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Estimating the Size and Timing of the Maximum Amplitude of Solar Cycle 24

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Abstract A simple statistical method is used to estimate the size and timing of maximum amplitude of the next solar cycle (cycle 24). Presuming cycle 23 to be a short cycle (as is more likely), the minimum of cycle 24 should occur about December 2006 (±2 months) and the maximum, around March 2011 (±9 months), and the amplitude is 189.9 ± 15.5, if it is a fast riser, or about 136, if it is a slow riser. If we presume cycle 23 to be a long cycle (as is less likely), the minimum of cycle 24 should occur about June 2008 (±2 months) and the maximum, about February 2013 (±8 months) and the maximum will be about 137 or 80, according as the cycle is a fast riser or a slow riser.

Key words: Sun: activity — Sun: sunspot — Sun: prediction

1 INTRODUCTION

Long-term sunspot activity is usually expressed in Wolf relative sunspot numbers (or simply, sunspot numbers). Study of variations in sunspot numbers is not only important for understanding the evolutional physical process in the atmosphere of the Sun, but also important for knowing the so-called "space weather", because they provide information or clues to variations of the solar-terrestrial environment (Hong 1990; Javaraiah 2003; Li et al. 2002b). A vast amount of research, which is mainly based on the analyses of the historical data of sunspot observations, has been carried out to understand the underlying mechanism of the 11-year sunspot cycles and then further to predict the cycles well in advance, including the size and timing of the maximum amplitude (Hathaway, WIlson & Reichmann 1999, 2001; Kane 2001; Li et al. 2002b).

Previously, Brown (1984) has described various methods and predictions as applied to cycle 21. Similarly, Kunches (1993) and Li et al. (2001) showed methods and predictions as applied to cycle 22, Kane (2001) and Li et al. (2001) presented methods and predictions as applied to cycle 23. It is apparent that successfully predicting the long-term solar activity for an unfolding cycle remains a very challenging task (Wang et al. 2002a, 2002b). In this paper, we propose

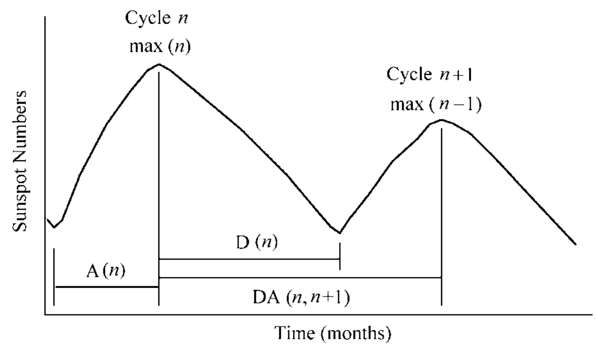

Fig. 1 Illustrating the definitions of $A(n)$, $D(n)$, $DA(n, n+1)$, Max(n) and Max($n+1$).

a simple statistical technique that allows for the prediction of the characteristic parameters of solar activity for cycle 24, including the timing of its minimum and maximum, and the size of its maximum amplitude.

2 PREDICTION OF SOLAR ACTIVITY OF THE FORTHCOMING SOLAR CYCLE 24

Here, some parameters of the time-amplitude profile of sunspot number cycles are introduced and illustrated in Figure 1. In the figure we denote the duration (in months) of the ascending phase (the time from the minimum to the maximum of the given cycle) of cycle n by $A(n)$, the duration of the descending phase (interval from the maximum to the next minimum) by $D(n)$, and time between the successive maxima of solar cycles n and $n+1$ by $DA(n, n+1)$. The cycle length of cycle n, the time from the minimum of the given cycle to the next minimum, denoted by $L(n)$, is $A(n) + D(n)$. The maximum amplitude of the monthly mean sunspot numbers of cycle n is denoted by Max(n), and similarly, Max($n+1$).

For the modern era sunspot cycles (cycle 10 to present), for which we have complete records of the daily sunspot relative number, there is a noticeable clustering of the individual cycles by period into two groupings (Wilson 1987; Wilson, Hathaway & Reichmann 1996): a short-period cycle group, averaging about 123 months in length and ranging from 121 to 126 months, and a long-period cycle group, averaging about 138.3 months in length and ranging from 134 to 142 months. An 8-month gap is found to separate the two groups (Wilson, Hathaway & Reichmann 1996), which is illustrated in Figure 2. It is the so-called "bimodality of the solar cycle". It must be pointed out that in the figure two points are marked for solar cycle 22. It is well-known that May 1996 marks the mathematical minimum of cycle 23, and conventionally the beginning time of solar cycle 23 is regarded as May 1996. From the perspective of curve fitting, however, the minimum should probably be regarded as occurring slightly later in time. For example, October 1996 marks the consensus minimum determined by the international group of solar physicists in NASA, who used October 1996 as the minimum of cycle 23 in their prediction of solar activity charts (Wilson et al. 1998; Li et al. 2002b). Establishing the time of minimum between cycles 22 and 23 is complicated because there were two periods of low solar activity during 1996 (Harvey & White 1999). To resolve this problem, Harvey & White (1999) studied the time of minimum in terms of the historical basis for defining this fiducial point in the solar cycle using several measures of solar activity. They suggested that the minimum between cycles 22 and 23 occurred in *September 1996* and not *May 1996*. Here, in the figure, May 1996 and September 1996 are

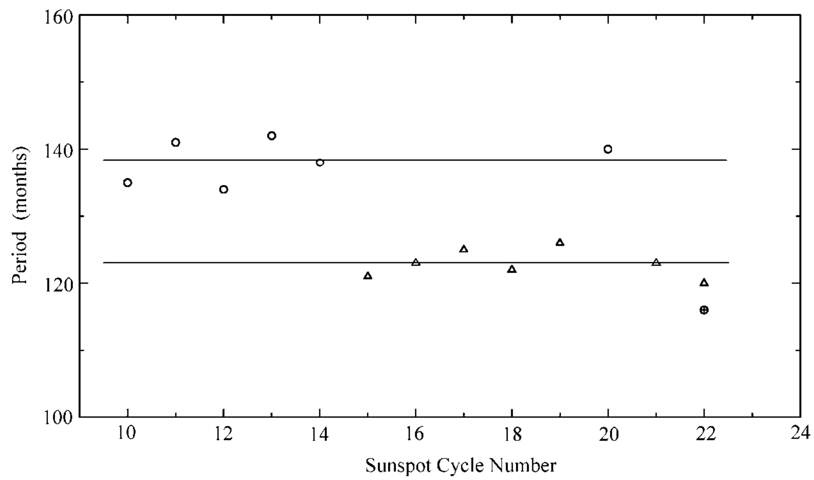

Fig. 2 Cycle length versus cycle number, spanning the modern era cycles. Cycle lengths appear to cluster near 138.2 and 123 months, suggesting a bimodal distribution of solar cycle lengths. Short cycles are marked by triangles, long cycles, by circles. For cycle 22, both May 1996 and September 1996 are used to mark the ending of the cycle.

both used to mark the ending time of cycle 22, and using September 1996 to mark the ending time of cycle 22 seems to match much better the bimodality of the solar cycle. So, in what follows we shall use September 1996 as the minimum of cycle 23 in our detailed prediction of solar activity in cycle 24.

Figure 3 shows the scatter plot of descent duration $D(n)$ versus ascent duration $A(n)$ for cycles 10 to 22. Obviously, one can observe that the descent duration is inversely related to the ascent duration, but only in terms of the inferred bimodal grouping (Wilson, Hathaway & Reichmann 1996). For short cycles, the relation may be clearly represented by a straight regression line $D(n) = 120.65 - 0.95A(n)$ with correlation coefficient –0.963, which is statistically significant at the 99% level of confidence, and the standard error of the regression line is 1.8 months. For long cycles, a statistically significant inverse correlation exists at 99.9% level of significance, $D(n) = 159.36 - 1.43A(n)$ (correlation coefficient –0.978, standard error of the regression line 1.7 months). If the beginning time of solar cycle 23 is taken to be May 1996, then the descent duration of cycle 22 should be smaller, the point representing cycle 22 should be further away from the regression line shown in the figure, and the regularity of the bimodality of the solar cycle will be weakened. September 1996 marking the ending time of cycle 22 seems much better to match the relationship shown in the figure. Thus, from this viewpoint, September 1996 also appears to be a better marker of the minimum of cycle 23. For cycle 23, $A(23) = 43$ months. According to the two regression lines, the descent duration of cycle 23 should be $D(23) = 79.8 \pm 1.8$ or 98.0 ± 1.7 months, according as cycle 23 is a short or a long cycle, and correspondingly, cycle 24 should start in December 2006 (± 1.8 months) or in June 2008 (± 1.7 months). Li et al. (2002a, 2002b) estimated the minimum time of cycle 24 being 2007.2 ± 1 year at the 68% level of confidence. MSFC Solar Physics Branch members, Wilson, Hathaway, and Reichmann, have studied the sunspot record for characteristic behavior that might help in predicting future sunspot activity, and they suggested that the start time of cycle

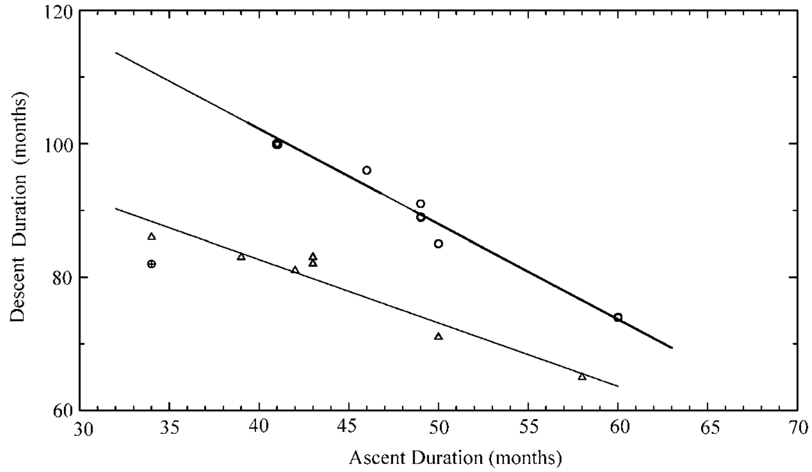

Fig. 3 Scatter plot of descent duration versus ascent duration for the modern era sunspot cycles (10–22). Short-period cycles are denoted by triangles and long-period cycles by circles, and their two regression lines are drawn to show the bimodal classification of solar cycles. The symbol ⊗ shows the case if May 1996 is taken to mark the ending time of cycle 22. The triangle over the symbol ⊗ shows the case if September 1996 is taken to be the ending time of cycle 22.

24 should be August 2006 (for details, see *http://science.nasa.gov/ssl/pad/solar/sunspots.htm*). So, it appears that cycle 23 should possibly be a short-period cycle.

A physical or statistical link between two successive cycles may possibly enable one to predict some parameters of the following cycle using the parameters of the preceding cycle (Thompson 1993; Wang et al. 2002a). Wang et al. (1986, 1992, 1993 and 2002a) found such a link, i.e., for a certain solar cycle and its similar cycles (similar cycles are that with similar characteristic parameters, such as the minimum and maximum values and their timing), $D(n)$ is strongly correlated with $DA(n, n+1)$. It is well known that for cycle 23 the minimum smoothed monthly sunspot number is about 8.8, the maximum smoothed monthly sunspot number is 120.8, and the ascent duration is 43 months. For a certain cycle, if its maximum smoothed monthly sunspot number is located in $120.8 \times (1 \pm 20\%)$, its ascent duration is in the range of $43 \times (1 \pm 20\%)$, and its minimum smoothed monthly sunspot number is not far away from 8.8, then the cycle is called a similar cycle of cycle 23. Based on these criteria cycles 2, 4, 11, 17, and 20 are found to be similar cycles of cycle 23. For these similar cycles of cycle 23, $D(n)$ is also strongly correlated with $DA(n, n+1)$, which is shown in Figure 4 and expressed the regression line, $DA(n, n+1) = -35.705 + 1.941D(n)$ (standard error 4.8 months), with a correlation coefficient $r = 0.991$, which is statistically significant at the 99% level. If cycle 23 is a short cycle, $D(23) = 79.8 \pm 1.8$ months, then according to the relation between $DA(n, n+1)$ and $D(n)$, $DA(23, 24) = 119.2 \pm 8.9$ months, thus, $A(24) = DA(23, 24) - D(23) = 39.4 \pm 11$ months, the maximum time of cycle 24 should be March 2011 (±8.9 months). If cycle 23 is a long cycle, $D(24) = 98.0 \pm 1.7$ months, then we have $DA(23, 24) = 154.4 \pm 8.1$ months, and hence, $A(24) = DA(23, 24) - D(23) = 56.4 \pm 9.8$ months, and the maximum time of cycle 24 should be February 2013 (±8.1 months).

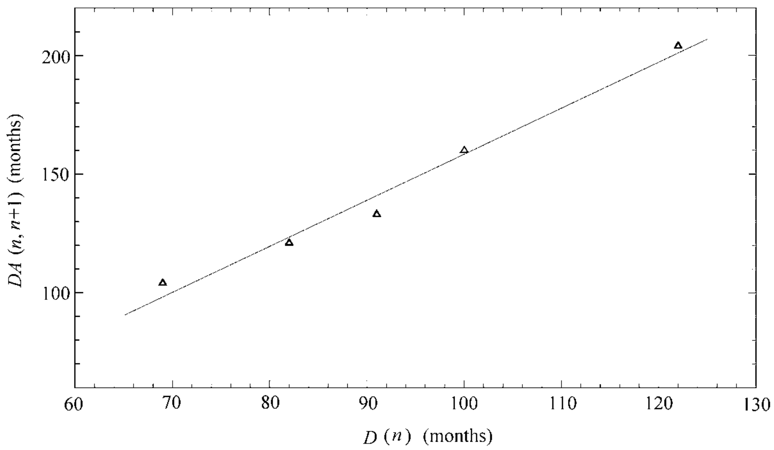

Fig. 4 Correlation between $D(n)$ and $DA(n, n+1)$ for similar cycles of cycle 23 when September 1996 is used as the minimum time of cycle 23.

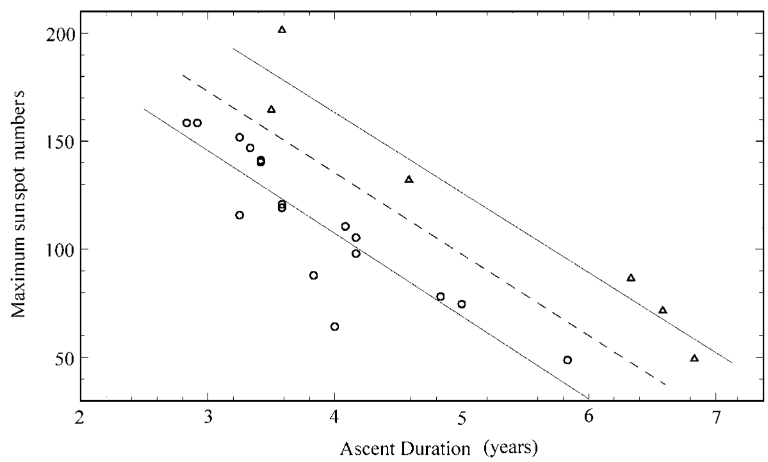

Fig. 5 Scatter plot of maximum smoothed monthly sunspot numbers versus ascent durations for solar cycles 1 to 23. The dashed line $\mathrm{Max}(n) = 286 - 37.66A(n)$ divides the cycles into two groups: the fast rising cycles (triangles), and the slow rising cycles (circles). The two starting times of cycle 23 are marked with different symbols.

Figure 5 shows the scatter plot of maximum smoothed monthly sunspot numbers versus ascent durations for solar cycles 1 to 23. It is found from the figure that solar cycles 1 to 23 may be divided into two groups by the line $\mathrm{Max}(n) = 286 - 37.66A(n)$: in one group, cycles have a fast ascent velocity i.e., the sunspot number reaches a value greater than $286-37.66A(n)$ for the ascent duration $A(n)$, and these cycles are called "fast rising cycles" (FRCs); similarly, cycles of the other group located below the line have lower ascent velocities, are called "slow rising cycles" (SRCs). All FRCs are odd number cycles (cycles 1, 5, 7, 9, 19, and 21). Maximum smoothed monthly sunspot number is inversely related to ascent duration in terms of the above grouping (Wang et al. 2002a). For FRCs, the relation is represented by the regression line $\mathrm{Max}(n) = 311.321 - 37.016A(n)$ (s.e.= 15.5, correlation coefficient −0.968, statistically significant at the 99% level). For SRCs, a highly significant inverse correlation is found to exist at the 99.9% level of significance: $\mathrm{Max}(n) = 260.557 - 38.305A(n)$ (correlation coefficient −0.885, s. e. 15.4). If cycle 23 is a short cycle with $A(24) = 39.4$ months, then we have $\mathrm{Max}(24) = 189.9 \pm 15.5$, if cycle 24 is an FRC, or $\mathrm{Max}(24) = 135.9 \pm 15.4$, if cycle 24 is an SRC. If cycle 23 is a long-period cycle with $A(24) = 56.4$ months, then we have, similarly $\mathrm{Max}(24) = 137.2 \pm 15.5$ or 80.3 ± 15.4 according as cycle 24 is an FRC or an SRC. May 1996 and September 1996 are both used to mark the starting time of cycle 23, and the latter seems to match much better the relationships shown in the figure.

3 CONCLUSIONS AND DISCUSSION

In this paper, a statistical method is proposed to estimate the size and timing of maximum amplitude for solar cycle 24. It is well known that the modern era sunspot cycles (cycle 10 to present) can be divided into two groups (Wilson 1987; Wilson, Hathaway & Reichmann 1996): a short cycle group, and a long-period cycle group, separated by an 8-month gap. This separation is known as the "bimodality of the solar cycle" (Wilson 1987). If September 1996 is used as the minimum time of cycle 23, and if cycle 23 is a short cycle, then cycle 24 should start in December 2006 (±1.7 months), and reach its maximum in March 2011 (±8.9 months). Also, the maximum amplitude of the cycle is 189.9 ± 15.5 or 135.9 ± 15.4 according as cycle 24 is an FRC or an SRC. However, if cycle 23 is a long cycle, cycle 24 should start in June 2008 (±1.8 months), reach maximum in February 2013 (±8.1 months); and the maximum amplitude of cycle 24 is 137.2 ± 15.5; or 80.3 ± 15.4, according as the cycle is an FRC or an SRC.

Based on the "bimodality of the solar cycle", September 1996 seems more suitable to mark the minimum of cycle 23. Consequently, if May 1996 is used to mark the minimum time of cycle 23, all relationships given in the present study will become weaker. According to the predictions given by MSFC Solar Physics Branch members Wilson, Hathaway, and Reichmann and Li et al. (2002a, 2002b), it appears that cycle 23 should possibly be a short cycle. In addition, the first spotless day during the decline phase of cycle 23 occurred in January 2004. Wilson (1995) has shown that the last several cycles all behaved quite similarly, indicating that minimum for cycle 24 should occur in less than 3 years, before 2007, presuming cycle 23's behavior mimics that of the last several cycles. Also, Hathaway et al. (2004) have found a significant predictive behavior linking cycle maximum two cycles in advance with the drift velocity of the meridian flow at cycle maximum, implying that cycle 24 will be larger than average size and cycle 23, and consequently will be of shorter length, since statistically shorter cycles tend to follow cycles of larger than average size. If this is true, then, not only the predicted beginning time of cycle 24 given here is in agreement with those results given by Li et al. (2002a, 2002b) and MSFC Solar Physics Branch members, but also the predicted maximum time of cycle 24 given here is in agreement with the predicted maximum time of cycle 24 given by Schatten (2002) and Sello (2003). Schatten (2002) predicted that peak amplitude for the monthly smoothed sunspot number in the next solar cycle 24 is near 96 ± 25, peaking in April 2011 (Sello 2003), just 1 month earlier than our prediction. Sello (2003) predicted that the peak amplitude for the monthly smoothed sunspot number in the next solar cycle 24 is 115 ± 21, peaking also in the year 2011. Duhau (2003) predicted that the maximum amplitude of cycle 24 is 87 ± 23.5. These predicted values of the maximum amplitude of cycle 24 suggested that cycle 24 is perhaps an SRC. This indicates that the maximum amplitude of 135.9 ± 15.4 for cycle 24 would be more plausible and of a higher probability than the other value, 189.9 ± 15.5. Time will tell.

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