# Fallback Disk-Involved Spin-Down of Young Radio Pulsars \*

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**Abstract** Disks originating from supernova fallback have been suggested to surround young neutron stars. Interaction between the disk and the magnetic field of the neutron star may considerably influence the evolution of the star through the so called propeller effect. There are many controversies about the efficiency of the propeller mechanism proposed in the literature. We investigate the fallback disk-involved spin-down of young pulsars. By comparing the simulated and measured results of pulsar evolution, we present some possible constraints on the propeller torques exerted by the disks on neutron stars.

**Key words:** stars: magnetic fields — stars: neutron — pulsars: individual: PSRs B0532+21, B0833+45, B0540-69, B1509-58

# **1 INTRODUCTION**

Pulsars are considered to be spinning, magnetized neutron stars. Radiation and particles carry away energy which originates in the rotational kinetic energy of the star (Pacini 1967, 1968; Gunn & Ostriker 1969). The rotational power of the pulsar is generally assumed to decrease with time in a power-law,

$$I\dot{\Omega} = -K\Omega^n,\tag{1}$$

where I is the moment of inertia,  $\Omega$  the angular velocity, and n the braking index of the neutron star. In the standard magnetic dipole radiation (MDR) model (Gold 1968; Pacini 1968), n = 3. However, the values of n measured so far are all less than 3 (Lyne et al. 1988; Kaspi et al. 1994; Lyne et al. 1996; Livingstone et al. 2005). Various models have been put forward to reconcile this discrepancy, including (1) change of the moment of inertia I for rapidly spinning neutron stars (Glendenning et al. 1997; Chubrian et al. 2000), (2) wandering of the magnetic axis (Macy 1974), (3) non-dipolar field structure, (4) secular field strength evolution (Blandford, Applegate & Hernquist 1983; Blandford & Romani 1988), and (5) quadrupole gravitational radiation (Ostriker & Gunn 1969), etc.

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Recent X-ray observations show that some young pulsars, such as the Crab and Vela pulsars, may have the jet configuration, which suggests the existence of a disk surrounding the neutron star (Blackman & Perna 2004 and references therein). Theoretical investigations also indicate that some young pulsars could be surrounded by a supernova fallback disk (Michel & Dessler 1981, 1983; Michel 1988; Chatterjee, Hernquist & Narayan 2000; Yusifov et al. 1995; Alpar 2001). Along this line, Menou, Perna & Hernquist (2001a) have explored a disk model for the spin-down of young radio pulsars, in which the neutron star loses rotational energy not only by emitting MDR, but also by torquing a supernova fallback disk. The predicted age, braking index and the third frequency derivative for the Crab pulsar are in agreement with the reported values. Marsden et al. (2001) considered a similar model to explain the discrepancies between a pulsar's true age and its characteristic age.

In this paper we construct a model of a fallback disk assisted spin-down in young radio pulsars. We describe the model in Section 2, and apply it to four young pulsars in Section 3. In Section 4 we study the statistical properties of young pulsars with surrounding fallback disks by Monte Carlo simulation. A discussion and our conclusions are presented in Section 5.

#### 2 FALLBACK DISK MODEL FOR YOUNG PULSARS

We consider a young neutron star surrounded by a disk formed through fullback of part of the supernova ejecta. Menou et al. (2001a) assumed that the inner radius of the disk is always equal to the light cylindrical radius  $R_{\rm LC} = c/\Omega$  of the pulsar, where material is flung away due to the propeller effect. Since the stellar magnetic field lines become open at  $R_{\rm LC}$ , whether the magnetic field - disk interaction can cause a efficient spin-down torque is quite questionable. We instead take the inner disk radius to be at the magnetospheric radius, where the magnetic pressure is balanced by the ram pressure in the disk,

$$R_{\rm m} = \left(\frac{\mu^2}{\dot{M}\sqrt{2GM_*}}\right)^{2/7},\tag{2}$$

where  $\mu = B_* R_*^3$  is the magnetic moment,  $B_*$ ,  $R_*$ , and  $M_*$  are the magnetic field strength, radius, mass of the neutron star, respectively, and  $\dot{M}$  is the mass inflow rate within the disk. The magnetic field is assumed to be initially dipolar. However, the existence of the disk can reconstruct the magnetic field configuration. According to Sturrock & Smith (1968) and Roberts & Sturrock (1973), at the inner radius of the disk, the field lines changes from closed to open, so that  $B \propto r^{-3}$  for  $R_* \leq r \leq R_{\rm m}$ , and  $B \propto r^{-2}$  for  $R_{\rm m} \leq r \leq R_{\rm LC}$ . So the expression of the magnetic field strength at the light cylinder can be derived to be

$$B_{\rm LC} = B_* R_*^3 R_{\rm m}^{-1} R_{\rm LC}^{-2}.$$
 (3)

The magnetic spin down torque is then (Sturrock 1971)

$$T_{\rm mag} = \frac{1}{2} R_{\rm LC}^3 B_{\rm LC}^2 = \frac{1}{2} B_*^2 R_*^6 R_{\rm m}^{-2} R_{\rm LC}^{-1}.$$
 (4)

Throughout this paper we assume that the mass and radius of the neutron star are  $M_* = 1.4 M_{\odot}$ and  $R_* = 10$  km, respectively.

Since no mass accretion occurs in radio pulsars, the inner disk radius must be larger than the so called corotation radius, i. e.  $R_{\rm m} > R_{\rm co} \equiv (GM/\Omega^2)^{1/3}$ . The infalling material is stopped at the magnetosphere by the centrifugal barrier, which prevents material from accreting onto the neutron star. The ejected material may carry away the angular momentum of the neutron star due to the propeller effect (Illarionor & Sunyaev 1975), and decelerate its spin. There are many proposed propeller spin-down models, with large differences in the spin-down efficiency (Davies & Pringle 1980). The spin-down torque by the propeller action can be expressed in a general form as,

$$T_{\rm p} = -\dot{M}R_{\rm m}^2\Omega_{\rm K}(R_{\rm m}) \Big[\frac{\Omega}{\Omega_{\rm K}(R_{\rm m})}\Big]^{\gamma},\tag{5}$$

where  $\Omega_{\rm K}(R_{\rm m})$  is the Keplerian angular velocity at  $R_{\rm m}$ ,  $\gamma$  is a parameter reflecting the various mechanisms and efficiencies of the propeller effect with its value ranging from -1 to 2 (Mori & Ruderman 2003). When  $\gamma = 0$  and 1, the corotating material is assumed to be ejected at the escape velocity (Davidson & Ostriker 1979) or the rotating velocity of the magnetosphere (Shakura 1975) at  $R_{\rm m}$ , respectively. The resulting angular momentum loss can be accordingly represented by the propeller torques,  $T_{\rm p} = -\dot{M}R_{\rm m}v_{\rm esc}(R_{\rm m}) = -\dot{M}R_{\rm m}(2GM/R_{\rm m})^{1/2} \simeq -\dot{M}R_{\rm m}^2\Omega_{\rm K}(R_{\rm m})$ , and  $T_{\rm p} = -\dot{M}R_{\rm m}^2\Omega_{\rm L}$ . It is also possible to use energy considerations to determine the propeller torque. Assuming that the rotational energy of the neutron star is deposited in the gas so that it attains escape velocity, we have  $I\Omega\dot{\Omega} = (1/2)\dot{M}v_{\rm esc}^2(R_{\rm m}) = -\dot{M}R_{\rm m}^2\Omega_{\rm K}^2(R_{\rm m})$ , and  $T_{\rm p} = -MR_{\rm m}^2\Omega_{\rm K}^2(R_{\rm m})/\Omega$ . This is the  $\gamma = -1$  case (Illarionov & Sunyave 1975). The case of  $\gamma = 2$  corresponds to the subsonic propeller stage considered by Davies, Fabian & Pringle (1979), in which energy is assumed to be dissipated at a rate of  $-\dot{M}R_{\rm m}^2\Omega_{\rm K}^2(R_{\rm m})[\Omega/\Omega_{\rm K}(R_{\rm m})]^3$ , resulting in  $T_{\rm p} = -\dot{M}R_{\rm m}^2\Omega_{\rm K}(R_{\rm m})[\Omega/\Omega_{\rm K}(R_{\rm m})]^2$ .

Note that in Eq. (5)  $\dot{M}$  changes with time. Cannizzo et al. (1990) showed that after the supernova fallback, there is a transient spreading phase of timescale  $t_0$  during which the accretion is nearly constant,  $\dot{M}(t) \simeq \dot{M}(t_0)$ , after which  $\dot{M}$  declines in a power law with time,

$$\dot{M}(t) = \dot{M}(t_0)(t/t_0)^{-\alpha},$$
(6)

with  $t_0 \simeq 300$  s and  $\alpha = 1.25$  (Francischelli, Wijers & Brown 2002).

The evolution of the star's rotation is determined by the sum of the energy outflow and the propeller spin-down torque,

$$I\dot{\Omega} = T_{\rm mag} + T_{\rm p} = -\frac{1}{2}B_*^2 R_*^6 R_{\rm m}^{-2} R_{\rm LC}^{-1} - \dot{M}(t) R_{\rm m}^2 \Omega_K(R_{\rm m}) \Big[\frac{\Omega}{\Omega_{\rm K}(R_{\rm m})}\Big]^{\gamma}.$$
 (7)

The braking index n, the second braking index m, and the characteristic age  $T_{\rm Ch}$  are defined correspondingly as

$$n(t) \equiv \frac{\dot{\Omega}\Omega}{\dot{\Omega}^2},\tag{8}$$

$$m(t) \equiv \frac{\Omega \Omega^2}{\dot{\Omega}^3},\tag{9}$$

and

$$T_{\rm Ch} \equiv -\frac{\Omega}{2\dot{\Omega}}.$$
 (10)

In the following we use the measured values of these parameters for specific pulsars to constrain the possible range of the parameter  $\gamma$  in Eq. (5), as well as the magnetic field strength  $B_*$ , initial accretion  $\dot{M}(t_0)$  and the initial spin period  $P_i$  of the pulsar.

# **3 APPLICATION TO FOUR YOUNG PULSARS**

The braking indices have been measured for five young pulsars, the Crab pulsar (PSR B0531+21), PSR B1509-58, PSR B0540-69, the Vela pulsar (PSR B0833-45) and PSR J1119-6127. All of the five pulsars are associated with supernova remnants (SNRs), but we will only focus on the first four, for which the SNR ages can be estimated with some confidence. The physical parameters for these pulsars are summarized in Table 1 (see also Menou et al. 2001a;

Crab 1509 - 580540 - 69Vela P (ms)33.5150.950.389.3  $T_{\rm Ch}$  (yr) 12581691166411300  $1800^{b}$  $800^{b}$  $1.4\times10^{4\ \rm c}$  $T_{\rm SNR}$  (yr) 950 $1.4\pm0.2$  $2.509\pm0.005$  $2.839 \pm 0.003$  $2.01\pm0.02$ n $10.23\pm0.03$  $18.3 \pm 2.9$ m

Table 1 The measured parameters for four young pulsars <sup>a</sup>

<sup>a</sup> References: for PSR 1509-58 see Livingstone et al. (2005); for other pulsars, see Menou et al. (2001a), Alvarez & Carramiñana (2004) and references therein.

<sup>b</sup> Chevalier (2005).

<sup>c</sup> Wallerstein & Silk (1971).



**Fig. 1** Allowed range of  $\gamma$  as a function of ages in terms of  $T_{\text{SNR}}$  for four young pulsars (a: The Crab pulsar, b: The Vela pulsar, c: PSR 0540–69 and d: PSR 1509–58.  $T_{\text{SNR}}$  is the age of the SNR associated with the pulsar).

Alvarez & Carramiñana 2004). By solving Eqs. (7)–(10), we have searched the allowed range of  $B_*$  and  $\dot{M}(t_0)$  for  $\gamma = -1, 0, 1$  and 2 respectively, so that the calculated values of  $T_{\rm Ch}$ , n and m (if available) at present are to within 10% of the measured. For the Crab pulsar, only the  $\gamma = 1$  model works with  $B_* = 1.5 \times 10^{11} - 7.1 \times 10^{12}$  G and  $\dot{M}(t_0) = 3.5 \times 10^{29} - 1.5 \times 10^{25}$  g s<sup>-1</sup>. For PSR B1509–58 and the Vela pulsar, there is no  $\gamma$  model meeting the requirements. For PSR B0540–69 we obtained  $B_* = 1.4 \times 10^{11} - 1.6 \times 10^{11}$  G, and  $\dot{M}(t_0) = 6.9 \times 10^{20} - 1.1 \times 10^{29}$  g s<sup>-1</sup> only when  $\gamma = 2$ .

The  $\gamma = 1$  model seems to be preferred since the age estimation of the Crab Nebula is much more accurate than the others. Compared with PSR B0540-69, the measured second braking index m of the Crab pulsar also leads to tighter constraints on the parameters. However, the pulsar's spin-down mechanism in practice may be much more complicated than the present model implies. So we relax the calculated values of n, m, and  $T_{\rm Ch}$  to within 20%. We also let  $\gamma$  change continuously from -1 to 2. Figure 1 shows the allowed range of  $\gamma$  for the pulsars, plotted against the dimensionless age  $T/T_{\rm SNR}$ . The combined results for the Crab and Vela pulsars and PSR B1509-58 suggest that  $\gamma \lesssim 1$  is plausible, if their estimated SNR ages are reasonable. Especially for the Vela pulsar, the small value of  $n \simeq 1.4$  requires  $\gamma < 1.4$ .

#### 4 MONTE CARLO SIMULATIONS

To examine to what extent the fallback disks affect the evolution of young radio pulsars, we carried out Monte Carlo simulations of the evolution of  $2 \times 10^6$  neutron stars based on the spin-down model presented in Section 2. We adopted typical values for the initial parameters of newborn neutron stars (see Chatterjee, Hernquist & Hernquist 2000; Li 2002). The initial magnetic field  $B_*$  is chosen from a log normal distribution of mean 12.5 and standard deviation 0.3. We assumed that all pulsars were born with a surrounding supernova fallback disk, the initial masses of which  $\log(M_0 t_0/M_{\odot})$  were distributed uniformly between -6 and -2. We set the initial spin periods  $P_i$  to be distributed uniformly between 10 ms and 100 ms. We stopped the calculations at a fiducial time of  $10^4$  yr, since the fallback disk would become thermally and viscously unstable at the time of several  $10^4$  yr (Menou et al. 2001b). We counted the number of pulsars with  $R_{\rm co} < R_{\rm m} < R_{\rm LC}$  within  $10^3$  and  $10^4$  yr for various propeller models. For  $\gamma = -1, 0, 1$  and 2, the emerging proportion of disk-fed pulsars is 21.1%, 24.0%, 38.7% and 51.6% at age  $10^3$  years, and  $9.3\%,\,9.7\%,\,15.4\%$  and 25.3% at age  $10^4$  years. Obviously a disk is easier to survive for the propeller mechanisms with relatively large  $\gamma$ . The reason is that a more efficient spin-down can cause the neutron star's spin period to increase more rapidly, so that the condition  $R_{\rm LC} > R_{\rm m}$  can be satisfied for a longer time.

In Figure 2 we plot the histogram of the spin periods for disk-fed pulsars. The solid lines correspond to the age of  $10^3$  yr and the hatched regions for  $10^4$  yr. We note that the average value of log P increases with  $\gamma$ . In the case of  $\gamma = 2$ , most neutron stars have P > 1 s, in contrast



Fig. 2 Histograms of the periods for disk-fed pulsars in the propeller phase for various  $\gamma$  models. The solid lines correspond to the age of  $10^3$  years and hatched regions for  $10^4$  years.

with the periods of the four pulsars studied here. Moreover, the periods predicted are so large that the neutron stars may have expired or have very low radio emission because they may have already passed the death line for radio pulsars (e.g. Ruderman & Sutherland 1975). This further strengthens our speculation that, if the fallback disks exist around young pulsars, the propeller mechanism with  $\gamma = 2$  may not be a viable proposition. This is not unexpected, since the subsonic propeller torque actually works only when  $R_{\rm m} \leq R_{\rm co}$  (Davies & Pringle 1980), which may not be applicable to radio pulsars. The  $\gamma = 1$  model has the same problem, though not as severe as with  $\gamma = 2$ . For the other two models P is most likely distributed around 0.1 s, longer than the initial values by roughly one order of magnitude. Since  $10^3 - 10^4$  yr are much shorter than the current ages of most radio pulsars, these periods can be practically taken as the initial periods for pulsars driven by MDR. Recent pulsar analysis by Vranesevic et al. (2004) has shown that about 40% of the pulsars may be injected into the population with initial periods of 0.1 - 0.5 (see also Vivekanand & Narayan 1981). The fallback disk involved evolution may be one of the reasons to account for this fact.

The distributions of the calculated braking indices n are shown in Figure 3 at the ages of  $10^3$  and  $10^4$  yr. The solid, dashed, and dotted lines correspond to  $\gamma = -1$ , 0, and 1, respectively. In the  $\gamma = 1$  case the values of n are strongly clustered around  $\sim 1 - 2$ . However, for  $\gamma = -1$  and 0, the distributions are much wider, though in the latter small values of n seem to be more populated. Current measured values of n for five pulsars are all less than 3. If this trend is confirmed by future measurements of other young pulsars, models with  $\gamma \leq -1$  could be excluded.

Similar to Figure 3, Figure 4 shows the histogram of the ratio of  $T_{\rm Ch}$  and T at ages of  $T = 10^3$ and  $10^4$  years. It is well known that some SNR ages deviate from the characteristic ages of the pulsars inside, which can be (partly) attributed to disk-involved neutron star evolution (e.g., Marsden et al. 2001). From the figure we see that when  $\gamma = -1$ ,  $T_{\rm Ch}$  is most likely to be larger than T, while  $T_{\rm Ch}$  is generally less than T when  $\gamma = 1$ , with  $\gamma = 0$ , the situation is in between.



Fig. 3 Histograms of the braking indices n.



**Fig. 4** Histograms of the ratio of  $T_{\rm Ch}$  and the true ages T.

# 5 DISCUSSION AND CONCLUSIONS

A new disk-involved spin-down model for young pulsars is presented in this paper. The influence of the fallback disks on the spin evolution of pulsars shows up in two aspects. First, it causes the magnetic fields of the neutron stars to redistribute and deviate from the pure magnetic dipolar form. Secondly, it carries away the rotational energy of the neutron stars with the operation of the propeller effect. The motivation of our work is also two-fold. First, we try to examine whether the disk-involved pulsar evolution can reproduce the observed timing characteristics of young radio pulsars. Secondly, we may derive the overall features of pulsar populations if the fallback disks indeed exist. Although there have been many investigations on the propeller effect, the spin-down torque by a disk on rapidly rotating neutron stars is still unknown and there are many controversies in the literature. We hope that some observed quantities of radio pulsars may provide valuable information on the propeller efficiency. The available data of the four young pulsars and our Monte Carlo simulations show that the models with  $\gamma \leq -1$  and  $\gamma \geq 2$  are probably not viable, implying that the propeller effect may operate through exchange of angular momentum rather than energy between the magnetized neutron star and accreting gas (see however, Wang & Robertson 1985). More timing measurements of young pulsars are required before a firm conclusion can be reached.

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