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Simulating Pairs of HFQPOs from Micro-Quasars with Equivalent Circuit

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Abstract Pairs of HFQPOs observed from some micro-quasars may be resulted from screw-instability in magnetosphere around the central black holes. Equivalent circuit with inductor is used to simulate these HFQPOs. Some feature of the observed pairs can be explained with this model.

Key words: Pairs of HFQPOs, micro-quasar, magnetosphere, screw-instability, equivalent circuit

1 INTRODUCTION

Pairs of high frequency quasi-periodic oscillations (here after HFQPO) has been observed from microquasar XTE 1550–564, GRO 1655–40, GRS1915+105. These pairs of HFQPOs has such feature: a. the HFQPOs are observed at X-ray band; b. three pairs of HFQPOs have commensurate frequencies in a 3:2 ratio; c. the frequencies of observed pairs of QPOs vary with the mass of central black hole M as $\nu \sim 1/M$ (Remillard et. al 2002; Remillard et.al 2003).

These HFQPOs may be connected with the magnetosphere around the central black hole. It is well known that magnetosphere plays an important role in the formation of jets from micro-quasars. In BZ process(Blandford and Znajek 1977) the magnetosphere connecting the central black hole and astrophysical load can transfer the rotating energy of the central black hole to the load; and in magnetic coupling (here after MC) process (Blandford R D 1999; Li 2002), the magnetosphere connecting the hole and accretion disk can transfer rotating energy from the hole to the disk. The appearance of BZ process means the coexistence of BZ process and MC process (hereafter CEBZMC. Wang et. al 2003). In CEBZMC, the magnetosphere around the black hole is divided into two regions: BZ region and MC region. In both regions, the magnetic field has both poloidal component and toroidal component. For such magnetic field configuration, it is known from the Kruskal-Shafranov criterion that screw instability will occur in the magnetosphere, where the toroidal magnetic field becomes so strong that the magnetic field line turns around itself about once (Kadomtsev 1966; Bateman 1978; Li 2000). Because of the tension the magnetic field line tends to shrink, the toroidal magnetic field will then decrease, so the magnetosphere can gain a temporal stability. However, the toroidal magnetic field will recover, which might lead to the next screw-instability to occur. Thus a QPO is formed. As

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screw-instability can occur in both BZ region and MC region, it is possible that screw-instability can result in pairs of QPOs.

2 EQUIVALENT CIRCUIT

Here we introduce an equivalent circuit to simulate the QPO (Yao et. al 2004a, 2004b).

Compared with the equivalent circuit used before (Wang et. al 2002), inductors are introduced into the circuit. The inductor comes from the self-inductance of the circuit. It is important for the behavior of the circuit in non-stationary case.

When the current in the circuit is strong enough, the toroidal magnetic field will be strong and the screw-instability will occur. This leads to the vanishing of the current. Then the current begin to increase. The self-inductance of the circuit will delay the recovering of the current. The current in the circuit recovers as

$$I = \frac{\varepsilon}{R}(1 - e^{-\frac{t}{\tau}})\,, \tag{1}$$

where $\tau = L/R$, and L, R are the self-inductance of the magnetosphere and the resistance of the circuit respectively.

After an interval of 3τ, the current in the circuit will recover to 95% of its maximum value, which may lead to the next occurrence of screw-instability. We use 3τ to estimate the period of the corresponding QPO.

3 DISCUSSION

3.1 Can such QPO be observed at X-ray band?

The power of the QPO mainly comes from the power of the respective process (BZ or MC) of the magnetosphere where screw-instability can occur. If the power per unit area of the corresponding process in the screw-instable region is strong enough, the QPOs can be observed at X-ray band.

In BZ process, the power per unit area is (MacDonald and Thorne 1982)

$$\alpha S_E^P = \Omega^F I B^P / 2\pi\,, \tag{2}$$

where B^P is the strength of poloidal magnetic field and

$$I = \frac{1}{2}[\Omega^F(\Omega^H - \Omega^F)\varpi^2 B_\perp]\Big|_{\text{Horizon}}\,. \tag{3}$$

Because of the existence of screw-instability, the astrophysical load can't be too far from the black hole. It is easy to work out that when the strength of magnetic field near the hole is about 10^4 T, the corresponding QPO can be observed at X-ray band.

In MC process, screw-instability also occurs not far away from the hole. When the strength of magnetic field at the accretion disk is about 10^4 T, the corresponding QPO can be observed at X-ray band too.

3.2 Can pairs of QPOs have commensurate frequencies in a 3:2 ratio?

Here is the procedure we estimate the frequencies of the QPOs resulted from screw-instability(Yao et. al 2004a, 2004b).

At first, a simplified poloidal configuration of the whole magnetosphere –including both BZ region and MC region –is presumed. The toroidal magnetic field is worked out from the poloidal component and the position where screw-instability can occur is determined from Kruskal-Shafranov criterion. The inductor and resistor of the sub-circuit where screw-instability occurs are worked out from the configuration of the magnetosphere. It is noted that the horizon has a surface resistivity $R_H = 377\Omega$, and the impedance matching condition is adopted in BZ region. Then the relax time of the sub-circuit is worked out and the period of the QPO can be obtained.

For a given black hole magnetosphere system, the frequency of MC region is fixed. On the other hand, because the height of the astrophysical load in BZ region is unknown, the frequency of the QPO there has a frequency range rather than a fixed value. How ever, in condition that the magnetosphere in BZ region where instability occurs should be observable while should not block the formation of the jet, the frequency of QPO in BZ region is roughly fixed. It is interesting that in such case the pair of QPOs from BZ region and MC region do show a 3:2 ratio.

3.3 Why frequencies of pairs HFQPOs scale inversely with the mass of central black hole?

In our model, if the configurations of the magnetosphere are similar as M varies, then from $\tau = L/R$, where $L \propto M$ and R remains unchanged as M varies, it is clear that the frequencies of such QPOs vary with M as $\nu \sim 1/M$. Here we prove that the figurations of the magnetosphere are similar as M varies.

The magnetosphere around a black hole is approximated to be force free, and the configuration should satisfied the GS equation(MacDonald D and Thorne 1982))

$$\nabla \cdot \{\frac{\alpha}{\varpi^2}[1 - \frac{(\Omega^F - \omega)^2 \varpi^2}{\alpha^2 c^2}]\nabla\Psi\} + \frac{(\Omega^F - \omega)}{\alpha c^2}\frac{d\Omega^F}{d\Psi}(\nabla\Psi)^2 + \frac{16\pi^2}{\alpha\varpi^2 c^2} I \frac{dI}{d\Psi} = 0\,, \tag{4}$$

where Ψ is the magnetic flux, and I,$\Omega^F$are functions of Ψ.

Obviously, Eqn. (4) is a non-linear partial differential equation.

First we discuss the situation in MC region.

When $M = M_0$, we suppose the solution of the equation is

$$\left\{\Psi_0(r,\theta), \Omega_0^F(r,\theta), I_0(r,\theta)\right\}, \tag{5}$$

the boundary condition here is

$$\begin{cases} \Psi_0(\theta)|_{r_H} = \int\limits_{\theta}^{\theta_0} B_0 2\pi\varpi\rho d\theta|_{r_H} \\ I_0(\theta) = \frac{(\Omega^H - \Omega^F)}{2}\varpi_H d\Psi/d\lambda \end{cases} \text{at the horizon} \tag{6}$$

and

$$\begin{cases} B_{D0}^p = B_0 \frac{\varpi_H}{\varpi_{ms}}(\frac{r}{r_{ms}})^{-n} \\ \Psi_0(r, \theta = \frac{\pi}{2}) = \int\limits_{r_{ms}}^{r} B_{D0}^p 2\pi\varpi\sqrt{g_{rr}}dr \end{cases} \text{on the accretion disk.} \tag{7}$$

When $M = pM_0$, we can prove that

$$\left\{\Psi_0(r/p,\theta), \frac{\Omega_0^F(r/p,\theta)}{p}, \frac{I_0(r/p,\theta)}{p}\right\}, \tag{8}$$

are solution of Eqn. (4) and boundary condition now is

$$\begin{cases} \Psi(\theta)|_{r_H} = \Psi_0(\theta)|_{r_H} \\ I(\theta) = \frac{(\Omega^H - \Omega^F)}{2}\varpi_H d\Psi/d\lambda \end{cases} \text{at horizon} \tag{9}$$

and

$$\begin{cases} B_D^p = \frac{B_0}{p^2}\frac{\varpi_H}{\varpi_{ms}}(\frac{r}{r_{ms}})^{-n} \\ \Psi(r,\theta=\frac{\pi}{2}) = \int\limits_{r_{ms}}^{r} B_D^p 2\pi\varpi\sqrt{g_{rr}}dr \end{cases} \text{on the accretion disk.} \tag{10}$$

When $M = M_0$, GS equation is supposed to be

$$\nabla\cdot\Big\{\frac{\alpha_0}{\varpi_0^2}\Big[1-\frac{(\Omega_0^F-\omega_0)^2\varpi_0^2}{\alpha_0^2c^2}\Big]\nabla\Psi_0\Big\}+\frac{(\Omega_0^F-\omega_0)}{\alpha_0c^2}\frac{d\Omega_0^F}{d\Psi_0}(\nabla\Psi_0)^2+\frac{16\pi^2}{\alpha_0\varpi_0^2c^2}I_0\frac{dI_0}{d\Psi_0}=0\,. \tag{11}$$

When $M = pM_0$, we substitute $\{\Psi, \Omega^F, I\}$ with (8) into Eqn. (4), we get

$$\begin{array}{l} \nabla\cdot\{\frac{\alpha}{(p\varpi_0)^2}[1-\frac{(\Omega_0^F-\omega_0)^2\varpi_0^2}{\alpha_0^2c^2}]\nabla\Psi_0\}+\frac{(\Omega_0^F-\omega_0)/p}{\alpha_0c^2}\frac{d\Omega_0^F/p}{d\Psi_0}(\nabla\Psi_0)^2 \\ +\frac{16\pi^2}{\alpha_0(\varpi_0p)^2c^2}(I_0/p)\frac{(dI_0/p)}{d\Psi_0}=0 \end{array}. \tag{12}$$

Substitute r/p with $r1$, Eqn. (12) becomes

$$\begin{array}{l} \frac{1}{p^4}\{\nabla'\cdot\{\frac{\alpha_0}{\varpi_0^2}[1-\frac{(\Omega_0^F-\omega_0)^2\varpi_0^2}{\alpha_0^2c^2}]\nabla'\Psi_0\}+\frac{(\Omega_0^F-\omega_0)}{\alpha_0c^2}\frac{d\Omega_0^F}{d\Psi_0}(\nabla'\Psi_0)^2 \\ +\frac{16\pi^2}{\alpha_0\varpi_0^2c^2}I_0\frac{dI_0}{d\Psi_0}\}=0 \end{array}, \tag{13}$$

where ∇' is the differential functor of $\{r1, \theta, \phi\}$. Obviously, Eqn. (13) is consistent with Eqn. (11).

Substituting Eqn. (8) into Eqn. (9) and Eqn. (10), we can see that they are consistent with Eqn. (6), Eqn. (7).

From the discussion above, we can conclude that when the mass of central black hole varies, the poloidal configuration of the magnetosphere in MC region is similar.

In the same way, we can get the similar conclusion in BZ region.

Here we point out that for the poloidal configuration suggested in this paper, though Eqn. (4) is non-linear, the shape of the magnetic field line is independent of the absolute magnitude of the magnetic field.

4 FIT TO THE OBSERVATION

Table below shows the fit to the observation of HFQPO pairs from microquasar XTE 1550–564, GRO 1655–40, GRS1915+105. The frequencies of QPOs resulted from screw-instability of magnetosphere in MC region and BZ region are respectively used to mimic the lower and higher frequencies of observed pairs of HFQPOs. Observations have given the range of mass for each central black hole. Here, within the frame of our model, we get the range of a_* of the corresponding central black hole.

Table 1 Fit to the Observations

Source	$M/M_\odot$	a_*	f(Hz)
XTE 1550–564	8.4∼10.8	0.67∼0.8	184,276
GRO 1655–40	6.0∼6.6	0.78∼0.83	300,450
GRS1915+105	10∼18	0.55∼0.84	113,168

5 CONCLUSIONS

Screw-instability in magnetosphere around black hole may result in pairs of HFQPOs. Such mechanism can explain some feature of the pairs of HFQPOs observed from some micro-quasars.

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