1.
$$L \propto R_{\rm s}^2 T_{\rm s}^4, \tag{A4}$$
where $R_{\rm s}$ and $T_{\rm s}$ are the radius and the temperature of the hydrogen burning shell, respectively.

2.
$$T_{\rm s} \approx T_{\rm c}, \tag{A5}$$
where $T_{\rm c}$ is the core temperature and we premise the core is isothermal. According to our calculations, this assumption is accurate well.

3.
$$R_{\rm s} \propto R_{\rm c}. \tag{A6}$$
Refsdal & Weigert (1970) first derived this relation from homology and it is roughly accurate during TPAGB phase according to our calculation.

4. For a WD
$$R_{\rm c} \propto M_{\rm c}^{-1/3}. \tag{A7}$$
Because the core in an AGB star has a very similar structure to a WD, Jeffery (1988) found that adopting this approximation has hardly any effect on the relation $L(M_{\rm c})$. This is because the core has a very high Fermi temperature (about 10^{11}K) and compared with this, the temperature of the core in an AGB star (about 10^8) or a WD (about 10^4) may be regarded as zero. The Fermi temperature is a diagnostic temperature and electrons in the core become non-degenerate if the temperature of core exceeds the Fermi temperature.

5.
$$\rho = \overline{\rho} \propto M_{\rm c} R_{\rm c}^{-3}. \tag{A8}$$

With these five assumptions and Equations (8), (9) and (10), we find

$$L \propto M_{\rm c}^{\frac{24\eta - 2}{3}}. \tag{A9}$$

Then $\eta = \frac{5}{24}$ or $5\delta = 32\alpha - 24$ gives a unique linear relation $L \propto M_{\rm c}$. This is the case when the pressure of the core is dominated by the extremely relativistic degenerate electron gas, i.e., $P \propto \rho^{4/3}$, $\alpha = 3/4$, and $\delta = 0$.

References

Branch D., Livio M., Yungelson L. R. et al., 1995, PASP, 107, 1019
Busso M., Gallino R., Wasserburg G. J., 1999, ARA&A, 37, 239
Eggleton P. P., 1971, MNRAS, 151, 351
Eggleton P. P., 1972, MNRAS, 156, 361
Eggleton P. P., 1973, MNRAS, 163, 279
Gilliland R. L., Nugent P. E., Phillips M. M., 1999, ApJ, 521, 30
Hachisu I., Kato M., Nomoto K., 1996, ApJ, 470, L97
Hachisu I., Kato M., Nomoto K. et al., 1999a, ApJ, 519, 314
Hachisu I., Kato M., Nomoto K., 1999b, ApJ, 522, 487
Hachisu I., Kato M., 2003a, ApJ, 588, 1003
Hachisu I., Kato M., 2003b, ApJ, 590, 445
Han Z., Posiadlowski Ph., Eggleton P.P., 1994, MNRAS, 270, 121
Han Z., Posiadlowski Ph., 2004, MNRAS, 350, 1301
Hillebrandt W., Niemeyer J.C., 2000, ARA&A, 38, 191
Iben I., 1977, ApJ, 217, 788
Iben I., 1983, ARA&A, 21, 271
Jeffery C.S., 1988, MNRAS, 235, 1287
Kippenhahn R., Weigert A., 1990, stellar structure and evolution, Springer-Verlag, p. 124 or 146 or 198
Kippenhahn R., 1981, A&A, 102, 293
Kovetz A., Prialnik D., 1994, ApJ, 424, 319
Kobayashi C., Tsujimoto T., Nomoto K. et al., 1998, ApJ, 503, L155
Langer N., Deutschmann A., Wellstein S. et al., 2000, A&A, 362, 1046
Leibundgut B., 2000, A&ARv, 10, 179
Li X. D., van den Heuvel E. P. J., 1997, A&A, 322, L9
Nelson C. A., Eggleton P. P., 2001, ApJ, 552, 664
Nomoto K., 1982a, ApJ, 253, 798
Nomoto K., 1982b, ApJ, 257, 780
Nomoto K., Thielemann F-K., Yokoi K., 1984, ApJ, 286, 644
Nomoto K., Kondo Y., 1991, ApJ, 367L, 19
Nomoto K., Iwamoto K., Kishimoto N., 1997, Science, 276, 1378
Nomoto K., Umeda H., Hachisu I. et al., 1999, in Truran J., Niemeyer T., eds., Type Ia Suppernova: Theory and Cosmology, Cambridge Univ. Press, New York, p.63
Nomoto K., Uenishi T., Kobayashi C. et al., 2003, in Hillebrandt W., Leibundgut B., eds, From Twilight to Highlight: The Physics of supernova, ESO/Springer serious “ESO Astrophysics Symposia” Berlin: Springer, p.115
Paczyński B., 1970, Acta Astron., 20, 47
Perlmutter S., Aldering G., Goldhaber G. et al., 1999, ApJ, 517, 565
Pols O.R., Tout C.A., Eggleton P.P. et al., 1995, MNRAS, 274, 964
Pols O.R., Tout C.A., Schröder K.P. et al., 1997, MNRAS, 289, 869
Pols O.R., Schröder K.P., Hurly J.R. et al., 1998, MNRAS, 298, 525
Prialnik D., Kovetz A., 1995, ApJ, 445, 789
Refsdal S., Weigert A., 1970, A&A, 6, 426
Regev O., Shara M.M., 1989, ApJ, 340, 1006
Riess A.G., Filippenko A.V., Challis P. et al., 1998, AJ, 116, 1009
Schröder K.P., Pols O.R., Eggleton P.P., 1997, MNRAS, 285, 696
Starrfield S., Timmes F.X., Hix W.R. et al., 2004, ApJ, 612, L53
Strolger L-G., Riess A.G., Dahlen T. et al., 2004, ApJ, 613, 200
Umeda H., Nomoto K., Kobayashi C. et al., 1999, ApJ, 522, L43
Umeda H., Nomoto K., 1999, ApJ, 513, 861
Yungelson L., Livio M., Tutukou A. et al., 1995, ApJ, 447, 656