Chin. J. Astron. Astrophys. Vol. 5 (2005), No. 6, 665–670
(*http://www.chjaa.org*)

Chinese Journal of
Astronomy and
Astrophysics

A Study on the Relationship between the Orbital Lifetime and Inclination of Low Lunar Satellites

Hai-Hong Wang[1,2] and Lin Liu[1,2]

[1] Department of Astronomy, Nanjing University, Nanjing 210093; *xhliao@nju.edu.cn*
[2] Institute for Space Environment and Astrodynamics, Nanjing University, Nanjing 210093

Received 2005 May 9; accepted 2005 August 26

Abstract A detailed theoretical analysis on the orbital lifetime and orbital inclination of a Low Moon-Orbiting satellite (LMOs) and the 'stable areas' of long orbital lifetime are given. Numerical simulations under the real force model were carried out, which not only validate the theoretical analysis and also give some valuable results for the orbit design of the LMOs.

Key words: celestial mechanics — Low-Moon-Orbiting satellite — orbital lifetime

1 INTRODUCTION

Due to the difference between the Moon's gravitational potential and the Earth's, the lifetimes of a Moon-Orbiting Satellite (MOs) and an Earth-Orbiting-Satellite (EOs) are very different. Problems on the orbital lifetime of high MOs (HMOs) and low MOs (LMOs) have been discussed in Matchal (1999), Mayer et al. (1994) and Wang et al. (2002, 2003). These mainly made use of numerical results to show the short orbital lifetime of polar orbit LMOs under the lunar odd zonal harmonic perturbations. In this paper we shall go more deeply into the theoretical relationships between the LMOs' lifetime and the orbital elements (especial the inclination i) using perturbations analytical solutions. Numerical simulations are also carried out under the full force model giving detailed useful orbital information on LMOs.

2 THE THEORETICAL ANALYSIS ON THE ORBITAL LIFETIME OF LMOS

Despite the absence of atmosphere on the Moon, LMOs will fall on the Moon when the Moon's nonspherical gravitational perturbation (NGP) has reduced the perifocus $r_p = a(1 - e)$ to the semimajor axis a_e. Now, there are only short period terms with small amplitudes (of the order of 10^{-4}) in the orbital semimajor axis a due to the perturbation of the Moon's dynamical oblateness of J_2 and they will not lead to appreciable variation of r_p. Thus, the reason why r_p diminishes greatly is due to the large amplitude long period terms Δe_l of the orbital eccentricity e.

The equation for the long period terms of e in the lunar-centered epoch celestial coordinate system due to the two main perturbations, the Moon's NGP and the Earth's gravitation, can be written as:

$$\begin{aligned}\frac{de_l}{dt} &= n(1-e^2)\sum_{l\geq 3}(\frac{J_l}{p_0^l})\sum_{p=1}^{(l-2+\delta)/2}(-1)^{(l-\delta)/2}(l-2p)F(i)\Big[\frac{1}{e}K(e)\Big]I(\omega)\\ &-n(1-e^2)\sum_{l\geq 2}\sum_{m=1}^{l}(\frac{1}{p_0^l})\sum_{p=1}^{l-1}(l-2p)F_{lmp}(i)\Big[\frac{1}{e}K(e)\Big]\Phi_{lmp}(\omega,\theta)\\ &+O(em'),\end{aligned} \tag{1}$$

where $J_2 = -C_{2,0}$ is the Moon's dynamical oblateness (of order 10^{-4}). The original expression of Eq.(1) was developed in Liu et al. (1998), here, some changes are made for the purpose of this paper and a dimensionless unit system is introduced with the lunar gravitational constant fixed at $\mu = GM = 1$. The 1st term and the 2nd term of Eq.(1) are the response to the zonal ($J_l = -C_{l,0}$) harmonic perturbations and tesseral harmonic perturbations and the 3rd term is the response to the Earth gravitational perturbation and they are all factored by e. In Eq.(1), $n = a^{-3/2}$ and the notation p_0 is used to distinguish it from the summing index p. The relative equations may be written as

$$\delta = \frac{1}{2}[1-(-1)^l] = \begin{cases} 1 & l \text{ is odd} \\ 0 & l \text{ is even} \end{cases}, \tag{2}$$

$$\begin{cases} \frac{1}{e}K(e) = \sum\limits_{\alpha(2)=(l-2p)}^{l-2} C_{lp\alpha}(\frac{1}{2})^{\alpha}e^{\alpha} = \begin{cases} O(e^0) & l \text{ is odd} \\ O(e) & l \text{ is even} \end{cases} \\ C_{lp\alpha} = \begin{pmatrix} l-1 \\ \alpha \end{pmatrix}\begin{pmatrix} \alpha \\ \frac{1}{2}(\alpha-(l-2p)) \end{pmatrix}, \quad \begin{pmatrix} n \\ m \end{pmatrix} = \frac{n!}{m!(m-n)!}, \end{cases} \tag{3}$$

$$\begin{cases} F(i) = -\sum\limits_{q=0}^{p}(-1)^q(\frac{1}{2})^{(2l-2p+2q-\delta_p)}C_{lpq}(\sin i)^{(l-2p+2q)} \\ \delta_p = \begin{cases} 0 & l-2p=0 \\ 1 & l-2p\neq 0 \end{cases} \\ C_{lpq} = \begin{pmatrix} l \\ p-q \end{pmatrix}\begin{pmatrix} 2l-2p+2q \\ l \end{pmatrix}\begin{pmatrix} l-2p+2q \\ q \end{pmatrix}, \end{cases} \tag{4}$$

$$I(\omega) = -(1-\delta)\sin(l-2p)\omega + \delta\cos(l-2p)\omega. \tag{5}$$

Note that $\theta = \Omega - S(t)$ in the function $\Phi(\omega,\theta)$ of the tesseral harmonic perturbation and $S(t)$ is the longitude of X axis in the lunar fixed coordinates system, which varies with the revolution of the Moon. The expression of $\Phi(\omega,\theta)$, inclination function $F_{lmp}(i)$ and the Earth gravitational perturbation term $O(em')$ are not given in this paper, because they can all be omitted as will be shown below.

Further analysis of Eq.(1) shows that the terms on the right are either $O(e^0)$ (i.e. terms like $e^0\cos\omega$) when l is odd, or $O(e)$ (i.e. terms like $e\sin 2\omega$) when l is even. The orbital eccentricity e is small (i.e. $e < 0.1$) for the orbital lifetimes under consideration. In fact, for the LMOs with mean altitude 100 km, r_p is near a_e, where a_e is $0.05 \sim 0.06$, and the LMOs will hit the Moon once e increases. Thus, perturbation terms in Eq.(1) with odd l need to be considered, while the tesseral harmonic perturbations are one order smaller than the odd zonal harmonic perturbations. As a result, in theoretical analysis only the odd zonal harmonic perturbations need to be retained in Eq.(1). The effect of the omitted terms can be considered in the numerical simulation under the full force model and the results have shown the reasonableness of the simplification. Retaining only terms like $O(e^0)$ which are confined to the odd zonal harmonic perturbation in Eq. (1), the simplified expression is

$$\frac{de_l}{dt} = \sin i \sum_{l(2)\geq 3} (-1)^{(l-1)/2}(\frac{1}{2})(l-1)(J_l/P_0^l)F^*(i)(n\cos\omega), \tag{6}$$

where

$$\begin{cases} F^*(i) &= \sum_{q=0}^{(l-1)/2} (-1)^q(\frac{1}{2})^{(l+2q)} C^*_{lpq}(\sin^2 i)^q \\ C^*_{lpq} &= \begin{pmatrix} l \\ (l-1)/2-q \end{pmatrix} \begin{pmatrix} l+2q+1 \\ l \end{pmatrix} \begin{pmatrix} 2q+1 \\ q \end{pmatrix} . \end{cases} \tag{7}$$

Here $l(2)$ means the summation in Eq.(6) is with stepsize 2, i.e. $l(2) = 3, 5, \cdots$. $\omega = \bar{\omega}(t) = \bar{\omega}_0 + \omega_c(t - t_0)$, ω_c is the secular coefficient of $\bar{\omega}(t)$, and if only the 1st secular coefficient due to J_2 is considered, then we have

$$\omega_c = \omega_1 = \frac{3J_2}{2p_0^2}n(2 - \frac{5}{2}\sin^2 i). \tag{8}$$

Integrating Eq. (6) leads to

$$\begin{aligned} \Delta e_l &= e_l(t) - e_l(t_0) \\ &= \sin i\Big\{\sum_{l(2)\geq 3} (-1)^{(l-1)/2}\frac{(l-1)}{3p_0^l}(\frac{J_l}{J_2})F^*(i)\Big\}\big[\sin\bar{\omega}(t) - \sin\bar{\omega}(t_0)\big]/(2 - \frac{5}{2}\sin^2 i)\,. \end{aligned} \tag{9}$$

It is easily seen that the amplitude of Δe is mainly determined by the ratio of the odd zonal harmonic coefficient $J_{2l-1}(l \geq 2)$ to J_2 and the character of the function $F^*(i)$ and we have

$$|\Delta e_l| \sim O(J_{2l-1}/J_2) \cdots F^*(i). \tag{10}$$

For the Earth orbiting satellite, because

$$O(J_{2l-1}/J_2) = 10^{-3} \tag{11}$$

the amplitude of eccentricity Δe_l is small. For the Moon orbiting satellite, because

$$O(J_{2l-1}/J_2) = 10^{-1} \tag{12}$$

it is possible for $r_p = a_e$ with sufficient increase of e. In fact, it is essentially determined by the character of $(J_{2l-1}/J_2)F^*(i)$.

Due to the character of the lunar nonspherical gravitational potential, the harmonic coefficients J_{2l-1} do no obviously decrease with increasing order l, and from the analysis of the function $(J_{2l-1}/J_2)F^*(i)$ we know that the modulus of long period variation $|\Delta e_l|$ has several maxima or minima for different inclinations i ($0° < i \leq 90°$), this means the LMOs' orbital lifetime is determined by both the odd zonal harmonic perturbations and the orbital inclination. The LMOs will not hit the Moon in a long period when the inclination i^* is near an area where the function $(J_{2l-1}/J_2)F^*(i)$ attains its minimum and this area is called the 'stable area' of orbital lifetime; but the LMOs will soon hit the Moon when the i^* is near an area where the function $(J_{2l-1}/J_2)F^*(i)$ attains its maximum, this area is called the 'unstable area'.

Although lunar libration could change lunar nonspherical gravitational potential, the order of magnitude of the change is only 10^{-7}. So, there are only short period terms in a and e due to the libration. Thus, the lunar libration is omitted in the above discussion. The effect of lunar libration is included in Zhang et al. (2005).

3 NUMERICAL SIMULATIONS AND SOME VALUABLE RESULTS FOR LMOS' ORBIT DESIGN

To verify the correctness of the theoretical analysis, numerical simulation were carried out for two cases for the LMOs:

(1) According to the analytical solution Eq.(9), the distribution of the maximum and minimum is found by calculating $|\Delta e_l|$ for i ranging from $0^\circ.5$ to $179^\circ.5$ in step of 1°.

(2) Considering the main perturbation (including lunar nonspherical gravitational perturbation, the Earth gravitational perturbation and the Sun gravitational perturbation), the perifocus altitude h_p for different inclinations i is given by the dynamical equations of the LMOs (the mean altitude is $\bar{h} = 100\,\mathrm{km}$, $e_0 = 0.001$). This is just the correlation between the LMOs' orbital lifetime and the inclination i.

Numerical calculation in case (2) verifies the correctness of the results given by the analytical solution, and identifies the relationship between the LMOs' orbital lifetime and the orbital inclination i. Therefore, it is clear that the result is mainly determined by the character of the lunar nonspherical gravitational harmonic potential.

In the above calculation, the lunar gravitation model LP75G of JPL which is similar to model LP165 of JPL is used. Moreover, only zonal harmonic terms with l in the range 30–45 are useful for case (1), and the full harmonic terms are useful with l is 75 and m is $0 - l$ for case (2).

The 'stable areas', i.e. where $|\Delta e_l|$ is very small, correspond to

$$i = 0^\circ,\ 27^\circ,\ 50^\circ,\ 77^\circ \text{ and } 85^\circ,$$

and Eq.(9) gives the corresponding magnitudes of $|\Delta e_l|$ as 0.0, 0.0053, 0.0010, 0.0042 and 0.0015. According to the character of $\sin i$, there are also 'stable areas' in the range $90^\circ - 180^\circ$, namely, at $95^\circ, 103^\circ, \cdots$.

The LMOs' orbital lifetime in the 'stable areas' is expected to be long in in case (2) calculated with the full force model, while it is very short in the 'unstable areas'. Numerical results are listed in Table 1 for different orbital inclinations i. In Table 1, min$h_p = 0.0$ or min$h_p \simeq 0.0$ indicates hitting the Moon's surface and T_c is the associated orbital lifetime. The maximum integrating interval is 10 years for all i_0 and the integration is stopped when $h_p = 0.0$. The

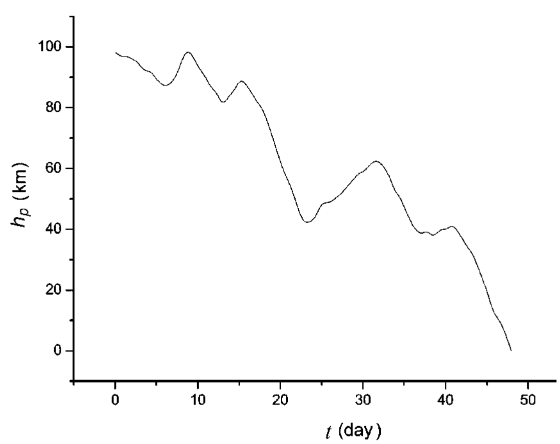

Fig. 1 Orbital lifetime of LMOs with initial inclination $i_0 = 40^\circ$.

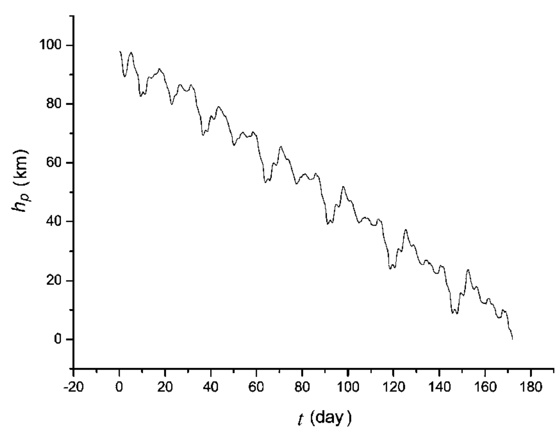

Fig. 2 Orbital lifetime of LMOs with initial inclination $i_0 = 90^\circ$.

perifocus altitude does not decrease greatly for LMOs in the 'stable area' (such as $i_0 = 85°$ and $95°$) in 10 years and the minimum altitude is still 60 km. Figures 1–4 show the h_p–t graph for $i_0 = 40°$, $90°$ and $85°$, $95°$.The lifetime is only 48 days and 172 days for $i_0 = 40°$ and $90°$, respectively, and the minimum altitude is 60 km for $i_0 = 85°$ and 68 km for $i_0 = 95°$.

Table 1 Orbit Lifetime of LMOs for Different Initial Inclinations i_0

i (deg)	T_c (day)	Max. e	Min. h_p (km)	i (deg)	T_c (day)	Max. e	Min. h_p (km)
1.0	2723.1	0.0362	33.9	60.0	88.2	0.0548	0.0
2.0	549.0	0.0414	24.6	61.0	88.1	0.0547	0.0
3.0	1852.7	0.0479	13.0	63.43	85.9	0.0545	0.0
4.0	273.2	0.0550	0.0	65.0	88.0	0.0546	0.0
5.0	49.5	0.0548	0.0	67.0	115.5	0.0547	0.0
7.5	42.9	0.0545	0.0	69.0	224.1	0.0523	3.9
10.0	42.5	0.0545	0.0	70.0	3347.5	0.0464	14.9
12.5	43.9	0.0547	0.0	71.0	3407.0	0.0406	25.5
15.0	43.9	0.0547	0.0	72.0	2453.3	0.0348	36.2
17.5	46.3	0.0548	0.0	73.0	1469.4	0.0333	39.0
20.0	77.0	0.0547	0.0	74.0	1498.4	0.0339	37.8
22.0	80.7	0.0532	3.2	75.0	1500.5	0.0340	37.8
24.0	80.0	0.0525	4.4	76.0	1449.0	0.0336	38.5
26.0	2543.1	0.0515	6.0	77.0	3383.8	0.0381	30.4
27.0	2219.9	0.0419	23.6	79.0	401.1	0.0544	0.0
28.0	2599.4	0.0264	52.1	80.0	320.6	0.0545	0.0
29.0	1404.4	0.0251	54.4	81.0	294.0	0.0545	0.0
30.0	2084.1	0.0453	17.3	82.0	294.7	0.0547	0.0
31.0	91.6	0.0547	0.0	83.0	403.2	0.0544	0.0
33.0	46.2	0.0547	0.0	84.0	2941.4	0.0419	23.0
35.0	44.5	0.0546	0.0	85.0	1711.7	0.0220	59.6
37.0	45.2	0.0546	0.0	86.0	3401.8	0.0414	23.6
39.0	47.4	0.0548	0.0	87.0	308.8	0.0523	4.0
40.0	47.9	0.0547	0.0	88.0	174.6	0.0542	0.3
41.0	48.4	0.0546	0.0	89.0	171.3	0.0545	0.0
43.0	48.3	0.0548	0.0	90.0	172.0	0.0545	0.0
45.0	49.7	0.0544	0.7	91.0	193.0	0.0546	0.0
47.0	72.7	0.0545	0.0	92.0	226.7	0.0546	0.0
49.0	177.9	0.0546	0.0	93.0	309.9	0.0546	0.0
50.0	2522.0	0.0545	0.0	94.0	1133.9	0.0392	28.1
51.0	1908.1	0.0337	38.5	95.0	1102.0	0.0172	68.3
52.0	211.9	0.0547	0.0	96.0	1557.2	0.0253	53.6
54.0	88.9	0.0546	0.0	97.0	1118.8	0.0464	14.5
56.0	83.1	0.0547	0.0	98.0	236.2	0.0546	0.0
58.0	84.6	0.0546	0.0				

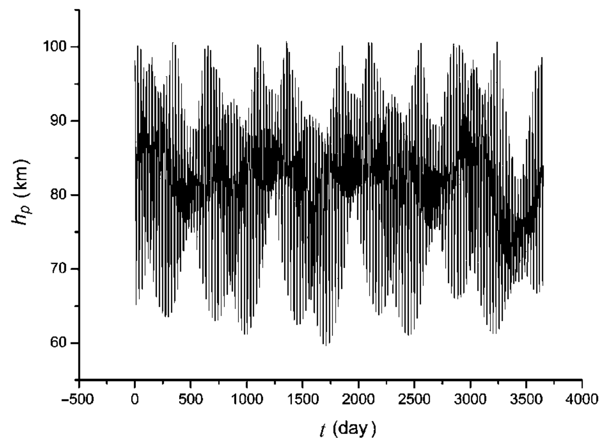

Fig. 3 Orbital lifetime of LMOs with initial inclination $i_0 = 85°$.

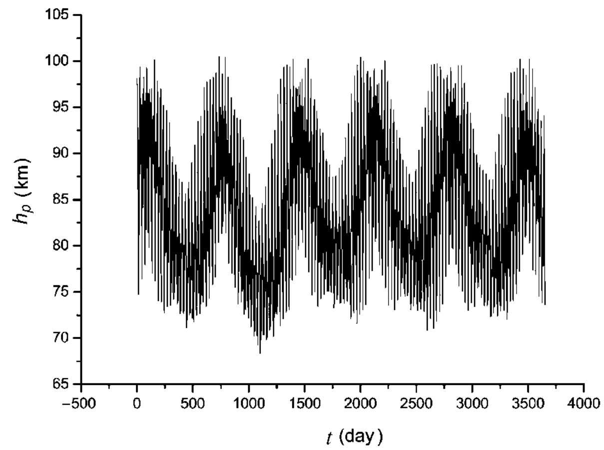

Fig. 4 Orbital lifetime of LMOs with initial inclination $i_0 = 95°$.

4 CONCLUSIONS

Numerical results listed above have validated the correctness of the theoretical analysis and presented the relationship between the LMOs' orbital lifetime and orbital inclination. Considering the smallness of the 'stable areas' (for the selection of i_0) and small perturbing terms omitted in the theoretical analysis of this paper, more refined calculations may be necessary in orbit design. Furthermore, an orbital manoeuver is always necessary with the existing of orbit injection errors.

References

Liu L., Wang J.-S., Chin. Astron. Astrophys., 1998, 22(3), 328

Marchal C. L., 52th International Astronautical Congress, Amsterdam, The Netherlands, 1999

Meyer K. W., Buglia J. J., Desai P. N., NASA Technical Paper, 3394, 1994

Wang X., Liu L., Chin. Astron. Astrophys., 2002, 26(4), 489

Wang X., Liu L., Chin. Astron. Astrophys., 2003, 27(1), 107

Zhang W., Liu L., Chin. Astron. Astrophys., 2005, 29(4), 438