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Estimates of AGN Black Hole Mass and Minimum Variability Timescale *

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Abstract Black hole mass is one of the fundamental physical parameters of active galactic nuclei (AGNs), for which many methods of estimation have been proposed. One set of methods assumes that the broad-line region (BLR) is gravitationally bound by the central black hole potential, so the black hole mass can be estimated from the orbital radius and the Doppler velocity. Another set of methods assumes the observed variability timescale is determined by the orbital timescale near the innermost stable orbit around the Schwarzschild black hole or the Kerr black hole, or by the characteristic timescale of the accretion disk. We collect a sample of 21 AGNs, for which the minimum variability timescales have been obtained and their black hole masses (M_σ) have been well estimated from the stellar velocity dispersion or the BLR size-luminosity relation. Using the minimum variability timescales we estimated the black hole masses for 21 objects by the three different methods, the results are denoted by $M_{\rm s}$, $M_{\rm k}$ and $M_{\rm d}$, respectively. We compared each of them with M_σ individually and found that: (1) using the minimum variability timescale with the Kerr black hole theory leads to small differences between M_σ and $M_{\rm k}$, none exceeding one order of magnitude, and the mean difference between them is about 0.53 dex; (2) using the minimum variability timescale with the Schwarzschild black hole theory leads to somewhat larger difference between M_σ and $M_{\rm s}$: larger than one order of magnitude for 6 of the 21 sources, and the mean difference is 0.74 dex; (3) using the minimum variability timescale with the accretion disk theory leads to much larger differences between M_σ and $M_{\rm d}$, for 13 of the 21 sources the differences are larger than two orders of magnitude; and the mean difference is as high as about 2.01 dex.

Key words: galaxies: active – galaxies: nuclei – galaxies: fundamental parameters

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1 INTRODUCTION

One of the aims of studying active galactic nuclei (AGNs) is to unify certain classes of AGNs. In a unified scheme different AGNs are understood in terms of variations in a few principal parameters (Barth et al. 2003). In accretion and jet theories, the fundamental parameters determining the appearance and behavior of the system are the black hole mass $M_{\rm H}$, the mass accretion rate $\dot{M}$, and the black hole angular momentum J, which are usually expressed in dimensionless forms as $m_8 \equiv M_{\rm H}/10^8 M_\odot$ ($M_\odot$ the solar mass), $\dot{m} = \dot{M}/\dot{M}_{\rm Edd}$ ($\dot{M}_{\rm Edd}$ the accretion rate that produces one Eddington luminosity), and $j = J/J_{\rm max}$ ($J_{\rm max} = GM_{\rm H}^2/c$ the angular momentum of a maximal Kerr black hole), respectively (Meier 2002). Obviously, determining the mass of a super-massive black hole in AGNs is an important step in this attempt. Many methods have been proposed to estimate the masses of the central black holes of AGNs. One set of methods assumes that the observed variability timescale is determined by the orbital timescale near the innermost stable orbit around the Schwarzschild black hole (Miller et al. 1987; Xie et al. 1987; Fan et al. 1999) or the Kerr black hole (Xie et al. 2002), or by the characteristic timescale of accretion disk (Fan et al. 1999). Another set of methods assumes that the broad-line region (BLR) is gravitationally bound by the central black hole potential, so that the black hole mass can be estimated from the orbital radius and the Doppler velocity (Woo & Urry 2002; Blandford & Mckee 1982; Peterson 1993; Mclure & Dunlop 2001; Vestergaard 2002; Kaspi et al. 2000; Barth et al. 2003). However, estimates of the black hole mass by different methods for the same object have larger differences, as pointed out by Barth et al. (2003). Therefore, it is very important to make a detailed comparison of the mass estimates by the different methods. In this paper we will report results of such a comparative study.

The sample we use in this paper includes 21 AGNs, for which rapid variability timescales have been obtained and their black hole masses have also been well estimated from stellar velocity dispersion and the BLR size - luminosity relation of Woo & Urry (2002). These data are listed in Table 1: column (1) gives the IAU name; column (2) the class of the source; column (3) the redshift; columns (4), (5) and (6) the logarithm of the minimal variability timescale, its band and the relevant reference; column (7) the bolometric luminosity (in erg $\rm s^{-1}$) given by Woo & Urry (2002); column (8) the Doppler factor, column (9) the black hole mass estimated using stellar velocity dispersion or the BLR size - luminosity relation of Woo & Urry (2002).

2 BLACK HOLE MASSES FROM MINIMUM VARIABILITY TIMESCALES

Large-amplitude, rapid optical variability is one of well-known identifying characteristics of blazars. The minimum timescale of variation may be used to place constraints on the size of the emitting region, even though it may have a complex structure. If one assumes that the variations are produced close to the black hole, the black hole mass can be estimated by the variability timescale (Miller et al. 1989; Xie et al. 1987).

2.1 Black Hole Mass Estimates based on the Schwarzschild Black Hole Theory

It is well-known that the maximum rotation frequency and the fundamental vibration frequency of any Newtonian polytrope are both of order $((GM/R^3)^{1/2})$ (Weinberg 1972; Xie et al. 1987). If the minimum variability timescale is $\Delta t_{\rm min}$, we have

$$\left(\frac{GM}{R^3}\right)^{\frac{1}{2}} = \frac{1}{\Delta t_{\rm min}}, \tag{1}$$

which can be written as

$$R = \frac{1}{\sqrt{2}}\sqrt{\frac{2GM}{c^2 R}}\, c\Delta t_{\rm min}. \tag{2}$$

Table 1 Observational Data and the Results Derived for 21 Blazars

Source	Class	z	$\log \Delta t$	band*	Ref.†	$\log L_{\rm bol}$♯	δ	$\log(M_\sigma/M_\odot)$♯	$\log(M_{\rm k}/M_\odot)$	$\log(M_{\rm s}/M_\odot)$	$\log(M_{\rm d}/M_\odot)$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
0317+183	XBL	0.190	3.75	O	X04a	45.50	1.37	8.0	8.0	7.9	6.51
0420−014	FSRQ	0.915	4.65	O	X04a	47.0	1.81	9.03	9.0	8.8	7.5
0548−322	XBL	0.069	3.54	O	X04a	45.6	1.60	8.15	7.8	7.67	6.91
1101+384	XBL	0.031	3.56	O	X88	45.7	1.65	8.29	8.0	7.69	6.45
1137+660	FSRQ	0.652	4.72	O	X91	46.85	1.64	9.36	9.1	8.8	7.4
1244−255	FSRQ	0.633	4.72	O	X91	46.48	1.38	9.04	8.9	8.8	7.4
1253−055	FSRQ	0.536	3.69	O	X04a	46.1	1.87	8.43	8.0	7.8	6.9
1355−416	FSRQ	0.313	4.78	O	X91	46.0	1.35	9.73	9.00	8.88	7.48
1510−089	FSRQ	0.361	3.50	O	X04a	46.90	2.94	8.65	8.1	7.67	6.57
1514−241	RBL	0.049	2.95	O	X04a	44.50	1.30	8.10	7.2	7.08	5.74
1538+149	FSRQ	0.605	3.44	O	X90	46.4	2.4	7.82	7.81	7.54	6.66
1641+399	FSRQ	0.595	3.90	O	X99	46.89	2.44	9.4	8.5	8.0	6.78
1652+398	XBL	0.034	4.10	O	X04a	45.3	1.10	9.21	8.3	8.1	6.57
1807+698	RBL	0.051	3.38	X	X04a	44.70	1.13	8.51	7.6	7.5	6.08
2005−489	XBL	0.071	3.84	X	X04a	45.20	1.20	9.03	8.1	8.0	6.59
2128−123	FSRQ	0.501	4.88	O	X91	46.76	1.46	9.61	9.10	8.98	7.58
2141+175	FSRQ	0.213	3.69	X	X91	46.5	2.24	8.74	8.24	7.8	6.60
2200+420	RBL	0.069	3.44	O	X04a	45.0	1.20	8.23	7.7	7.57	6.19
2251+158	FSRQ	0.859	3.84	O	X04a	47.27	2.98	9.17	8.25	8.0	6.65
2254+074	RBL	0.190	3.68	O	X04a	46.00	1.80	8.62	8.0	7.8	6.55
2344+514	XBL	0.044	4.00	X	Gi00	45.00	1.00	8.8	8.2	8.1	6.70

References for Table 1:

* O: optical, X: X-rays;

† X88: Xie et al. (1988); X90: Xie et al. (1990); Xie91: Xie et al. (1991); X99: Xie et al. (1999); X04a: Xie et al. (2004a); Gi00: Giommi et al. (2000);

♯ These data come from Woo & Urry (2002) except 1538+149, whose M_σ comes from Wu et al. (2002) but $\log L_{\rm bol}$ from Xie et al. (2004b).

According to the general theory of relativity, the stable interior radius is $3R_{\rm s}$, $R_{\rm s} = 2GM_{\rm s}/c^2$ being the Schwarzschild radius of the central black hole with mass $M_{\rm s}$, so that we have

$$M_{\rm s} = 1.36 \times 10^4 \Delta t_{\rm min} (M_\odot). \quad (3)$$

From this equation we derive the black hole mass estimates for the 21 blazars listed in column (11) of Table 1.

2.2 Black Hole Mass Estimates on the Basis of the Kerr Black Hole Theory

Assuming that the central super-massive black hole is a maximal Kerr black hole, and that the innermost edge of the disk does not necessarily coincide with the marginally stable orbit ($r = r_{\rm ms}$), but can be much closer to the hole; as close, in fact, as the marginally bound orbit ($r = r_{\rm mb}$). The varying region can now be smaller than the least stable circular orbit. This condition could be violated if fluctuations are produced, e.g., by some plasma instabilities in a small region in or above the accretion disk or in a jet, that momentarily release energies comparable to the overall energy. The frequency of circular orbit motion gives a good estimate of a minimal possible periodicity of flux generated at a given point on the surface of the disk (Abramowicz & Nobili 1982), i.e.,

$$\Delta t_{\rm min} = \tau \frac{r_{\rm G}}{c} = 0.98 \times 10^{-5} \tau \Big(\frac{M_{\rm k}}{M_\odot}\Big) \,{\rm s}, \tag{4}$$

where $M_{\rm k}$ denotes the mass of the central Kerr black hole, $r_{\rm G} = 2GM_{\rm k}/c^2$, τ is a dimensionless parameter depending on the location of the region that provides the time variation ($r_* = r/r_{\rm G}$) and on the Kerr parameter a, which characterizes the rotation of the black hole ($a = 0$ for zero rotation, $a = 1$ for the maximum possible rate): $\tau \equiv \pi(r_*^{3/2} + a)$. The locations of the horizon, marginally bound, and marginally stable orbits for the non-rotating black hole are given by $r_* = 2$, 4, and 6, respectively; for the black hole rotating at the maximum speed, all three locations are given by $r_* = 1$. Therefore, the parameter τ must obey $\tau > \tau_{\rm min} = 2\pi$. If $\tau_{\rm max} < 2\pi$, observations exclude the possibility of the black hole model (Abramowicz & Nobili 1982). Adopting $\tau = \tau_{\rm min} = 2\pi$ and without considering relativistic beaming, one can deduce an upper limit to $M_{\rm k}$ from Equation (4) as

$$M_{\rm k} = 1.62 \times 10^4 \Delta t_{\rm min} (M_\odot), \tag{5}$$

where $\Delta t_{\rm min}$ is the minimal timescale of AGN variability. If we consider the relativistic beaming effect, based on the relation $\Delta t_{\rm min}({\rm in}) = [\delta/(1+z)]\Delta t_{\rm min}({\rm ob})$, Equation (5) then becomes (Xie et al. 2002)

$$M_{\rm k} = 1.62 \times 10^4 \frac{\delta}{1+z} \Delta t_{\rm min} (M_\odot), \tag{6}$$

where $\Delta t_{\rm min}$ is the observed minimal timescale of AGN variability, δ the Doppler factor and z the redshift. According to the argument presented in Xie et al. (1991a, 2001a, 2001b, 2003 and 2004a), the Doppler factor, δ, should be given by

$$\delta \geq \Big(\frac{5.0 \times 10^{-43} \Delta L_{\rm ob}}{0.057 \times \Delta t_{\rm min,ob}}\Big)^{\frac{1}{4+\alpha}}, \tag{7}$$

where α is the spectral index. The results of Equation (6) for our sample are listed in column (10) of Table 1.

2.3 Black Hole Mass Estimates using the Accretion Disk Theory

The variability discussed here could be directly related to shock processes in a jet. If we take the variability timescale as a measure of the size, R, of the emission region, then R in the jet obeys the inequality,

$$R \leq c \Delta t_{\rm min} \frac{\delta}{1+z} \,{\rm cm}, \tag{8}$$

where $\Delta t_{\rm min}$, in units of second, is the doubling time scale (Fan et al. 1999).

On the basis of the observations of the same kind of typical events of both optical (Xie et al. 1998) and TeV γ-rays bursts (Gaidos et al. 1996) we have found in our previous paper (Xie et al. 1998), that $\sim 200R_{\rm s}$ ($R_{\rm s}$ being the Schwarzschild radius of the central black hole) is perhaps an important critical point, which had been expected based on the standard thin accretion disk theory by Sunyaev (1975). Because when $R < 200R_{\rm s}$, the electrons in the accretion flow become ultra-relativistic. On the other hand, the mixture of relativistic electrons and non-relativistic protons has an adiabatic index $\gamma < 5/3$, with which the transition to supersonic accretion regime is possible in the region of $R < 200R_{\rm s}$ (Sunyaev 1975). As can be seen, both observation and theory of accretion disk have shown that the $200R_{\rm s}$ is an important critical point. Based on

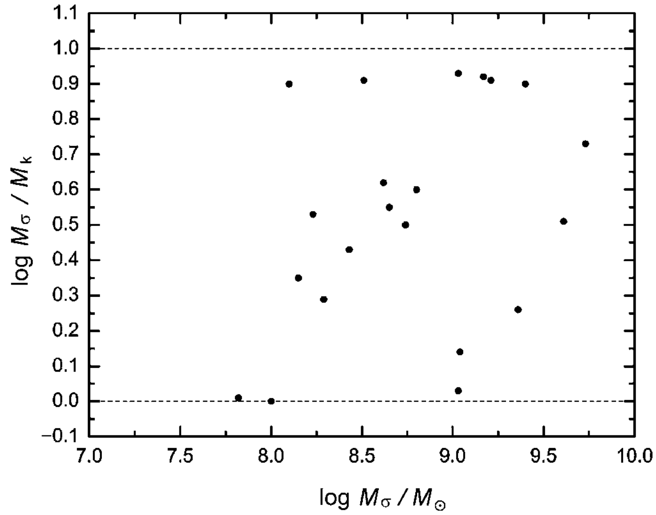

Fig. 1 A comparison between M_σ and M_k. Equality of the two is represented by the lower dashed line.

the assumption that $200R_\mathrm{s}$ is the critical point of the accretion disk, Fan et al. (1999) obtained the black hole mass

$$M_\mathrm{d} = 5 \times 10^2 \frac{\delta}{1+z} \Delta t_\mathrm{min} (M_\odot). \tag{9}$$

The results for the 21 blazars are listed in column (12) of Table 1.

3 COMPARISON OF M_σ WITH M_k, M_s AND M_d

As addressed above there are three ways of estimating the black hole mass from the minimum variability timescale, based on three different theoretical models. It can be easily seen from Equations (3), (6), and (9), or more specifically from columns (10), (11), and (12) of Table 1, that there are rather large differences. In order to determine which one is the most reliable, we shall compare each one with the mass estimates from the BLR size-luminosity relation or stellar velocity dispersion of Woo & Urry (2002), given in column (9) and taken to represent the true values.

3.1 Comparison of the Black Hole Mass Estimates M_σ and M_k

There are two completely independent ways of estimating the black hole mass, one based on the BLR size-luminosity relation or stellar velocity dispersion of Woo & Urry (2002, hereafter the Method-WU), and the other from the minimum variability timescale and the Kerr black hole theory by Xie et al. (2002). A comparison between the two is made in Figure 1, in which equality is represented by the lower dashed line.

We can see that they agree relatively well. The largest difference between them is less than one order of magnitude. The mean of the differences between them is very small; the mean difference in logarithm is

$$\left\langle \log \frac{M_\sigma}{M_\mathrm{k}} \right\rangle \approx 0.53. \tag{10}$$

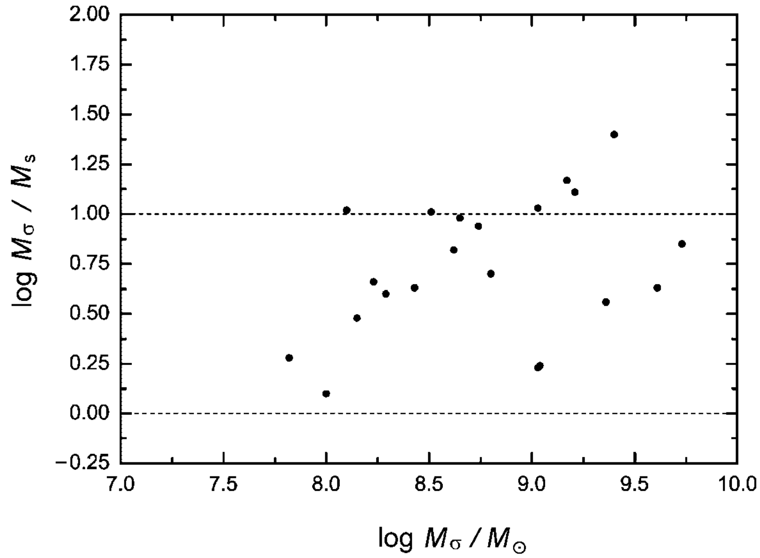

Fig. 2 A comparison between M_σ and $M_{\rm s}$. Equality of the two is expressed by the lower dashed line, difference of one order of magnitude, by the upper dashed line.

Furthermore, the standard deviation of the logarithm of the M_σ to the $M_{\rm k}$ ratio is only about 0.31

$$\sigma_{\log \frac{M_\sigma}{M_{\rm k}}} = \sqrt{\frac{\sum \left(\log \frac{M_\sigma}{M_{\rm k}} - \left\langle \log \frac{M_\sigma}{M_{\rm k}} \right\rangle \right)^2}{N}} \approx 0.31. \qquad (11)$$

Obviously the estimates $M_{\rm k}$ compare well with the estimates M_σ.

3.2 Comparison of Black Hole Mass Estimates between M_σ and M_s

The black hole mass estimate from the Method-WU is also independent of that from the minimum variability timescale based on the Schwarzschild black hole theory by Miller et al. (1989) and Xie et al. (1987). The results of comparing the two are shown in Figure 2.

Figure 2 shows, for 6 of the 21 sources, the difference is more than one order of magnitude. The mean difference in logarithm is as much as

$$\left\langle \log \frac{M_\sigma}{M_{\rm s}} \right\rangle \approx 0.74. \qquad (12)$$

The differences in this case are much larger than in the previous case. However, the standard deviation of the logarithm of the M_σ to $M_{\rm s}$ ratio is nearly the same as was found in Section 3.1,

$$\sigma_{\log \frac{M_\sigma}{M_{\rm s}}} = \sqrt{\frac{\sum \left(\log \frac{M_\sigma}{M_{\rm s}} - \left\langle \log \frac{M_\sigma}{M_{\rm s}} \right\rangle \right)^2}{N}} \approx 0.34. \qquad (13)$$

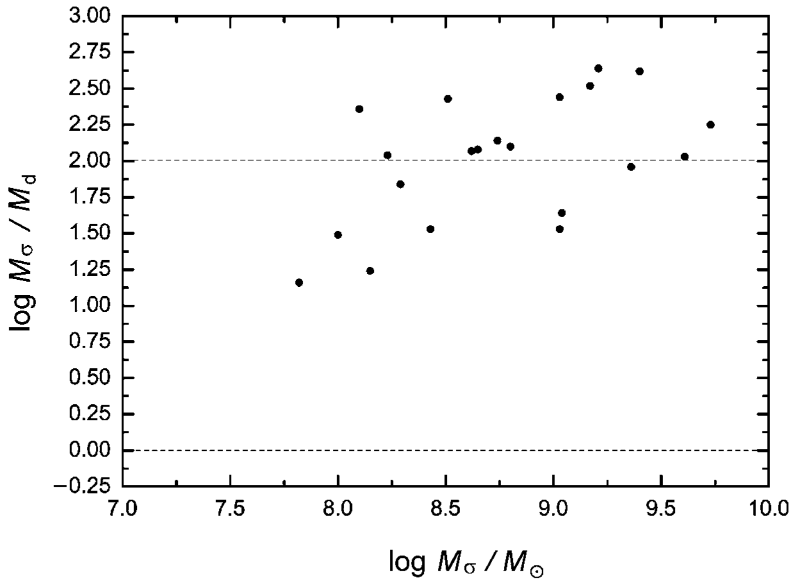

Fig. 3 A comparison between M_σ and $M_{\rm d}$. The lower dashed line represents equality, the upper dashed line, a two orders of magnitude difference.

3.3 Comparison of Black Hole Mass Estimates between M_σ and $M_{\rm d}$

Figure 3 shows the comparison between another two completely independent estimates of black hole mass, one from the Method-WU, and the other from minimum variability timescale in the accretion disk theory (Fan et al. 1999).

Figure 3 shows that even the smallest difference is more than one order of magnitude, and for 13 of the 21 sources, the differences are more than two orders of magnitude. The mean difference is as much as

$$\left\langle \log \frac{M_\sigma}{M_{\rm d}} \right\rangle \approx 2.01, \quad (14)$$

and is the largest of the three cases. The standard deviation of the logarithm of the M_σ to the $M_{\rm d}$ ratio is

$$\sigma_{\log \frac{M_\sigma}{M_{\rm d}}} = \sqrt{\frac{\sum \left(\log \frac{M_\sigma}{M_{\rm d}} - \left\langle \log \frac{M_\sigma}{M_{\rm d}} \right\rangle \right)^2}{N}} \approx 0.43. \quad (15)$$

It is very small and is almost the same as that we found in Sections 3.1 and 3.2.

4 SUMMARY AND CONCLUSIONS

We estimated the black hole masses of 21 AGNs from the minimum variability timescale on the basis of three theoretical models. For these sources, rapid optical variabilities have been reliably established and their black hole masses have also been well (and independently) estimated by the stellar velocity dispersion and the BLR size-luminosity relation (Woo & Urry 2002). Direct comparisons in Figures 1, 2, and 3 show that : (1) The minimum variability timescale and the Kerr black hole theory give reliable black hole mass estimates, the differences between M_σ and

M_k are very small for most of the sources, the mean differences is only about 0.53. (2) Using the minimum variability timescale and the Schwarzschild black hole theory leads to somewhat larger differences between M_σ and M_s, for 6 of the 21 sources, the differences are larger than one order of magnitude and there is a somewhat larger difference of 0.74 between the $\log M_\sigma$ and $\log M_s$. (3) The method of the minimum variability timescale and the accretion disk theory leads to much larger differences between M_σ and M_d. The differences are more than one order of magnitude for all of the 21 sources and more than two orders of magnitude for 13 of them. The mean difference in logarithm is as much as 2.01.

Our results seem to show that the method of black hole mass estimates through the minimum variability and the Kerr black hole theory is reliable. Thus, it could be expected that most of black holes of AGNs are probably Kerr black holes. It is obviously of interest to apply this method to a larger sample of AGNs at the highest known redshifts.

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