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A New Method to Determine Epochs of Solar Cycle Extrema *

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Abstract A weighted average method is proposed to determine the epochs of solar cycle extrema and hence the solar cycle lengths. Comparing to the previous methods, this method has the advantage that the extremum epochs are easily and uniquely determined.

Key words: Sun: activity — Sun: sunspots — Sun: general

1 INTRODUCTION

It is well known that solar cycle length is a key parameter to describe the inherent properties of solar magnetism, such as the dynamo action (Eddy 1976; Dicke 1978; Landscheidt 1999) and the latitude's migration of magnetic activity (Kane & Trivedi 1980; Wilson 1987; Li et al. 1998, 2000, 2001a, b, 2002; Zhan et al. 2003). Furthermore, it has been revealed that the cycle length is closely related to the global climate (Reid 1987; Friis-Christensen & Lassen 1991; Donahue & Baliunas 1992; Hoyt & Schatten 1993; Butler 1994; Wilson 1998).

Conventionally, the solar cycle length is defined as the time difference between two successive sunspot minima. This length depends on how sunspot numbers (SN, hereafter) are averaged or smoothed. Some authors have used additional parameters to determine epochs of solar cycle extrema. These parameters include first spotless day (Wilson 1995), number of spot groups, 10.7-cm radio flux, total magnetic flux, Ca II K index, He I 1083 equivalent width, total irradiance, and the number of active regions (Harvey & White 1999 and references therein).

A median cycle length was defined by Mursula & Ulich (1998). The great advantage of this definition is that the median times so defined are almost independent of the exact locations of sunspot minima, and is defined to within a few days.

Figure 1 illustrates the 181-day running mean SNs from 1975 to 1987. In this figure are labelled two successive minimum times (min), the maximum time and the median time. Here the median time means that the sum of SNs between two successive minima is equally divided into two parts (Mursula & Ulich 1998). The median time is usually later than the maximum time since the duration of the descending phase is usually longer than that of the ascending phase.

In this paper, we propose a new method of determining the extremum epochs with a weighting function. We then derive two types of cycle lengths, between the minima and between the maxima. These will be described in Section 2. The effects of smoothing windows are analyzed in Section 3, and our conclusions are presented finally in Section 4.

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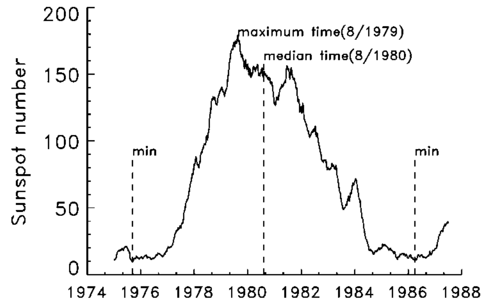

Fig. 1 Daily SNs (1975–1987) smoothed with the time window of 181 days. Two minimum times (min), a maximum time, and a median time are marked.

2 EPOCHS OF EXTREMA

Early Wolf SNs are thought to be overestimated (Eddy 1976; Kane 1999), so we use the more reliable data from 1818 to 2004. The daily SNs, available from *http://www.ngdc.noaa.gov/stp/SOLAR/getdata.html*, are smoothed with a time window of 181 days.

Usually there are several dates when the SNs have the same values around a given extremum, while we have to select one particular epoch. In order to reasonably determine the epoch, a weighted average method is proposed. The method is as follows.

Figure 2 illustrates the 181-day running mean SNs (1980–1990) including a minimum R_0, where $R_{\rm m}$ is the maximum from the previous to next cycle. We suggest a parameter, δ, to measure the scale of the interval $[R_0, R_0 + \delta]$. The value of δ is proportional to the difference between $R_{\rm m}$ and R_0, i.e.,

$$\delta = L \cdot (R_{\rm m} - R_0), \tag{1}$$

where L is the scaling factor. Firstly we set $L = 0.1(10\%)$.

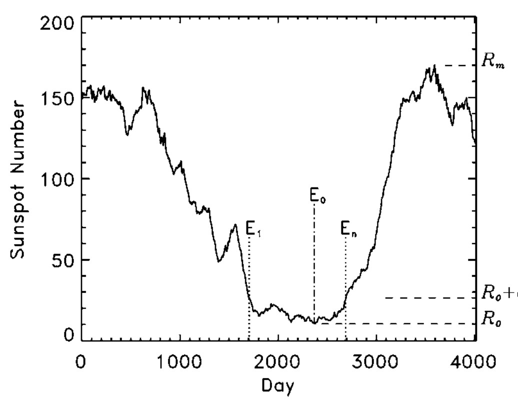

Fig. 2 181-day running mean SNs (1980–1990) including a minimum and two maxima. In the figure are labelled the minimum (R_0), maximum ($R_{\rm m}$), $\delta = 0.1(R_{\rm m} - R_0)$, the epochs ($E_1, E_n$) when SNs satisfy the condition $R = R_0 + \delta$. The minimum epoch, E_0, is derived from Eq. (2).

Table 1 Parameters from Weighted Average and Official Definition

Cycle	$E_0^{\rm d}$	$O_o^{\rm a,d}$	$T_{\rm min}$	$O_{\rm min}^{\rm b}$	$E_{\rm m}{}^{\rm d}$	$O_{\rm m}^{\rm a}{}^{,\rm d}$	$T_{\rm max}$	$O_{\rm max}^{\rm b}$	$T_{\rm med}^{\rm b}$	$T_{\rm W}^{\rm c}$
	yr.mon	yr.mon	yr	yr	yr.mon	yr.mon	yr	yr	yr	yr
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
7	1823.2	1823.5	10.84		1829.6	1829.11	7.58			
8	1833.12	1833.11	9.62	9.6	1837.1	1837.3	10.81	10.9		
9	1843.8	1843.7	12.52	12.5	1847.11	1848.2	12.50	12.0		
10	1856.2	1855.12	11.07	11.2	1860.5	1860.2	10.11	10.5	10.4	11.25
11	1867.3	1867.3	11.72	11.7	1870.6	1870.8	13.57	13.3	12.2	11.75
12	1878.11	1878.12	10.93	10.7	1884.1	1883.12	9.48	10.2	10.7	11.25
13	1889.11	1890.2	12.21	12.1	1893.7	1894.1	12.35	12.9	12.6	11.83
14	1902.1	1902.1	11.21	11.9	1905.11	1906.2	11.71	10.6	11.1	11.58
15	1913.4	1913.7	10.16	10.0	1917.7	1917.8	10.78	10.8	10.1	10.00
16	1923.6	1923.7	10.28	10.2	1928.5	1928.4	9.25	9.0	10.4	10.08
17	1933.9	1933.9	10.49	10.4	1937.8	1937.4	10.00	10.1	10.1	10.42
18	1944.3	1944.2	10.06	10.1	1947.8	1947.5	10.30	10.4	9.9	10.17
19	1954.4	1954.4	10.44	10.6	1957.11	1958.3	11.33	11.0	11.3	10.50
20	1964.9	1964.10	11.29	11.6	1969.3	1968.11	10.63	11.0	11.1	11.67
21	1975.12	1976.6	10.46	10.3	1979.11	1979.12	10.07	9.7	9.8	10.25
22	1986.6	1986.9	10.07	9.6	1989.12	1989.7	10.60			
23	1996.7	1996.4			2000.7	(2000.5)				

[a] From Letfus (1994) except the last item in bracket.
[b] From table 1 of Mursula (1998).
[c] From table 1 of Wilson (1993).
[d] In units of calender year and month.

Let $E_i (i = 1, 2, ..., n)$ be the epochs when the values of SN(R_i) fall in the interval $[R_0, R_0 + \delta]$. Then we define the minimum epoch as

$$E_0 = \frac{1}{\sum_{i=1}^{n} w_i} \sum_{i=1}^{n} E_i w_i, \tag{2}$$

where $w_i = 1/(R_i - R_0)$ is the weight of E_i. When $R_i = R_0$, w_i is taken to be $3w'$, w' being the maximum weight of $\{w_i\}_{R_i \neq R_0}$. The min-min cycle length is defined as the time difference between two successive minima,

$$T_{\rm min} = E'_0 - E_0. \tag{3}$$

Similarly, the maximum epoch $E_{\rm m}$ is defined by

$$E_{\rm m} = \frac{1}{\sum_{i=1}^{n} w_i} \sum_{i=1}^{n} E_i w_i, \tag{4}$$

where E_i is the epoch of R_i, $R_{\rm m} - \delta \leq R_i \leq R_{\rm m}$, and $w_i = 1/(R_{\rm m} - R_i)$ is the weight of E_i. When $R_i = R_{\rm m}$, w_i is taken as $3w'$, with w' the maximum weight of $\{w_i\}_{R_i \neq R_{\rm m}}$. The max-max cycle length is defined as

$$T_{\rm max} = E_{\rm m} - E''_{\rm m}, \tag{5}$$

where $E''_{\rm m}$ and $E_{\rm m}$ are the two successive maximum epochs.

With Equations (2)–(5), the extremum epochs are easily and uniquely determined with data of SNs alone. In Table 1, Columns 2–9 list the weighted minimum epoch (E_0) and the corresponding official epoch $(O_{\rm o})$, the min-min cycle length $(T_{\rm min})$ and the official length $(O_{\rm min})$, the weighted maximum epoch $(E_{\rm m})$ and the official epoch $(O_{\rm m})$, the max-max cycle length $(T_{\rm max})$ and the official length $(O_{\rm max})$, respectively. For a comparison, Columns 10–11 list the median length $(T_{\rm med})$ from Mursula & Ulich (1998), and the min-min cycle length $(T_{\rm W})$ from Wilson (1993).

It should be noted, from Figure 1 and Table 1, that the weighted maximum epoch in Cycle 21 (Nov. 1979) is three months behind the (official) maximum time (Aug. 1979). This is caused by its longer descending phase duration. This maximum epoch is close to that of monthly maximum (Dec. 1979), but 9 months earlier than the median time (Aug. 1980).

Figure 3 depicts the differences between the weighted and official minimum epochs, $\Delta E_0 = E_0 - O_{\rm o}$ (solid), and the differences between the weighted and official maximum epochs, $\Delta E_{\rm m} = E_{\rm m} - O_{\rm m}$(dashed). The mean absolutes of ΔE_0 and $\Delta E_{\rm m}$ are 1.47 and 2.82 months, respectively. The minimum epochs differ from those of Harvey & White (1999, table 1) by about 2 months on average.

Figure 4 displays a comparison among the weighted-average, official (Letfus 1994), Wilson's (Wilson 1993), and median (Mursula & Ulich 1998) cycle lengths ($T_{\rm min}$, $O_{\rm min}$, $T_{\rm W}$, $T_{\rm med}$). The mean difference of $\Delta_{\rm min} = T_{\rm min} - O_{\rm min}$ (solid), is about 2.15 months, which is close to and slightly less than the mean $\Delta_{\rm W} = T_{\rm W} - O_{\rm min}$(dashed), 2.28 months. While the mean $\Delta_{\rm med} = T_{\rm med} - O_{\rm min}$ (dotted) is about 5.1 months.

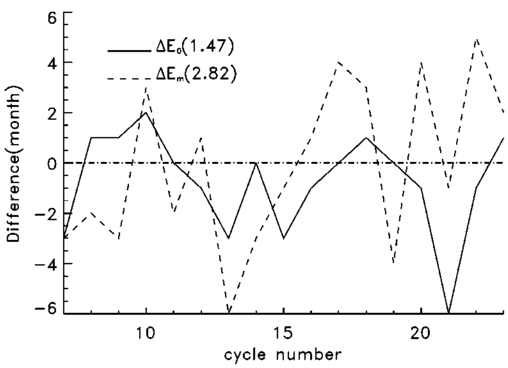

Fig. 3 Differences $\Delta E_0 = E_0 - O_{\rm o}$ (solid line), and $\Delta E_{\rm m} = E_{\rm m} - O_{\rm m}$ (dashed line). The mean absolute values are 1.47, 2.82 months, respectively.

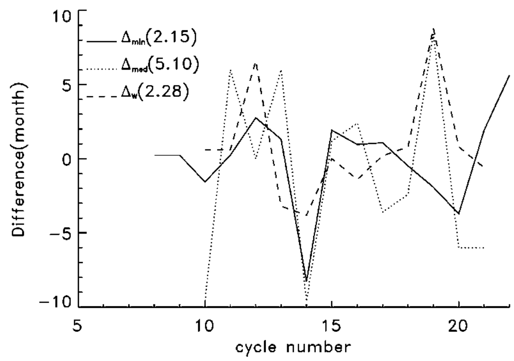

Fig. 4 Differences $\Delta_{\rm min} = T_{\rm min} - O_{\rm min}$ (solid line), $\Delta_{\rm W} = T_{\rm W} - O_{\rm min}$ (dashed line), and $\Delta_{\rm med} = T_{\rm med} - O_{\rm min}$ (dotted line). The mean absolute of $\Delta_{\rm min}$, $\Delta_{\rm W}$ and $\Delta_{\rm med}$ are 2.15, 2.28 and 5.10.

Now, we consider the effects of different values of $L(1\%, 2\%, 3\%, 4\%, 5\%, 6\%, 7\%, 8\%, 9\%, 10\%, 20\%, 30\%, 40\%$ and $50\%)$ on determination of the minimum epochs. For each value of $L_i (i = 1, 2, \cdots, 14)$ and for each cycle n, we calculate the minimum epochs, $E_0(L_i, n)$ from Equation (2), and

$$\overline{E}_0(n) = \frac{1}{N_L} \sum_{i=1}^{N_L} E_0(L_i, n), \tag{6}$$

where $n = 7, 8, \cdots, 23$ is the cycle number, $N_L = 14$ is the number of L_i and $\overline{E}_0(n)$ is the mean minimum epoch of cycle n. The mean absolute difference between $E_0(L_i, n)$ and $\overline{E}_0(n)$ for a given L_i can be obtained from

$$\delta E_0(L_i) = \frac{1}{N_c} \sum_{n=7}^{23} \left| E_0(L_i, n) - \overline{E}_0(n) \right|, \tag{7}$$

where $N_c = 17$ is the number of cycles from 7 to 23.

Similarly, the maximum epochs for each L_i each cycle n, $E_{\rm m}(L_i, n)$, are computed from Equation (4), and then the mean maximum epoch of cycle n is computed as

$$\overline{E}_{\rm m}(n) = \frac{1}{N_L} \sum_{i=1}^{N_L} E_{\rm m}(L_i, n). \tag{8}$$

The mean absolute difference between $E_{\rm m}(L_i, n)$ and $\overline{E}_{\rm m}(n)$ for a given L_i can be obtained with

$$\delta E_{\rm m}(L_i) = \frac{1}{N_c} \sum_{n=7}^{23} |E_{\rm m}(L_i, n) - \overline{E}_{\rm m}(n)|. \tag{9}$$

Figures 5 depicts the results of $\delta E_0(L)$ and $\delta E_{\rm m}(L)$ as a function of L. It should be noted that both $\delta E_0(L)$ and $\delta E_{\rm m}(L)$ are smaller than one month when $L \leq 40\%$. $\delta E_0(L)$ reaches its minimum, 0.18 month, at $L = 0.09$, and $\delta E_{\rm m}(L)$ reaches its minimum, 0.32 month, at $L = 0.10$. In other words, both $\delta E_0(L)$ and $\delta E_{\rm m}(L)$ reach their minimum values near $L = 0.1$. For this reason, we set $L = 0.1$.

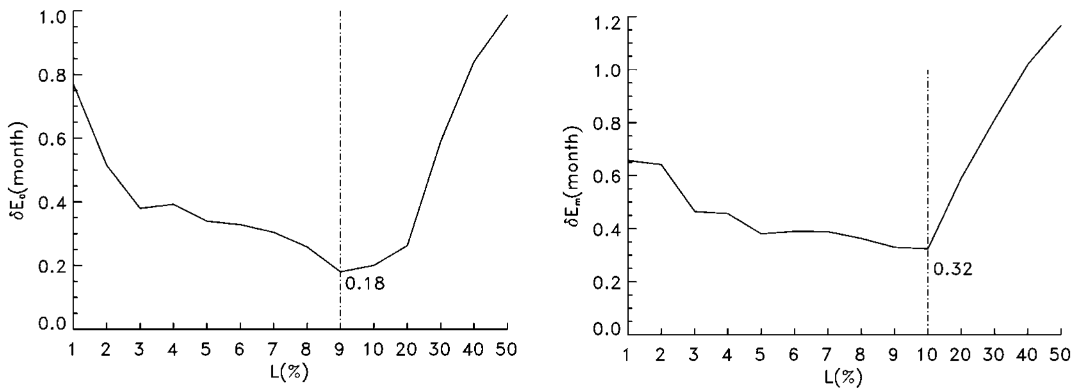

Fig. 5 $\delta E_0(L)$ and $\delta E_{\rm m}(L)$ as a function of L, both reach their minimum values, 0.18 and 0.32 months, at $L = 0.09$ and 0.10 (dot-dashed vertical lines), respectively.

3 EFFECT OF SMOOTHING WINDOW

For different applications, SNs need to be smoothed with different smoothing windows (SW, hereafter). We usually take 13 months as the SW in monthly data processing and 6 months in daily data processing. Different SWs will lead to some differences in the cycle length. Here we consider ten SWs of 7, 15, 27, 55, 81, 121, 151, 181, 241 and 365 days, and then make a comparison among the cycle lengths determined by these SWs.

For a given SW, the extremum epochs are determined by Equation (2) and the cycle lengths, by Equation (3). Thus, there are 10 sets of cycle lengths ($T_{\rm min}$). Figure 6 shows these cycle lengths in units of days. It can be seen that these lines are close to each other. The mean difference among these cycle lengths is less than 2 month. In other words, the cycle length, $T_{\rm min}(s)$, does not vary significantly with different SWs.

The intensity (integrated SNs in a cycle) for a given SW is defined as

$$R_{\rm sum} = \sum_{E=E_0(i)}^{E_0(i+1)} R(E), \tag{10}$$

where $E_0(i)$ and $E_0(i+1)$ are two successive minimum epochs. Figure 7 shows the results of $R_{\rm sum}$ for the ten different SWs.

It should be noted that almost all the 10 lines of $R_{\rm sum}$ for different SWs converge to the same line. This result can be understood as a natural consequence because $R_{\rm sum}$ is the sum of SNs between two successive solar cycle minima and is almost independent of the SW. The mean difference among these intensities is only $408.39(0.18\%)$ and can generally be neglected.

Now we obtain the mean intensity in a cycle,

$$R_{\rm mean} = R_{\rm sum}/T_{\rm min}, \tag{11}$$

where $T_{\rm min}$ is in units of days. $R_{\rm mean}$ reflects the mean size of the cycle. Figure 8 shows the results of $R_{\rm mean}$ determined by ten SWs. The mean difference of $R_{\rm mean}$ is $0.67(1.19\%)$ for different SWs. It should be noted that the 10 lines of $R_{\rm mean}$ from different SWs nearly converge to a certain line. It implies that the mean intensities, $R_{\rm mean}$, are nearly independent of the SW.

Figure 9 is a scatter plot of $R_{\rm mean}$ vs. $R_{\rm sum}$. It shows that the two parameters are highly correlated ($r = 0.9743$). Their regression equation is,

$$R_{\rm mean} = -3.3678 + 0.0002703 R_{\rm sum}. \tag{12}$$

Wilson (1988) noted that the maximum amplitude is highly correlated with either $R_{\rm mean}$ or $R_{\rm sum}$. The Wilson result is confirmed by Equation (12).

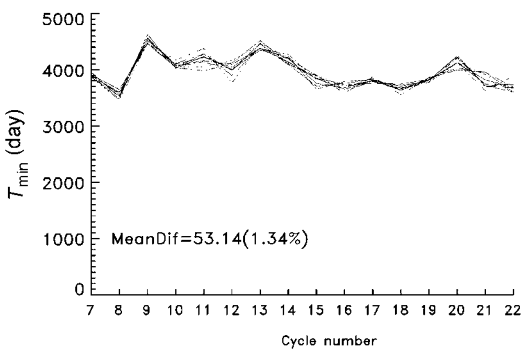

Fig. 6 The abscissa represents the cycle number. The ordinate represents cycle lengths ($T_{\rm min}$). Ten lines represent different series of $T_{\rm min}$ from different SWs. The mean difference of the 10 lines with respect to the mean line is 53.14 days (1.8 months, 1.34%).

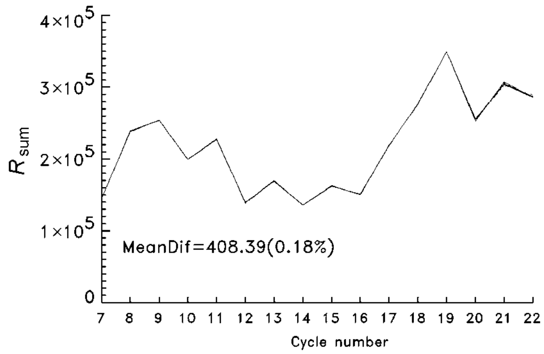

Fig. 7 Ten lines represent different series of intensities ($R_{\rm sum}$) from different SWs. Almost all these lines converge to the same line. The mean difference between them is $408.39(0.18\%)$.

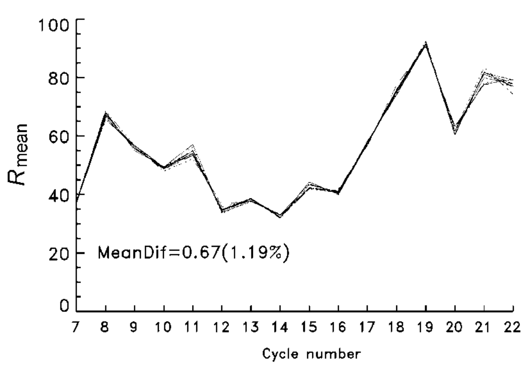

Fig. 8 Ten lines represent different series of mean intensities ($R_{\rm mean}$) from different SWs. The mean difference of the 10 lines with respect to the mean line is $0.67(1.19\%)$.

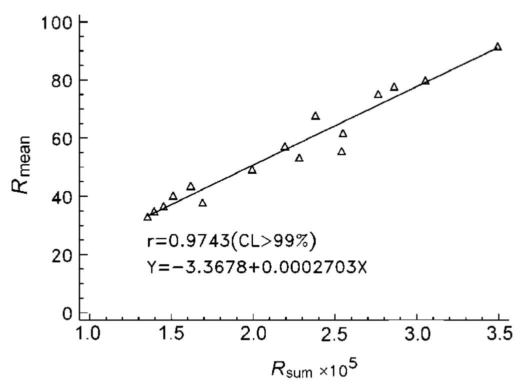

Fig. 9 Scatter plot of $R_{\rm mean}$ vs. $R_{\rm sum}$. Their linear regression equation is $R_{\rm mean} = -3.3678 + 0.0002703 R_{\rm sum}$, with correlation coefficient $r = 0.9743$, at confidence level $> 99\%$.

From Table 1 and the figures above, it can be seen that the epochs determined with the weighted average method do not vary significantly for different SWs. These epochs give reasonable indications of the extremum epochs of the sunspot cycles.

4 CONCLUSIONS

The extremum epochs of the solar cycle are key parameters for solar activity. In an effect to reasonably determine the extremum epochs, some authors have used additional parameters besides sunspot numbers (Harvey & White 1999 and references therein). Here we propose a simple method using the sunspot numbers alone. We used many different smoothing windows and obtained consistent results. The solar cycle lengths derived from our newly defined extremum epochs are in agreement with those derived from conventional definition. Comparing with the previous methods, this method has the advantage that the epochs of extrema are easily and uniquely determined.

Based on the weighted epochs of solar cycle extrema, we can investigate the relationship between the maximum amplitudes and cycle lengths (Du et al. 2006; Du 2006a, b; Du & Du 2006) or the relationship between the descending and ascending times (Du 2006c).

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