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On the Evolution of the Apparent Size of Gamma-Ray Burst Remnants *

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Abstract The remnants of two gamma-ray bursts, GRB 030329 and GRB 041227, have been resolved by Very Long Baseline Interferometry observations. The radio counterparts were observed to expand with time. These observations provide an important way to test the dynamics of the standard fireball model. We show that the observed size evolution of these two events cannot be explained by a simple jet model, rather, it can be satisfactorily explained by the two-component jet model. It strongly hints that gamma-ray burst ejecta may have complicated structures.

Key words: gamma rays: bursts — ISM: jets and outflows

1 INTRODUCTION

Since their discovery in 1967 (Klebesadel et al. 1973), gamma-ray bursts (GRBs) have been one of the most mysterious phenomena in astrophysics (Fishman & Meegan 1995). The first detection of X-ray, optical and radio afterglows of some well-localized GRBs in 1997 has opened up a new era in the field (Costa et al. 1997). Since then, redshifts have been successfully measured for tens of GRBs, and most long classical GRBs are believed to be of cosmological origin. The so called fireball model (Goodman 1986; Paczyński 1986; Mészáros et al. 1994; Fenimore et al. 1996; Cheng & Lu 2001, and reference therein) is strongly favored, which satisfactorily explains the general features of GRB afterglows.

According to the fireball model, a yet unclear mechanism gives birth to an initially very small fireball ($\sim 10\,$km) which, however, had a huge amount of energy ($\sim 10^{52}$ erg). Consequently the strong internal radiative pressure will promptly accelerate baryons in the fireball to an extremely high speed, producing an ultra-relativistic shell (Wu et al. 2001). Many shells produced in this way with slightly different velocities will collide with each other at about $\sim 10^8$ km away from the central engine, and generate a GRB through internal shocks. At a distance of $\sim 10^{11}$ km, the shells merge into one main shell and plough into the circum-burst medium to produce a long-lasting external shock, giving birth to the observed multi-wavelength afterglows (Mészáros & Rees 1997; Waxman 1997a, b; Tavani 1997; Sari 1997; Huang et al. 1998a, b; Dai & Lu 1998a, b; Mao & Wang 2001a, b; Dai & Wu 2003). Further studies revealed that at least some GRB ejecta should not be isotropic, but should be highly collimated (Rhoads 1997, 1999; Huang et al. 2000a, b; Lu et al. 2000; Ma et al. 2003).

For a long time, measuring the strength of the afterglows (either by gaining the afterglow light curves or by obtaining the spectra) is the sole way to examine the fireball model. Things began to change in

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2004. We know that GRBs are produced by stellar objects. However, lying at cosmological distances, their angular sizes are extremely small, but recently the remnant of a famous event, GRB 030329, at a redshift of only $z = 0.1685$ (Greiner et al. 2003a), was luckily resolved by Very Long Baseline Interferometry observations (Taylor et al. 2004, 2005). The expansion of this GRB remnant was recorded, which gives an independent and valuable way to check the fireball model. More recently, the size evolution of another event, GRB 041227, a giant flare from the soft gamma-ray repeater (SGR) 1806-20, was also observed (Gaensler et al. 2005; Cameron et al. 2005; Granot et al. 2006), and it provides further constraints on the theoretical understanding of GRBs.

Analytical results show that the evolution of the remnants of GRBs 030329 and 041227 is roughly consistent with the fireball model (Oren, Nakar & Piran 2004; Granot et al. 2004). However, those analyses are still very coarse, and also there are some problems in the theoretical explanations. For example, the observed behavior deviates from the theoretical predictions of the simple fireball model. In this paper, we study the dynamical evolution of GRB remnants numerically. We find that the observed size evolution of these two GRB remnants can be satisfactorily explained by the two-component jet model. The structure of present paper is that we first describe our model in Section 2, numerical results are then presented in Section 3 for GRBs 030329 and 041227, and Section 4 gives a discussion and our conclusions.

2 MODEL

Let us consider a GRB fireball of radius R. Note that due to the relativistic effect and the equal arrival time surface effect (Waxman 1997c; Sari 1998; Panaitescu & Mészáros 1998), R (or $R \sin\theta$ in case of a jet with a half opening angle θ) itself is not the apparent size observed by us. The evolution of the apparent size of a GRB remnant can be derived analytically under some simplified assumptions (Oren, Nakar & Piran 2004). For example, when an isotropic fireball decelerates adiabatically in a homogeneous medium, the apparent radius evolves as $R_\perp \propto t^{5/8}$ in the relativistic phase, and as $R_\perp \propto t^{2/8}$ in the Newtonian phase. For a jet the apparent radius is $R_\perp \propto t^{5/8}$ when its Lorentz factor (γ) is larger than $1/\theta$, and is $R_\perp \propto t^{1/4}$–$t^{1/2}$ when $\gamma < 1/\theta$, depending on the lateral expansion speed. However, as mentioned in the previous section, the observed size evolution of GRBs 030329 and 041227 cannot be explained satisfactorily by a simple isotropic fireball or a simple jet. In our study we will examine numerically the observations in the light of the two-component jet model.

It was once believed that GRB jets are homogeneous conical outflows. However, recently people begin to realize that they actually may have complicated structures. For example, the energy per unit solid angle may in fact be a power-law or a Gaussian function on the angular distance from the jet axis. The two-component jets are also a special kind of structured jets, which have attracted much attention among researchers (Berger et al. 2003; Huang et al. 2004; Wu et al. 2005; Gao & Wei 2005). The two-component jet has two components: a narrow ultra-relativistic outflow and a wide but mildly relativistic ejecta. At first glance, the two-component jet model still seems to be too coarse, but it strikingly gives a reasonable explanation to the multi-band observations of GRB 030329. As suggested by Berger et al. (2003), the γ-ray and early afterglow emission of GRB 030329 came from the narrow component, while the radio and optical afterglow beyond 1.5 days should be produced by the wide component. Recently it was further suggested by Huang et al. (2004) that the observed rapid rebrightening in the afterglow of XRF 030723 can also be explained by a two-component jet viewed slightly off-axis.

In this paper, we designate the initial half opening angle of the narrow component (suffix N) and the wide component (suffix W) by $\theta_{0,\mathrm{N}}$ and $\theta_{0,\mathrm{W}}$. We further designate their isotropic equivalent kinetic energies by $E_{\mathrm{N,iso}}$ and $E_{\mathrm{W,iso}}$, and initial Lorentz factors by $\gamma_{0,\mathrm{N}}$ and $\gamma_{0,\mathrm{W}}$. We assume that the two components are coaxial.

We use the model developed by Huang et al. (2000b, c) to describe the dynamical evolution of each component. In this model, the evolution of the bulk Lorentz factor is given by (Huang, Dai & Lu 1999a, b),

$$\frac{d\gamma}{dm} = -\frac{\gamma^2 - 1}{M_{\mathrm{ej}} + \epsilon m + 2(1-\epsilon)\gamma m}, \tag{1}$$

where m is the mass of swept-up interstellar medium (ISM), M_{ej} is the initial mass of the ejecta, and ϵ is the radiative efficiency. Equation (1) has the virtue of being applicable in both the ultra-relativistic and the non-relativistic phases (Huang et al. 1999a, b). The lateral expansion of the outflow is described realistically by (Huang et al. 2000a, b),

$$\frac{d\theta}{dt} = \frac{c_{\rm s}(\gamma + \sqrt{\gamma^2 - 1})}{R}, \tag{2}$$

with

$$c_{\rm s}^2 = \hat{\gamma}(\hat{\gamma} - 1)(\gamma - 1)\frac{1}{1 + \hat{\gamma}(\gamma - 1)}c^2, \tag{3}$$

where $c_{\rm s}$ is the co-moving sound speed, and $\hat{\gamma} \approx (4\gamma + 1)/(3\gamma)$ is the adiabatic index. For simplicity, we assume that the ejecta is adiabatic, which means the radiative efficiency in Equation (1) is $\epsilon \equiv 0$. The evolution of the radius and swept-up mass is given by

$$\frac{{\rm d}R}{{\rm d}t} = \beta c\gamma(\gamma + \sqrt{\gamma^2 - 1})\,, \tag{4}$$

$$\frac{{\rm d}m}{{\rm d}R} = 2\pi R^2(1 - \cos\theta)nm_{\rm p}, \tag{5}$$

where $\beta = \sqrt{\gamma^2 - 1}/\gamma$, n is the number density of the interstellar medium (ISM) surrounding the jet, and $m_{\rm p}$ is the mass of proton.

Note that R in Equation (4) is the rest frame radius of the shock, and is not the observed apparent radius. At any given observer's time t, the apparent radius is the maximum lateral size of an equal arrival time surface determined by (Moderski et al. 2000; Panaitescu & Mészáros 1999)

$$t = \int \frac{1 - \beta\cos\Theta}{\beta c}dr \equiv {\rm const}, \tag{6}$$

where Θ is the angle from the jet axis. In the next section, we solve Equations (1)–(6) numerically to determine the size evolution of a two-component jet, and compare our theoretical results with observations.

3 COMPARISON WITH OBSERVATIONS

3.1 GRB 030329

GRB 030329, which occurred on 2003 March 29, is a watershed event in the field of GRBs. This burst is very bright and is actually in the top 1% of all detected GRBs (Vanderspek et al. 2004). Lying at a redshift of $z = 0.1685$ (Greiner et al. 2003a), it is the closest classical GRB to date. For the first time, an unambiguous underlying type Ic supernova was revealed spectroscopically about one week after the trigger (Hjorth et al. 2003). The connection between GRBs and core-collapse supernovae is thus firmly established, shedding light on the previously unclear nature of long GRBs. A very detailed polarization light curve for the optical afterglow was obtained (Greiner et al. 2003b), casting light on the physics of GRB afterglows. GRB 030329 is also the event with the most copious afterglow data, thanks to its extremely bright afterglow. For example, the observed R-band afterglow light curve compiled by Lipkin et al. (2004) comprised 1644 points, spanning from $\sim 0.05\,$d to $\sim 80\,$d.

Another encouraging fact from GRB 030329 is that its remnant was resolved in radio wavelength by Very Long Baseline Interferometry observations, leading to a direct measurement of the size of a cosmological GRB remnant for the first time (Taylor et al. 2004, 2005). These observations provide an independent and valuable way to test the standard fireball model. Evidently, a simple jet model cannot explain the observed size evolution of GRB 030329 satisfactorily. We note that a two-component jet model has been suggested to account for the multiband afterglow behaviors of GRB 030329 by Berger et al. (2003). Here we go further to use this model to fit the size evolution. We assume that the two components are co-axial (Frail et al. 2000; Ramirez-Ruiz, Celotti & Rees 2002). Our best results are presented in Figure 1. In our calculations, we have taken $E_{\rm N,iso} = 2 \times 10^{53}$ erg, $\theta_{0,\rm N} = 0.1$, $\gamma_{0,\rm N} = 200$ for the narrow component; and $E_{\rm W,iso}$=8×10^{52} erg, $\theta_{0,\rm W} = 0.3$, $\gamma_{0,\rm W} = 10$ for the wide component. The number density n of the ISM is $0.2\,{\rm cm}^{-3}$.

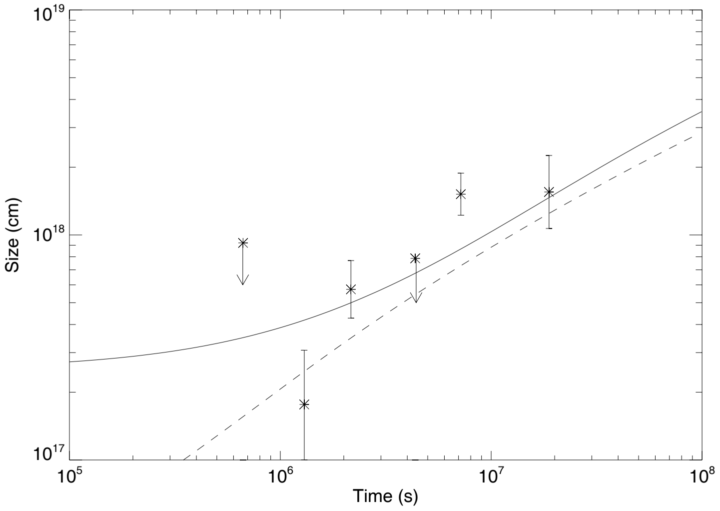

Fig. 1 Observed size evolution of GRB 030329 and our best fit with the two-component jet model. The dashed line corresponds to the narrow component and the solid line to the wide component. The observational data are taken from Taylor et al. (2004, 2005).

Figure 1 shows clearly that the observed size evolution of the remnant of GRB 030329 cannot be explained by a single jet model. The main difficulty is that the first observed data point at $\sim 1.4 \times 10^{6}$ s corresponds to a too small scale as compared with the other three points. However, in the two-component jet model, the observations can be explained more reasonably: the first observed point reflects the size of the narrow component, while the three later points reflect that of the wide component.

3.2 GRB 041227

There are actually two kinds of GRBs. One is the so called classical GRBs which comprise the majority of usual GRBs, the other is soft GRBs, which are much softer in photon energy and are repeatedly emitted by soft γ-ray repeaters (SGRs). SGRs are believed to be magnetars, neutron stars with periods of a few seconds but with dipole magnetic fields of $\sim 10^{15}$ G (Cheng & Dai 2002; Lu & Zhang 2004; Woods & Thompson 2005). In 1998, soon after the discovery of afterglows from classical GRBs, Huang et al. (1998c) argued that SGR bursts can also produce relativistic fireballs. Basing on the standard fireball model they predicted that SGR bursts should also have afterglows. The 1998 August 27 giant flare from SGR 1900+14 is the first detected SGR burst with afterglow (Frail et al. 1999). Radio afterglow from this event is consistent with emission from an initially subrelativistic blastwave due to the burst activity (Cheng & Wang 2003). Among the five SGR sources identified to date, three have been observed to produce one giant flare each, the brightest one being GRB 041227 from SGR 1806-20 on 2004 December 27 (Hurley et al. 2005).

Abundant observational data are available for the radio afterglow of GRB 041227 (Granot et al. 2006; Cameron et al. 2005; Gaensler et al. 2005; Gelfand et al. 2005). The early radio flux ($t < 20$ d) can be well explained by emission from a collimated blastwave (Wang et al. 2005). The observed obvious rebrightening beginning at $t \sim 30$ d can be interpreted as emission from another wider ejecta (Dai et al. 2005). Thus this giant flare may also be due to a two-component jet. Of special interest is the fact that the remnant of GRB 041227 was also resolved, providing the second valuable sample that can be used to test the dynamics directly. The observed size increased slowly as $t^{0.04 \pm 0.15}$ on days 7–10 (Cameron et al. 2005), but increased faster later (Granot et al. 2006).

We have fitted the observed size evolution in the framework of the two-component jet model. We take the following parameters: $E_{\rm N,iso} = 5 \times 10^{45}$ erg, $\theta_{0,\rm N} = 0.125$, $\gamma_{0,\rm N}$=10 for the narrow component; and

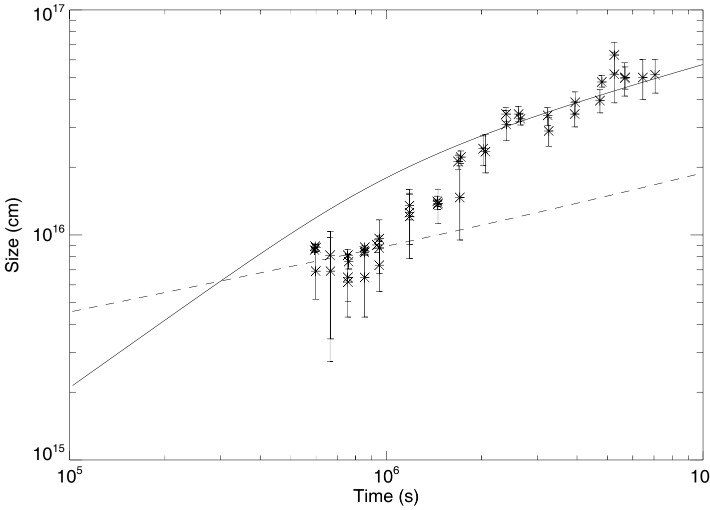

Fig. 2 Observed size evolution of the giant flare of GRB 041227 from SGR 1806-20, and our best fit by using the two-component jet model. The dashed line corresponds to the narrow component and the solid line to the wide component. The observational data are taken from Granot et al. (2006), Cameron et al. (2005), and Gaensler et al. (2005).

$E_{\rm W,iso}$=1×10^{45} erg, $\theta_{0,\rm W} = 0.7$, $\gamma_{0,\rm W} = 1.2$ for the wide component. The number density n of the ISM is $0.01\,{\rm cm}^{-3}$. The result is shown in Figure 2. Again from this figure, it is clear that the observations cannot be satisfactorily explained by a single jet. In the two-component jet model, however, the early observations ($t < 2 \times 10^6$ s) can be well explained by the narrow component, and the late observations ($t > 2 \times 10^{6}$ s) can be accounted for by the wide component. In this figure, the observed transition occurs at about $t \sim 2 \times 10^{6}$ s, consistent with the emergence of the second component indicated by observed afterglow light curves.

In Figure 2, it is interesting to note that the shape of the solid line is very similar to that of the observed points. One thus may conjecture that if the solid curve can be shifted toward the upper-right, then it may fit all the observed data. This shift can be effected by reducing the initial half opening of the jet and reducing the ISM density at the same time. This means a single jet may also be capable of explaining the observed apparent size evolution of the giant flare. However, there are a few problems in this single-jet explanation. First, the shapes of the solid curve and the data points are not completely identical, thus the fit will not be completely satisfactory even if the solid curve is shifted as desired. Secondly, the shift will definitely need to further reduce the ISM density significantly. Note that in Figure 2, however, the number density is already as low as $0.01\,{\rm cm}^{-3}$. We believe that a density much lower than this value, although still possible, is not reasonable. Thirdly and most importantly, a single-jet model cannot explain the observed radio afterglow light curves of GRB 041227, especially the rebrightening beginning at ~ 30 d. In the single-jet model, a rebrightening can be produced either through an energy-injection event (Huang et al. 2006) or a sudden density increase of the ISM. However, since the observed rebrightening is so remarkable, if it were due to an energy injection, the injected energy would be unbelievably large. A sudden density increase is also not ideal in producing such an obvious rebrightening (Huang et al. 2006). Combining all these analyses, we believe that the two-component jet model is still the preferred choice for GRB 041227.

4 CONCLUSIONS AND DISCUSSION

The observed size evolution of the remnants of GRBs 030329 and 041227 provides an important and independent way to examine the dynamics of GRB fireballs. In this study, we have shown that the observations cannot be explained by a simple fireball model or a simple jet model, but can be well interpreted by the two-component jet model. Interestingly, this is true for both GRB 030329 and GRB 041227. It should not be regarded as a mere coincidence. Instead, it strongly indicates that GRB jets may have complicated structures, consistent with results derived from the measurements of the afterglow flux (i.e., afterglow light curves) in some GRBs.

It can also be noted that in our calculations, the parameters of GRB 041227 are quite different from those of GRB 030329. The initial Lorentz factors are $\gamma_{0,\rm N} = 10$ and $\gamma_{0,\rm W} = 1.2$ for GRB 041227, both significantly less than those of GRB 030329 ($\gamma_{0,\rm N} = 200$, $\gamma_{0,\rm W} = 10$). Also the wide component of GRB 041227 is much wider ($\theta_{0,\rm W} = 0.7$, as compared with $\theta_{0,\rm W} = 0.3$ for GRB 030329). The difference correctly reflects the nature of the two kinds of GRBs. GRB 030329 is a classical GRB, which is more relativistic and fierce; but GRB 041227 is a less energetic SGR burst, whose speed is only trans-relativistic.

However, only two GRB remnants have been resolved so far. Additionally, these two GRBs both have some special characteristics: GRB 030329 is famous for its variabilities in the afterglow, while GRB 041227 is a SGR burst, not a typical classical GRB. Thus it is still largely uncertain how the remnant of a typical GRB will evolve. In the future, if more samples are available, then more definite conclusions can be obtained and important clues will be provided for our understanding of the enigma of GRBs.

Recently, the optical afterglow of GRB 030329 was re-examined by Huang et al. (2006). They found that while the two-component jet model can fit the overall afterglow light curve that spans from ~ 0.05 d to ~ 80 d, it cannot reproduce the rapid rebrightening which was observed at $t \sim 1.6$ d. They suggested that the observed rebrightening can be better interpreted by an energy-injection event. However, GRB 030329 *is* a very special and complex burst. It is very likely that we are dealing here with a compound model of a two-component jet *p*lus an energy-injection event. For a two-component jet with energy-injections, the evolution of its apparent size will likely be quite different. This problem will need further study.

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