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Evolution of the Level of Sunspot Activity in Solar Cycles
I. Evolution in the Descending Phase *

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Abstract Taking the 13-point smoothed monthly sunspot number, R_i, and the deviation of the 13 associated monthly sunspot numbers from the smoothed one, D_i, as a number-pair describing the global level of sunspot activity, the evolution of the level is statistically studied for the period from the month which is just 48 months before the minimum to the minimum in the descending phase, using the observed data of Solar Cycles 10 to 22. Our results show (1) for 46 months (94%) of the studied 49 months it is found that for a given month, the distribution of the 13 pairs which come from the 13 solar cycles on a $\log R_i \sim D_i$ plane may be fitted by a straight line with a correlation coefficient larger than the critical one at confidence level $\alpha = 5\%$, and for 36 months (73%) the fitting is even better, for $\alpha = 1\%$; (2) time variations of these two parameters and their correlations in the studied period can be described respectively by functions of time, whose main trends may be expressed by a linear or simple curvilinear function; (3) the evolutionary path of the level of sunspot activity may be represented by a logarithmic function as $\log R_i = 0.704 \ln D_i - 0.291$.

Key words: Sun — sunspots — active level evolution

1 INTRODUCTION

Sunspot activity is one of the most important magnetic activities on the Sun. The evolution of active level of sunspots as a whole may reflect the magnetohydrodynamic process under the solar surface, and displays the course of solar cycles and determines the characteristic values of a solar cycle (Bray & Loughhead 1979; Harvey & White 1999; Parker 1997, 2001). Meanwhile, solar irradiance may be affected by sunspot activity and so has a solar cycle variation and short-term changes (e.g., Pap 1980). In addition, various solar eruptive features such as CMEs, spray prominences, and solar flares are generally sunspot associated (Smith & Smith 1963; Severny et al. 1979; Wang 1985,1986; Shea & Smart 1993; Webb & Howard 1994). Sunspot activity may cause sensible change or disturbance on the surface and/or in the space environment of the earth (Herman & Goldberg 1978; Willson 1982; Speich 1989; Wilkinson et al. 1991; Allen & Wilkinson 1993; Webb 1993; Wang & Sun 2002).

Evidently, understanding the sunspot activity should be very useful for constructing a model of solar cycles and helpful for developing new methods for solar cycle predictions, and hence beneficial to space weather forecasts (cf. Thompson 1997). However, for understanding the detailed physics of solar activities including sunspots, there are quite a few difficult problems to be resolved. It has been pointed out that while

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theoretical understanding of solar activities has been able to provide some general ideas of their nature, there are unanswered questions and unknown physical processes found in the best observations need to be clarified (Parker 1985, 1997, 2001). Accordingly, a statistical study on the evolution of the global level of sunspot activity based on reliable observational data may give some useful restrictions on solar cycle models and provide a possibly different method to current ways of solar cycle predictions (see Ohl et al. 1979; Brown 1986; Kunches 1993; Thompson 1993; Obridko 1995; Lantos & Richard 1998; Hathaway et al. 1999; Kane 1999; Landscheidt 1999; Li et al. 2001; Wang et al. 2002a, b; Xu 2002 and references therein).

In this paper, we use a number pair consisting of the smoothed monthly sunspot number and its deviation from the smoothed trend to describe the global level (the level, hereafter) of sunspot activity in the descending phases of Solar Cycles 10 to 22. The results show some statistical restrictions on solar cycle studies, and describe a general evolutionary process of sunspot activity level during the last 49 months of a solar cycle.

2 METHOD AND RESULTS

Several papers have focused on studies of different indices such as group sunspot number, sunspot area etc. (e.g., Usoskin & Mursula 2003; Li 1999; Zhan et al. 2005). One of the purposes of these studies is to improve the description of sunspot activity. Now, there are always random differences between the unsmoothed and smoothed monthly sunspot numbers. While the unsmoothed number is obviously much better in describing the actual level of sunspot activity. However, the smoothed one is used widely in different fields. In this work, we shall use the number-pair (R_i, D_i) R_i being the 13-point smoothed monthly sunspot number for month i (see Eq. (1)), and D_i, the rms deviation of the 13 unsmoothed monthly sunspot numbers from R_i (Eq. (2)), to reflect the true level of sunspot activity for the month i,

$$R_i = [(S_{i-6} + S_{i+6})/2 + \sum Sj]/12\,, \qquad j = i-5, \cdots i+5 \tag{1}$$

$$D_i = \sqrt{[(R_i - (S_{i-6} + S_{i+6})/2)^2 + \sum (R_i - S_j)^2]/12}\,, \qquad j = i-5, \ldots i+5 \tag{2}$$

where S_i is the monthly sunspot number for month i.

Considering the reliability of the historic data (Usoskin & Mursula 2003), we use records of monthly sunspot numbers from Cycle 10 to Cycle 22 (13 cycles in all) in this study. The data come from Catalogues of Geomagnetic Storms, Key Data and Dates of Solar-Terrestrial Physics and *http://www.sec.noaa.gov/ftpdir/weekly/RecentIndices.txt*. R_{is} and D_{is} are calculated using Equations (1) and (2) for the period from the month 48 months before the solar minimum to the minimum (49 months in all) in the descending phases of the 13 solar cycles. Thus, for every given month of the studied 49 months we obtain 13 pairs of $(\log R_i, D_i)$. Note that in the following sections, a month in the descending phase is i months to the minimum will be referred to as month i (or called month No. i), corresponding to $t = i$ months, and the month of the minimum will be referred to $t = 0$.

2.1 Linear Correlation Between log R_i and D_i

For every set of the 13 number-pairs which belongs to the same numbered month (i.e., corresponding to the same t) of the 13 solar cycles, a regression analysis is made. It is found that for most of the 49 months the distribution of the 13 number-pairs may be fitted by a straight line on the $\log R_i \sim D_i$ plane, with regression equation of the form

$$\log R_i = M_i + N_i D_i\,, \quad i = 0, 1, 2 \ldots 48\,, \tag{3}$$

with regression coefficients M_i and N_i (for month i or $t = i$ months). For the studied 49 months, we thus have 49 linear equations. Then 49 correlation coefficients, G_s, 49 regression coefficients each of M_s and N_s, are obtained, which we arrange into three sequences, from $t = 0$ to $t = 48$. Comparisons of the obtained G will be done with two critical values, $G_c = 0.684$ for confidence level $\alpha = 1\%$ and $G_c = 0.553$ for $\alpha = 5\%$.

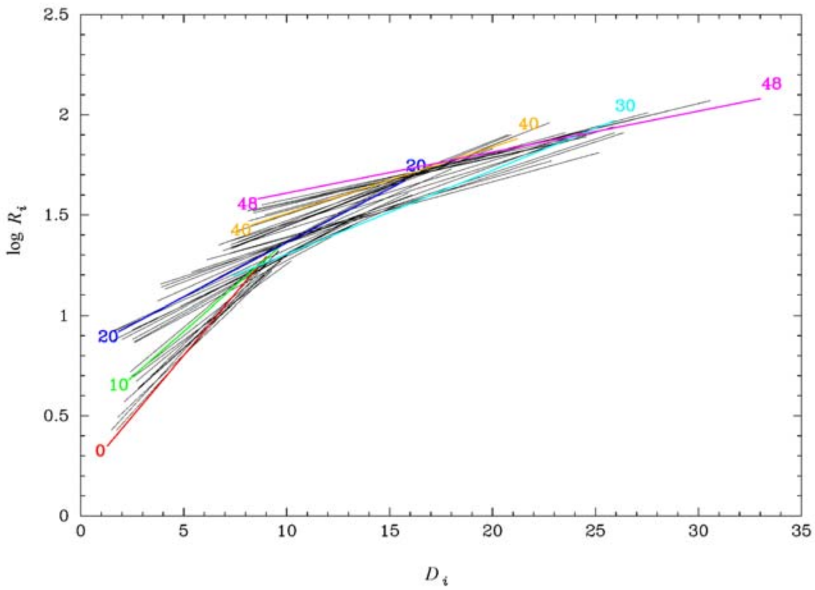

Fig. 1 Regression lines showing the linear correlation between $\log R_i$ and D_i during the descending phases of Solar Cycles 10 to 22. Numbers at the ends of some lines indicate the number of the corresponding month (see text).

It is found (1) that, for 46 of the studied 49 months ($\sim 94\%$), the obtained correlation coefficient G_s are greater than G_c with $\alpha = 5\%$, showing the 46 of the linear fittings are significant. This means that the linear correlation of $\log R_i$ with D_i is kept for most part of the descending phase during the evolution of the active level of sunspot activity; (2) that for 36 of the 49 months ($\sim 73\%$), the linear fitting is even better, with $G > G_c$ for $\alpha = 1\%$; (3) that of the 49 months, 10 ($\sim 20\%$) have a G which is greater than the 5% G_c and less than the 1% G_c, and only 3 of the 49 months ($\sim 6\%$) have G values less than the 5% G_c; and (4) that the parameters M, N and G are all time functions. In Figure 1, these fitting lines are shown, and some of them are selected to have their number of month marked. Recall that '0' means the month of minimum, 10, the month 10 months before the minimum, and so on.

What Figure 1 shows a general process and an important statistical characteristic of the evolution of sunspot activity level in the descending phase of solar cycles. The time variations of G, M and N will be analyzed in the following sections.

2.2 Time Variation of the Correlation Coefficient G

To know how the linear correlation between $\log R_i$ and D_i varies with time would be another important aspect of our study. In our case the number of the studied months is the same as the time in months, t (month), reckoned backward from the solar minimum. Hence, the time variation of the obtained correlation coefficient G is shown in Figure 2.

Figure 2 shows that the time variation of G seems to consist of a quasi-periodic variation of about 13 months superposed on a linear decreasing trend, as described by the following two equations:

$$G = g(t) + \text{quasi-periodic variation}, \tag{4}$$

where $g(t)$, obtained by a regression analysis, is

$$g(t) = 0.810 - 0.0033t, \tag{5}$$

with a statistically significant correlation coefficient of –0.483, t is the time in months.

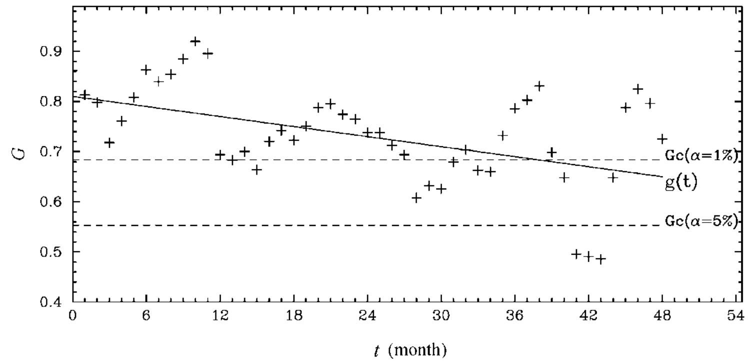

Fig. 2 Time variation of the correlation coefficient, G, between $\log R_i$ and D_i (crosses), in the descending phase. The general trend is shown by the solid line. The two dashed lines marked the two critical values at $\alpha = 1\%$ and 5%.

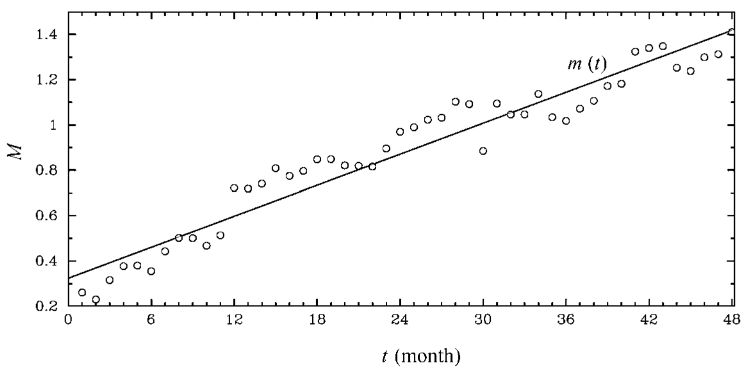

Fig. 3 Time variation of the regression coefficient M (circle). The solid line labelled $m(t)$ shows the general trend of the time variation. A quasi-periodic undulation around $m(t)$ can be seen.

Also, it can be seen in Figure 2 that the differences between the peaks and valleys of G can not be neglected. There are 10 values between the two dashed lines marking the $\alpha = 5\%$ and $\alpha = 1\%$ critical values and the three values (at $t = 41, 42, 43$) lying below the 5% dashed line can be attributed to the negative slope, or, more likely, to the large amplitude of the quasi-periodic variation. The general trend means that the closer the month is to the solar maximum, the smaller the correlation coefficient G will be.

2.3 Time Variation of the Regression Coefficients, M_i and N_i

The evolution path of the sunspot activity level in the studied period (see Fig. 1) seems to be largely related to the parameters M and N — important factors for determining the position of the fitting line. It would be interesting to see how M and N vary with time. The variations are shown in Figures 3 and 4, respectively.

One may see from Figure 3 that time variation of M can be resolved into a linear component and a quasi-periodic component, as given by Equations (6) and (7), and that the trend of the time variation of M is very different from that of G: G decreases with time while M increases with time. We have

$$M = m(t) + \text{quasi-periodic variation}, \tag{6}$$

where $m(t)$ obtained by a regression analysis is

$$m(t) = 0.324 + 0.023t \tag{7}$$

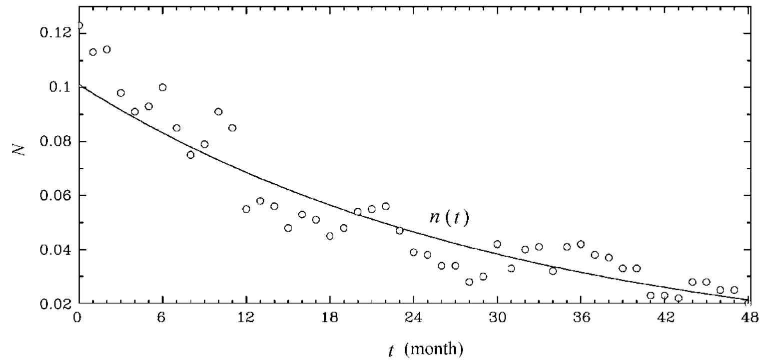

Fig. 4 Time variation of the regression coefficient N (circle). The curve labelled $n(t)$ represents the general trend of the time variation.

with a statistically significant correlation coefficient of 0.966.

It can be found in Figure 4 that, very different from M, the general trend of the time variation of N is a decrease with time that may be described by a nonlinear function of time. Quasi-periodic fluctuations exist around the curved trend. A similar analysis to that for M results in Equations (8) and (9)

$$N = n(t) + \text{quasi-periodic variation}, \tag{8}$$

where $n(t)$ obtained by a regression analysis is a nonlinear equation,

$$n(t) = 0.101\mathrm{e}(-0.032t)\,, \tag{9}$$

with a correlation coefficient of 0.939.

2.4 Time Variations of the Minimum, Median and Maximum of D_i during the Evolution

Knowledge of the limits, such as the maximum and minimum, the median of D_i and their time variations, should be very necessary factors, in our case, for a suitable description of the evolution of sunspot activity level. As mentioned above, for a given month, 13 D_{is} values can be acquired from the studied 13 solar cycles. Then the maximum, minimum and the median of the 13 D_{is} can be found, and they are denoted respectively by $D_{i,\max}$, $D_{i,\min}$ and $D_{i,m}$. This can be done for all of the studied 49 months, so we obtain the three sequences shown in Figure 5.

Despite the evident fluctuations about the solid lines in Figure 5, high correlation coefficients are obtained. The regression equations are,

$$\log D_{i,\min} = 0.180 + 0.018t, \tag{10}$$

$$\log D_{i,m} = 0.723 + 0.013t, \tag{11}$$

and

$$\log D_{i,\max} = 0.957 + 0.011t\,, \tag{12}$$

and the correlation coefficients are 0.919, 0.956 and 0.930, respectively, all of which are larger than the critical value.

Equations (10), (11) and (12) enable us to check on the evolutionary path. For example, the difference between $D_{i,\max}$ and $D_{i,\min}$ at any time in the evolution may be obtained using Equations (10) and (12) and hence the corresponding width of the evolutionary path in D_i can be estimated.

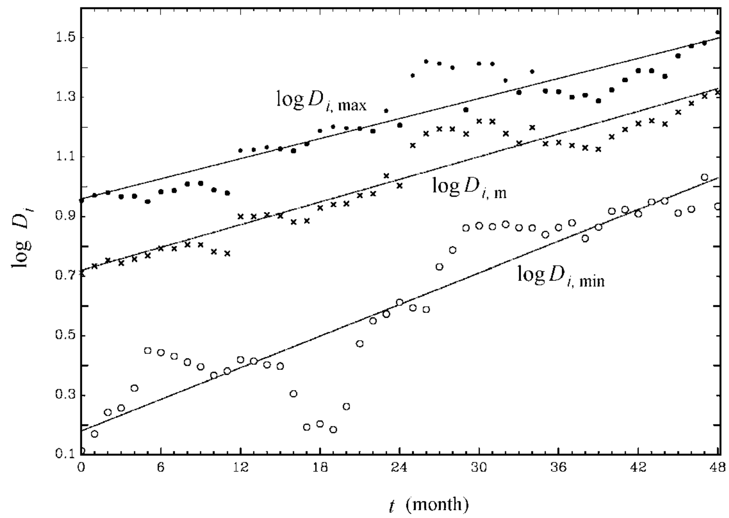

Fig. 5 Time Variations of $\log D_{i,\max}$ (points), $\log D_{i,m}$ (crosses) and $\log D_{i,\min}$ (circles). Solid lines are the linear fitting lines.

2.5 Time Variation of the Minimum, Median and Minimum of $\log R_i$ during the Evolution

No doubt, an investigation of extreme and median values of R_i and their time variations during the evolution must be of some significance for this statistical study on the evolution of the sunspot activity level.

First, according to values obtained for $D_{i,\min}$, $D_{i,m}$ and $D_{i,\max}$ above and using linear correlations obtained in Section 2.1, 49 $\log R_{i,\min}$, 49 $\log R_{i,ms}$ and 49 $\log R_{i,\max}$ corresponding to 49 months, can be calculated. Then, their time variations are obtained, as shown in Figure 6.

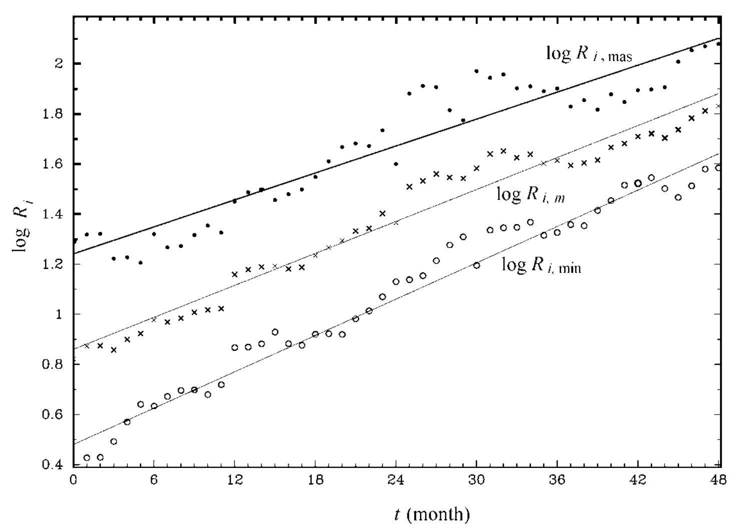

Fig. 6 Time variations of $\log R_{i,\min}$ (circles), $\log R_{i,m}$ (crosses) and $\log R_{i,\max}$ (points). Solid lines are the linear fitting lines.

Time variations of $\log R_{i,\min}$, $\log R_{i,m}$ and $\log R_{i,\max}$ shown in Figure 6 can be represented by linear fitting lines with different slopes. A regression analysis produces the following equations:

$$\log R_{i,\min} = 0.483 + 0.024t, \tag{13}$$

$$\log R_{i,m} = 0.861 + 0.021t, \tag{14}$$

and

$$\log R_{i,\max} = 1.24 + 0.018t. \tag{15}$$

The correlation coefficients for Equations (13), (14) and (15) are 0.985, 0.982 and 0.947, respectively, and all of them are larger than the critical one. Hopefully, the possible range of R_i at any time during the evolution can be estimated using Equations (13) and (15).

2.6 A Typical Correlation between R_i and D_i during the Evolution

Another subject we are interested in is what the correlation between D_i and R_i looks like during the evolution in our studied period. For this, a simple way could be to use the correlation between $D_{i,m}$ and $R_{i,m}$, to represent that between D_i and R_i. Then, to see how both $D_{i,m}$ and $R_{i,m}$ change on the evolution path would help us to set up a general conception of a quantitative relation between $D_{i,m}$ and $R_{i,m}$, as the level of sunspot activity evolves. A trial regression analysis provides us with a logarithmic correlation of $\log R_{i,m}$ and $D_{i,m}$ in the studied 49 months, as shown in Figure 7.

The correlation is shown by Equation (16),

$$\log R_{i,m} = 0.704 \ln D_{i,m} - 0.291\,, \tag{16}$$

with a correlation coefficient of 0.988.

However, it should be pointed out that while the fitting has such a high correlation coefficient, it shows only a quantitative relation of $\log R_{i,m}$ with $D_{i,m}$ and has nothing to do with any strict time sequence. Another feature shown in Figure 7 is that the distribution of the data pointed in $\log R_i \sim D_i$ plane seems to be clustered when $D_{i,m} \leq 17$.

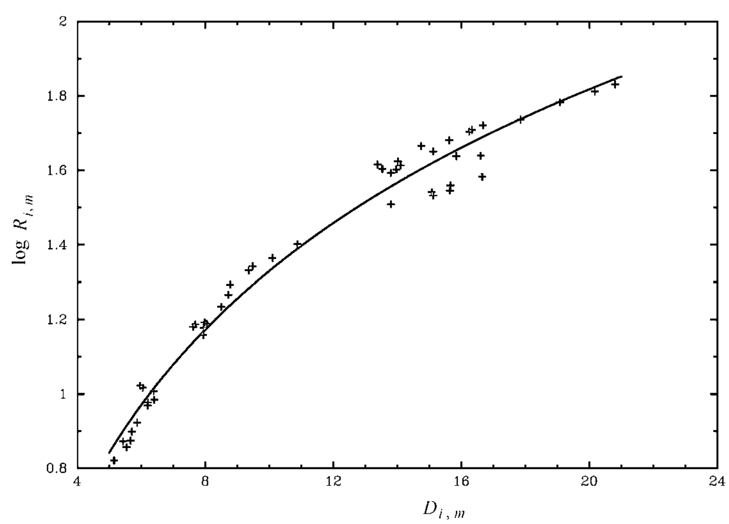

Fig. 7 Correlation between $\log R_{i,m}$ and $D_{i,m}$, indicated by crosses. Distribution of the crosses is fitted by a logarithmic curve (solid line).

3 CONCLUSIONS

(1) Evolutionary characteristics of sunspot activity level are statistically studied for the last 49 months in each solar cycle based on a number-pair composed of the 13-point smoothed monthly sunspot number and the deviation of the 13 observed monthly sunspot numbers from the smoothed one. Results obtained seem to be able to put some constraints on theoretical or model research of sunspot cycles. The evolutionary process of the activity level can be described by using 65 equations including 49 linear ones obtained in Section 2.1 and Equations (1) to (16) deduced in other sections.

(2) That the linear correlation between $\log R_i$ and D_i for month i is kept valid for most (∼94%) of the studied 49 months seems to be a new result on sunspot activity.

(3) Linear components of the time variations of the correlation coefficient, G, and the regression coefficients, M and N, show the general trends of these time variations, while the quasi-periodic components sometimes play a dominant role in determining the value of the coefficient.

(4) The time sequences of D_i's extrema and median are made based on observations. The width of the evolutionary path in D_i at any time can be obtained directly. While the time sequences of $\log R_i$'s maximum, minimum and median are made by using obtained equations. That shown in Figures 6 and 7 should be semi-empirical results.

(5) The curve shown in Figure 7 is a centric line of the evolutionary range of sunspot activity level in $\log R_i$ vs. D_i plane. Also it may be thought to be an ideal representative of the relation between $\log R_i$ and D_i in the evolution plane. It should be pointed out that in Figure 7, most of the parameter pairs around the curve appear in the order of the evolutionary time but some ones appear out of the order of the evolutionary time. Figure 7 might be thought to be a spatial correlation between $\log R_i$ and D_i.

(6) Finally, a study of the statistical properties obtained in this paper for the evolution of the global level of sunspot activity might be of significance in developing a method for solar prediction in the descending phase.

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References

Allen J. H., Wilkinson D. C., 1993, In: J. Hruska, M. A. Shea, D. F. Smart et al. eds., Solar-Terrestrial Prediction – IV, Vol. 1, p.75

Bray R. J., Loughhhead R. E., 1979, Sunspots, New York: Dover Inc.

Brown G. M., 1986, In: P. A. Simon, G. Heckman, M. A. Shea, eds., Solar-Terrestrial Predictions: Proceedings of a Workshop at Meudon, France, June 18–22, 1984, p.1

Catalogues of Geomagnetic Storms, Key Data and Dates of Solar-Terrestrial Physics, Kang-Kun Tschu, ed., 1982, Beijing: Institute of Geophysics, Chinese Academy of Sciences

Harvey K. L., White O. R., 1999, J. Geophys. Res., 104(A9), 19759

Hathaway D. H., Wilson R. M., Reichmann E. J., 1999, J. Geophys. Res., 104(A10), 22375

Herman J. R., Goldberg R. A., 1978, Sun, Weather and Climate, Washington: NASA Press

Kane R. P., 1999, Solar Phys., 189, 217

Kunches J. M., 1993, In: J. Hruska, M. A. Shea, D. F. Smart., eds., Solar-Terrestrial Predictions-IV, Vol. 2, p.198

Landscheidt T., 1999, Solar Phys., 189, 413

Lantos P., Richard O., 1998, Solar Phys., 182, 231

Li K. J., 1999, A&A, 345, 1006

Li K. J., Yun S. H., Gu X. M., 2001, A&A, 368, 285

Obridko V. N., 1995, Solar Phys., 156, 179

Ohl A. I., Ohl G. I., 1979, In: R. E. Donnelly, ed., Solar-Terrestrial Predictions Proceedings, NOAA, Boulder, Vol.2, p.258

Pap J., 1981, Solar Phys., 97, 21

Parker E. N., 1997, Solar Phys., 176, 219
Parker E. N., 2001, ChJAA, 1, 99
Severny A. B., Stepanyan N. N., Steshenko N. V., 1979, In: R. F. Donnelly, ed., Solar-Terrestrial Predictions Proceedings, NOAA, Boulder, Vol.1, p.72
Shea M. A., Smart D. F., 1993, In: J. Hruska, M. A. Shea, D. F. Smart, eds., Solar-Terrestrial Predictions – IV, Vol.2, p.48
Smith H. J. Smith E. V. P., 1963, Solar Flares, New York: The Macmillan Co.,
Speich D., 1989, In: R. M. Winglee, B. R. Dennis, eds., Max 91 Workshop #2: Developments in Observations and Theory for Solar Cycle 22, p.233
Thompson R. J., 1993, Solar Phys., 148, 383
Thompson R. J., 1997, In: G. Heckman, K. Marubashi, M. A. Shea, eds., Solar –Terrestrial Predictions-V, Tokyo: Hiraiso Center, p.3
Usoskin I. G., Mursula K., 2003, Solar Phys., 218, 319
Wang J.-L., 1985, Science in China (Series A), Vol.28, 12, 1308
Wang J.-L., Han C.-S., Xuan J.-Y. et al., 1986, Acta Astrophysica Sinica, 6(1), 34
Wang J.-L., Gong J.-C., Liu S.-Q. et al., 2002a, ChJAA, 2, 396
Wang J.-L., Gong J.-C., Liu S.-Q. et al., 2002b, ChJAA, 2, 557
Wang J.-L., Sun J., 2002, Quaternary Sciences, 22(6), 510
Webb D. F., 1993, In: J. Hruska, M. A. Shea, D. F. Smart et al., eds., Solar-Terrestrial predictions-IV, Vol.2, p.71
Webb D. F., Howard R. A., 1994, J. Geophys. Res., 99, 4201
Willson R. C.,1982, J. Geophys. Res., 87, 4319
Xu W., 2002, Science in China (Series A), 45(S), 3
Zhan L., Guo L., Zhao H., Hu L., 2005, Solar Phys., 232, 143