# The Gyrosynchrotron Radiation Spectrum in a Nonuniform Source $^\ast$

Ai-Hua Zhou

Purple Mountain Observatory, Chinese Academy of Sciences, Nanjing 210008; zhouah@pmo.ac.cn

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Abstract Effects of the energy spectral index  $\delta$ , low energy cutoff  $E_0$  and number density N of energetic electrons on gyrosynchrotron spectrum are investigated for a model source with a nonuniform magnetic field. It is found that the flux density  $S_{\nu\mp}$ of the x-mode and o-mode systematically increase with increasing  $E_0$ , N and with decreasing  $\delta$ . The peak frequency of the spectrum,  $\nu_p$ , also systematically increases as increasing  $E_0$  and N, but it may not depend on  $\delta$ . The gyrosynchrotron radiation in the nonuniform case is polarized predominately in the x-mode at  $\nu \geq 3$  GHz. A sense reversal of circular polarization also occurs but at much lower frequencies ( $\nu \leq 3$  GHz). The reversal frequency also increases with increasing  $E_0$  and N, but it perhaps is independent of  $\delta$ .

Key words: Sun: flares — Sun: radio radiation

## 1 INTRODUCTION

Radio emission from solar flares offers a number of unique diagnostic tools to address long-standing questions regarding energy release and particle acceleration and transport in magnetized plasma. For most microwave bursts incoherent gyrosynchrotron emission with mildly relativistic electrons plays a dominant role. So gyrosynchrotron emission has been studied for a long time (e.g., Ramaty 1969; Takakura 1972; Li et al. 1979; Alissandrakis & Preka-Papadema 1984; White & Kundu 1992; Benka & Holman 1992, 1994; Zhou et al. 1998; Zhou, Huang & Wang 1999; Holman 2003; Fleishman & Melnikov 2003a, b; Zhou, Su & Huang 2004, 2005; Huang, Zhou & Su 2005). It is noted, in some recent work, that a model of uniform source is usually assumed in such studies on the gyrosynchrotron emission spectrum. The uniform source assumption, however, can deviate from reality. For the case of inhomogeneous source several authors (e.g., Takakura & Sclalise 1970; Takakura 1972; Li et al. 1979; Alissandrakis & Oraka-Paradema 1984; Klein & Trottet 1984) computed the gyrosynchrotron radiation in an attempt to discuss its spectrum or spatial brightness distribution, but none of these studies considered the influence of the parameters of the energetic electrons.

In this paper, we will present a model calculation which takes account of the influence of different parameters of energetic electrons on the computed spectrum integrated over the whole emitting source. We also calculate the spectrum of gyrosynchrotron emission from nonuniform source, and compare it with previous results from uniform source.

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#### 2 MODEL CALCULATION

According to the general equation of radiative transfer, the flux density is

$$S_{\nu\mp} = \Omega \int_0^{\tau_{\nu\mp}} (\eta_{\nu\mp}/\kappa_{\nu\mp}) e^{-t_{\nu\mp}} dt_{\nu\mp} , \qquad (1)$$

where the subscripts '-' and '+' refer to the x-mode and o-mode, respectively,  $\Omega$  is the solid angle,  $\eta_{\nu}$  and  $\kappa_{\nu}$  are respectively the emission and absorption coefficient, and  $\tau_{\nu}$  is the optical depth.

Here, we take the energetic electron distribution to be isotropic in the pitch angle and to follow a power law in the energy

$$n(E) = GE^{-\delta} \,. \tag{2}$$

For our model source of a magnetic dipole field we have (Zhou, Su & Huang 2004),

$$S_{\nu\mp} = \frac{G\pi e^2 d}{3c} (2.8 \times 10^6 B_0)^{1/3} \nu^{2/3} \Omega \times 10^{19} \int_{s_0}^{s_m} \varsigma_{s\mp} s^{-2/3} e^{-\tau_{\nu\mp}(s)} ds, \tag{3}$$

where

$$\begin{split} \varsigma_{s\mp} &= \frac{1}{2 |\cos \theta|} \sum_{n>s \sin \theta}^{\infty} \int_{p_0}^{p_m} (a \pm b)^2 \frac{(\sqrt{1+p^2}-1)^{-\delta}}{(1+p^2)} dp, \\ G &= N(\delta-1) E_0^{\delta-1} \quad (\delta \neq 1). \end{split}$$

For the self absorption, the optical depth of the sth harmonic emission at a given frequency is (Zhou 2005)

$$\tau_{\nu\mp}^{\text{self}}(s) = \int_{h_{s(\nu)}}^{h_u} \kappa_{\nu\mp} dh, \tag{4}$$

where  $h_{s(\nu)}$  is the height of the sth harmonic emission for the given frequency,  $h_u$  is the height of the upper boundary of the radio source, and  $s_0$  and  $s_m$  are respectively the harmonics of the emissions from the lower boundary  $(h_d)$  and upper boundary  $(h_u)$  at frequency  $\nu$ .

The above equations depend on a number of parameters. For the determination of the flux density of gyrosynchrotron radiation in self absorption there are eight parameters (see Table 1), including the energy spectral index,  $\delta$ , the lower and upper energy cutoffs,  $E_0$  (keV) and  $E_m$  (MeV), the number density of energetic electrons, N (cm<sup>-3</sup>), the magnetic field at the photosphere,  $B_0$  (G) and the propagating angle,  $\theta$ . Once the parameters are defined the flux density at a given frequency can be calculated. The values of the eight parameters given in Table 1 are typical values of microwave bursts.

 Table 1
 Parameters used in the Calculations

δ	$E_0$	$E_m$	N	$B_0$ (G)	$\theta$ (°)	$h_u$ (cm)	$h_d$ (cm)
3	10	5	$1 \times 10^3$	3000	60	$6 \times 10^9$	$1.8\times10^9$

#### 3 NUMERICAL RESULTS

For the spectrum calculation we first estimated the lower cutoff for extraordinary mode,  $\nu_x$ ,  $\nu_x = \sqrt{\nu_p^2 + \nu_B^2/4 + \nu_B/2}$ , where  $\nu_p$  is the plasma frequency and  $\nu_B$  is the gyrofrequency. In our model calculation  $\nu_x$  is about 1.1 GHz. So the frequency range of the calculated flux and polarization spectra is 1.1 to 20 GHz. Calculations with different values of the parameters  $\delta$ ,  $E_0$ ,  $E_m$  and N and integrated over the whole emitting source were then carried out. Our calculations show that

the spectrum does not vary strongly with  $E_m$ , but it depends heavily on the energy spectral index  $\delta$ , the low-energy cutoff,  $E_0$ , and the number density N.

Figures 1–3 (each comprising three panels labelled a, b, c) show, over the frequency range 2 to 20 (or 50) GHz, the effect of varying the three parameters,  $\delta$ ,  $E_0$  and  $N_0$ , one at a time, on the x-mode flux (panels a), the o-mode flux (panels b) and the degree of circular polarization p (panels c).

$$p = (S_{\nu-} - S_{\nu+})/(S_{\nu-} + S_{\nu+}).$$
(5)

Figure 1 shows the effect of varying the energy spectral index  $\delta$  on the flux and polarization of the x-mode and o-mode. Now, Equation (3) shows that the flux at any given frequency is made up of contributions from a set of harmonic emission from  $s_0$  to  $s_m$ , so the flux densities of the x-mode and o-mode should vary smoothly with the frequency for both the optically thin and thick parts. As  $\delta$  decreases (i.e., as the spectrum becomes harder), the flux densities  $S_{\nu\mp}$  systematically increase, especially for the optically thin part, while the spectrum peak,  $\nu_p$ , remains constant. The radiation is polarized predominately in the x-mode for  $\nu > 1.1$  GHz frequencies. With decreasing frequency the degree of polarization, p, first increases at the optically thin part, peaks at  $\nu_p$ , then it falls off at frequencies  $\nu < \nu_p$ , and increases once gain at  $\nu \leq 5$  GHz.



**Fig. 1** Gyrosynchrotron emission spectrum over the frequency range 2 to 20 GHz for four different values (labelled) of the energy spectral index  $\delta$ , and fixed values of the shown in Table 1. Figures 1a and 1b for the flux density (in sfu) of the x-mode and o-mode, Figure 1c, the degree of circular polarization p.



**Fig. 2** Gyrosynchrotron emission spectrum over the frequency range 2 to 20 GHz for four different values of the low-energy cutoff  $E_0$  (keV).

Figure 2 shows that, with increasing low-energy cutoff  $E_0$  (keV), the flux densities  $S_{\nu\mp}$  and  $\nu_p$  systematically increase, while the polarization p decreases. The sense of circular polarization shows a reversal at  $\nu < 2$  GHz when the emission is dominated by the o-mode radiation and p is negative. The reversal frequency in the range 1.1 to 2 GHz increases with increasing  $E_0$ .

Figure 3 shows that, generally, with increasing number density N of energetic electrons, the flux densities  $S_{\nu\mp}$  systematically increase and so does  $\nu_p$ , but in the  $N = 1 \times 10^7$  cm<sup>-3</sup> case  $S_{\nu\mp}$  drops sharply at about 2 GHz. The polarization p decreases with increasing N. The sense of circular polarization is reversed at  $\nu \leq 3$  GHz. The sense reversal frequency in the range 1.1 to 3 GHz also increases with increasing N.



Fig. 3 Gyrosynchrotron emission spectrum over the frequency range 2 to 50 GHz for four different values of the number density N (cm<sup>-3</sup>).

#### 4 DISCUSSION

Equation (3) shows that integrated over a nonuniform source, the flux density of gyrosynchrotron emission at a given frequency is contributed by a sequence of harmonic emissions from  $s_0$  to  $s_m$ . So we expect the gyrosynchrotron emission spectrum to vary smoothly with the frequency in our model source with a nonuniform magnetic field for both the optically thin and optically thick parts. A smooth spectrum seems to agree with most of the observations in the microwave range. In the case of a uniform source, the emission at any given frequency is decided only by one particular harmonic emission. The emissions at lower frequencies in the optically thick part are decided by a lower harmonic emission. Large harmonic structures of lower harmonics (s < 10) appear in the emission and absorption coefficients of gyrosynchrotron radiation (Zhou & Huang 2001) and these may result in clear harmonic structures in the flux densities at lower frequencies (Benka & Holman 1992; Holman 2003), because such flux densities are in direct proportion to the emission coefficient and in inverse proportion to absorption coefficient in the case of uniform source. The flux densities  $S_{\nu \mp}$  systematically increase with decreasing energy spectral index  $\delta$ , increasing low-energy cutoff  $E_0$  (keV) and increasing number density N (cm<sup>-3</sup>) of energetic electrons.

The sharp drop in the flux densities,  $S_{\nu\mp}$ , at about 3 GHz, in the case of  $N = 1 \times 10^7$  cm<sup>-3</sup> (see Fig. 3) could have resulted firstly from self absorption of the gyrosynchrotron emission at lower frequencies and secondly, if the radio source for the low frequencies is not (geometrically) high enough (see Table 1),  $S_{\nu\mp}$  could diminish greatly also.

The gyrosynchrotron radiation in the nonuniform magnetic field model is polarized predominately in the x-mode at  $\nu \geq 3 \text{ GHz}$  (i.e., p > 0). The polarization p increases with decreasing frequency in the optically thin part, peaks at  $\nu_p$ , and then it falls off at  $\nu \leq \nu_p$ . A sense reversal of the circular polarization also occurs in the nonuniform case as in the uniform case (Benka & Holman 1992; Holman 2003), but at much lower frequencies ( $\nu \leq 3 \text{ GHz}$ ). We have found for the first time that the peak frequency of the gyrosynchrotron emission spectrum,  $\nu_p$ , systematically increases with increasing  $E_0$  and N, while probably independent of  $\delta$ .

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